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"TO SAIL AWAY IN FANCY"

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CARHART

PHYSICS

FOR

HIGH SCHOOL STUDENTS

BY

HENRY S. CARHART, LL.D.

PROFESSOR OF PHYSICS IN THE UNIVERSITY OF MICHIGAN

AND

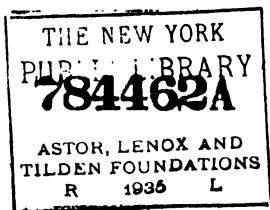
HORATIO N. CHUTE, M.S.

INSTRUCTOR IN PHYSICS IN THE ANN ARBOR HIGH SCHOOL

*NEW EDITION, THOROUGHLY REVISED AND
FURNISHED WITH NEW PROBLEMS*

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PREFACE.

THE present volume has been written with the same purpose that guided the authors in the preparation of their "Elements of Physics," a book which has long been received with marked favor. The advances in physics are so rapid, and the point of view from which many topics are now considered is so different from that of half a dozen years ago, that it seemed best to make an entirely new book rather than a revision of the former one. The order of the general subdivisions has been changed to that employed by one of the authors in his "University Physics," an order which has been approved by experience.

The authors hold firmly to the opinion that the class-room and the laboratory should be provided with separate books. The material which must be included in any text-book for the class-room makes by itself an ample volume; none of it can be omitted without serious sacrifice. The addition of quantitative experiments for laboratory practice must either overload the volume devoted primarily to the exposition of principles, or it will result in an incomplete and unsatisfactory treatment of the directions for the laboratory. A class-book and a hand-book for the laboratory should be prepared from different points of view. The former should be devoted to a clear exposition of principles by qualitative experiments and by precise didactic statements; the latter should contain somewhat minute directions for the quantitative experiments of the laboratory.

The authors' thanks are due to the publishers for their generous aid in furnishing so many excellent woodcuts to illustrate the book. These are not only faithful representations of actual apparatus, but they also possess artistic merit and make the book attractive.

PREFACE TO REVISED EDITION.

THE chief feature in this revision is the substitution of an entirely new set of problems in place of those of the former edition. Great care has been taken in their preparation, so as to secure variety, to arrange in graded series, and to reduce the arithmetical work of the solutions to the lowest limit. The number of problems is so great that for most classes the teacher will doubtless find it desirable to omit many of them. It is suggested that no more be assigned for solution than the average pupil can readily solve.

Important changes have been made in the chapter on the Mechanics of Solids with the view of lessening the difficulties for immature students. Physics is not an easy subject; and if by omitting the more difficult topics it is made so easy as to require no intellectual effort, the subject will have lost most of its value.

The authors hold to their former view that an elementary book should be restricted to settled facts and principles, avoiding both radical novelties in treatment and unconfirmed hypotheses which may properly find a place in advanced courses in physics.

H. S. C.

H. N. C.

ANN ARBOR, MICHIGAN,
May, 1907.

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HIGH SCHOOL PHYSICS.

INTRODUCTION.

WHEN we examine attentively the facts of nature about us, we study what are called physical as distinguished from mental phenomena. In this study it is necessary to assume the reality of this outer world of nature, and to assert that external objects exist apart from and independently of the mind of any one observing them. It is true that we become acquainted with the physical universe solely by means of our senses, but these alone do not enable us to decide whether the outer material world has a real existence or only the appearance of it. The final test of physical reality, or that the material world is as real as the mental one which gives us thoughts and feelings, is the fact that the physical world remains unchanged in quantity, or fixed in amount, however it may be measured. Tried by this test, there are only two *classes* of things in the physical world — *matter and energy*.

1. **Matter.** — It is only a colorless definition of matter to say that it occupies space. It is better described by its properties, to which the next chapter is devoted. Science is not yet able to tell what matter *is*, but the balance has demonstrated that it is invariable in amount, whatever form it may be made to assume.

A limited portion of matter is a *body*, and different kinds of matter, having distinct properties, are called *sub-*

stances. A gold coin, a drop of water, air enclosed in a vessel, are bodies. Gold, water, and air are substances, since each has properties distinct from the others.

2. Energy. — It is a fact of common observation that a body in motion can impart motion to another body, either by direct collision or otherwise. It is customary to say in such a case that the first body *does work* on the second. One body may also impart motion to another by virtue of its relative position. Thus, the weight of a clock when wound up gives motion to the pendulum and keeps it swinging against the resistance of the air and of friction. Whenever one body changes the motion or relative position of another, against a resistance opposing the change, the first body is said to do work on the second. Energy may be defined as the capacity for doing work. It is a grand doctrine of modern science that the energy of the physical universe is *conserved*, or is invariable in amount. This principle of the Conservation of Energy will become clearer when we have studied the various forms which energy may assume, and its conversion from one of these forms into another.

3. Physics. — In its most general aspect Physics may be defined as *the science of matter and energy*. It is desirable, however, to restrict this definition so as to exclude problems involving celestial bodies and their motions (Astronomy), discussions relating to the nature and interaction of different forms of matter (Chemistry), and the laws of matter and energy in living things (Biology). The distinction between Physics and Chemistry is a somewhat artificial one, and is becoming less clearly defined as science advances.

4. Physical Phenomena, Theory, Law. — Any change taking place in matter and not altering its composition is a *physical phenomenon*. The fall of a raindrop, the freezing of water, the melting of lead, a flash of lightning, the colors of the rainbow, the setting of the sun, the ringing of a bell, are all physical phenomena.

An *hypothesis* is a supposition advanced to explain phenomena. The more varied the phenomena it explains, the greater the probability of its truth. When the evidence in its support becomes large, it is raised to the rank of a *theory*; and, finally, it becomes a *law* when its truth is fully established.

Physical law expresses the constant connection between related phenomena. For example, it is found that if a quantity of gas, as air, be compressed at a constant temperature, its volume will decrease in the same ratio as the pressure is increased. This is the law of the compressibility of gases.

5. An Experiment consists in producing a phenomenon under conditions chosen and controlled by the experimenter. By making definite changes in the conditions one by one, the results may enable him, by a process of exclusion, to fix the conditions necessary to the phenomenon in question, and so to discover the law embracing it. The basis of all experimentation is the belief in the *constancy of nature*, as enunciated in the statement that "*under the same physical conditions the same physical results will always be produced, irrespective altogether of time and place.*"

CHAPTER I.

PROPERTIES OF MATTER.

6. **The Properties of Matter** are its peculiar qualities which serve to describe it and to define it provisionally. They are, in general, the different ways in which it presents itself to our senses. These properties are either *general*, that is, those common to all kinds of matter in whatever state or physical condition they may be; or *special*, those distinctive of some kinds of matter and conspicuously absent in others. Thus, all matter has *extension*, or occupies space; but a piece of window glass lets light pass through it, or is *transparent*; while a piece of slate does not transmit light, or is *opaque*. A watch spring recovers its shape after bending, or is *elastic*; while a strip of lead possesses this property in so slight a degree that it is classed as *inelastic*. Extension is a general property of matter, while transparency and elasticity are special properties.

7. **Extension.** — All matter occupies space or has dimensions. This general property is known as *extension*. The dimensions of a body are its length, breadth, and thickness, or all bodies occupy space of *three dimensions*. A sheet of writing paper or of gold-leaf appears, at first thought, to have only two dimensions, length and breadth; but its third dimension is relatively small, and if its thickness should actually become zero, it would cease to be a sheet of paper or a piece of gold-leaf.

8. Measurement of Extension. — The measurement of extension is made in terms of some unit of length, which is chosen arbitrarily. The units of area and of volume are the square and the cube respectively of the unit of length.

The two systems of units in common use are the *English* and the *Metric*. The former is generally used in Great Britain, Canada, and the United States, while the latter is almost exclusively used on the continent of Europe. The standard of length in the English system is the *Imperial Yard*. It was defined by Act of Parliament in 1855 as the distance between the transverse lines in two gold plugs in a certain bronze bar at 62° Fahrenheit deposited in the Office of the Exchequer. One-third of the yard is the *Foot*, and one thirty-sixth is the *Inch*.

In the Metric system the *Metre* is the standard unit of length. It is defined for the United States by the length of a certain platinum-iridium bar, at the temperature of melting ice, or 0° C. (Centigrade scale). This bar is preserved in the National Bureau of Standards.

The metre as a standard length is in no way superior to the yard. Its great advantage lies in the fact that its multiples and submultiples are all in the decimal system. It is therefore much more convenient to use than the yard, with its irrational subdivisions. Its universal adoption by civilized nations is only a question of time.

The metre (m.) is divided into 10 decimetres (dm.), the decimetre into 10 centimetres (cm.), and the centimetre into 10 millimetres (mm.). The only multiple of the metre of practical use is the kilometre (km.), equal to 1000 metres. It is the unit employed on the continent of Europe for such distances as we express in miles. One kilometre equals 0.6214 mile, or one mile equals 1.6093 kilometres.

The United States *Gallon* (231 cubic inches), and the

Litre (1000 cubic centimetres), are legal units of volume for liquid measure. Tables for the conversion of quantities from one system of units into the other will be found in the Appendix.

By Act of Congress in 1866, the use of the metric system of weights and measures became lawful throughout the United States, and the weights and measures in common use were defined in terms of the units of the metric system. By this same act the legal value of the metre in the United States is 39.37 inches, and the yard is now defined as being $\frac{3600}{3937}$ of a metre. When approximate values only are needed, the metre may be taken to be $39\frac{1}{2}$ inches, the centimetre as $\frac{2}{5}$ inch, the kilometre as $\frac{5}{8}$ mile, and the litre as $2\frac{1}{9}$ pints.

9. Mass. — *Mass* has often been defined as the quantity of matter in a body. It is in fact the *measure* of the body's inertia (§§ 13, 38), that is, of the resistance which a body offers to motion or change of motion. The mass of a body is independent of the kind of matter composing it. It is not weight (§ 51), because mass is independent of gravity. The mass of a ball of iron would be the same if the ball could be taken to the north pole, to the centre of the earth, or to the sun. But its weight at the earth's centre would be zero, and at the surface of the sun nearly twenty-eight times as great as at the earth's surface.

10. Measurement of Mass. — Two systems of measuring mass are in legal use in Great Britain and the United States. The standard units of mass are the *Avoirdupois Pound* for the British system, and the *Kilogramme* for the metric system. The *Ton* of 2000 pounds is the chief multiple of the pound in the United States; its submulti-

ples are the *Ounce* and the *Grain*. The *avoirdupois* pound is equal to 16 ounces, and to 7000 grains. The coinage of the United States is regulated by the "troy pound of the mint," containing 5760 grains.

For the United States the standard unit of mass in the metric system is the mass of a certain piece of platinum-iridium preserved in the National Bureau of Standards in Washington. This standard kilogramme and the standard metre in the same Bureau were prepared by an International Committee, and they are called "National Prototypes." The real prototypes are kept in the national archives in Paris. The kilogramme was originally designed to represent the mass of a cubic decimetre of pure water at 4° C., the temperature of the greatest density of water. The gramme is the thousandth part of a kilogramme, and is very nearly equivalent to the mass of a cubic centimetre of pure water at 4° C. The gramme (gm.) is divided into 10 decigrammes (dgm.), or into 1000 milligrammes (mgm.) One kgm. equals 2.2 lb. nearly.

11. Impenetrability.—Matter not only occupies space, but one portion of matter appears to occupy a portion of space to the entire exclusion of all other matter. This property of *impenetrability* means that no two portions of matter, however small, can occupy the same space at the same time. The volume of an irregular solid is sometimes measured by measuring the volume of liquid displaced when the solid is completely immersed in it. The method is based on this property of impenetrability. 29/5

12. Porosity.—Impenetrability does not preclude interpenetration of matter. Thus, a sponge or a piece of sandstone may absorb much water without change of volume.

All matter is probably spongelike or porous in structure, and the property corresponding to this structure is called *porosity*. These pores, whether visible or invisible, are in reality not a part of the space occupied by the material of the body. They may therefore be filled by some other material.

Experiment.—Into a long test-tube pour 27 cm³. (cubic centimetres) of water. Add carefully 23 cm³. of strong alcohol, tipping the test-tube so that the liquid flows down its walls. Mark the position of the surface of the alcohol, and then mix the two liquids thoroughly by shaking. The volume will shrink to 48.8 cm³.

The porosity of some metals is shown by the fact that gases pass through them. Palladium has the capacity of absorbing a large quantity of hydrogen, and carbon dioxide passes quite freely through red-hot cast iron. The salt glaze on earthenware is porous and is penetrated by some liquids. Agate, though extremely hard, is still porous; advantage is taken of this property to color it, some layers being more porous than others. Glass and other vitreous bodies are not known to be porous.

13. Inertia.—The most characteristic property exhibited by matter is *inertia*. In fact, inertia is the *only* property inherent in matter which has to do with motion. Inertia is the persistence of matter in whatever state of rest or of motion it may chance to be, and its resistance to any attempt to change that state. If a moving body be stopped, its arrest is always due to some influence outside of itself; and if a body at rest be set in motion, the motion must be imparted to it by some other body.

Many familiar phenomena are due to inertia: for example, the onward motion of a rider when his horse sud-

denly shies or stops; the oscillation of water in a pail when carried; the water flying from a revolving grindstone; the persistence with which the axis of rotation of a spinning top maintains its direction. The violent jar to a water pipe on suddenly closing the faucet is due to the inertia of the stream. Tall columns and detached chimneys are sometimes twisted around on their base by sudden gyratory earthquake movements. The sudden twist of the earth



Fig. 1.



Fig. 2.

under the column leaves it behind, and the slower return motion of the ground carries the column with it. Figure 1 is the picture of such a monument in India twisted on its base in this way by an earthquake.

Experiment.—Suspend a heavy weight (Fig. 2) by a string, and to the under side the weight attach a small bar *B* by a piece of the same string. If we pull *steadily* downward on *B*, the string will break above *A*. The tension in the upper string is greater than in the lower one because it has to support the weight *A* and resist the pull applied at *B*. If, however, we pull downward *suddenly* on *B*, the string will break below *A*. On account of the inertia of the heavy weight, the lower string breaks before the sudden pull reaches the upper string.

14. Elasticity.—Apply pressure to a rubber ball, stretch a common rubber band, bend a piece of watch spring, twist a stout cotton cord. In each case the form or the volume has been changed, and the body has been *strained*. A *strain* means either a change of size or a change of shape. As soon as the distorting force, or *stress*, has been withdrawn, these bodies recover their initial volume and dimensions. This property exhibited by matter of recovery from a strain on the removal of the stress is called *elasticity*. It is called *elasticity of form* when the body recovers its form on the removal of the force of distortion; and *elasticity of volume* when the temporary distortion is one of volume. Gases, vapors, and liquids possess perfect elasticity of volume; that is, they recover their initial volume when the initial pressure is restored; but they have no elasticity of form. In the case of solids the recovery from distortion is incomplete if the distortion is pushed beyond a limit which is different for different

substances. When permanent alteration is about to take place, the body has reached its *elastic limit*. Some solids, when subjected to long-continued forces, suffer elastic fatigue, yield slowly, and never recover their original form. Shoemaker's wax is an example.



* Fig. 3.

The elasticity of a body may be called forth by pressure, by stretching, by bending, or by twisting. The bounding ball and the common popgun are illustrations of the first; rubber bands are familiar examples of the second; bows and springs are instances of the third; the torsional

pendulum (Fig. 3) and the stretched spiral spring exemplify the fourth.¹

15. Hooke's Law. — The law of the distortion of elastic bodies was first announced by Hooke in 1676. It may be briefly expressed by saying that the stress of restitution is proportional to the strain; or, in other words, the force of restitution is proportional to the change of form. Illustrations will make the meaning of the law clearer. Clamp a flat steel bar by one end in a vise, the flat side horizontal. Load the free end with weights by means of a light scale pan, and observe the bending of the bar. The vertical deflection of the end of the bar should be noted with some convenient weight in the pan. Then double the weight and note the new deflection. It will be double the first one. The amount of the bending or distortion of the bar is proportional to the weight. Force is required to bend an inelastic bar like lead, but there is no elastic force of recovery of form when the bending force is removed.

Experiment. — Turn the weight of Fig. 3 through 15 degrees and release it, noting the period of the torsional vibration by observing the intervals in seconds between the successive passages of the pointer through the position of rest. Then turn the weight through 30 degrees and observe the period of vibration again. Repeat with a

¹ The *coefficient of elasticity of volume* is the quotient of the pressure applied by the compression produced by it. By pressure is meant the intensity of pressure, or the pressure on unit area; and by compression, the compression suffered by unit volume. Let p be the increase of pressure on unit area, and let the original volume V become $V - v$. Then $\frac{v}{V}$ is the compression for a unit of volume, and the coefficient of elasticity is $p \div \frac{v}{V} = \frac{pV}{v}$. A general definition of the coefficient of elasticity is the quotient of the stress called out in the body by the strain. A stress is a force and a strain is a distortion. Elasticity is measured by the ratio of the *internal stress* to the strain.

twist of 45 degrees, and so on. It will be found that the period of torsional vibration is the same whatever the twist of the weight and wire. The force tending to bring the weight back to its rest position is proportional to the angle of twist or to the distortion, and therefore the period is unaffected by the angular twist. With double the angle to swing through, there is double the force to cause the weight to swing in the same time.

Experiments of similar import may be made by stretching heavy rubber bands or strips. The amount of stretching will be proportional to the force applied.

16. Plasticity.—*Plasticity* is the inability of a body to recover from distortion produced by a stress. In so far as bodies are not *elastic* they are *plastic*, and elastic bodies are plastic beyond the limits where they cease to be perfectly elastic. Plastic bodies require force to change their shape, but they do not require the continued application of force to maintain the change. Elastic bodies spring back when the force of distortion is removed; plastic bodies do not. Bodies are classed as elastic if they have a large limit of elasticity, and plastic if their limit of elasticity is small. A bar of lead will vibrate when struck, showing that it is elastic; but its limit of elasticity is small, and it is therefore classed as a plastic body.

17. Cohesion.—All bodies are made up of very minute invisible particles called molecules. *Cohesion* is the attraction between the molecules of a body. It unites these molecules or particles together throughout the mass, whether the molecules be like or unlike. *Adhesion* is the molecular attraction uniting bodies by their adjacent surfaces. Cohesion holds together the molecules of a substance so as to form a larger mass than a molecule. It resists a force tending to break or crush a body. When two clean surfaces of white-hot wrought iron are brought

into intimate contact by hammering, they will cohere. If a clean glass rod be dipped into water and then withdrawn, a drop will adhere to it. Graphite is ground to a very fine powder and cleaned chemically by boiling with nitric acid and chlorate of potash. It is then washed, dried, and compressed in moulds by a powerful hydraulic press. The black powder is by this means converted into a solid slab which may be sawed into thin strips for lead pencils. The pressure causes the clean particles to cohere.

Experiment.—Suspend from one of the scale pans of a balance a perfectly clean glass disk by means of threads cemented at three points (Fig. 4). After counterpoising the disk, place below it a vessel of water and adjust the apparatus so that the disk touches the surface of the water when the beam of the balance is horizontal. Now add weights to the pan till the disk is separated from the water. An examination of the under surface of the glass plate will show that a film of water has been pulled away with the glass. The force of adhesion between the glass and the water is greater than the force of cohesion in the water.

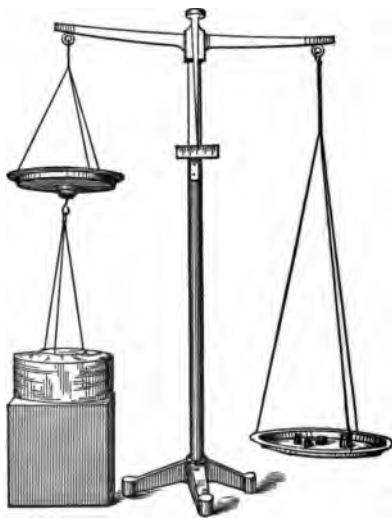


Fig 4.

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18. Molecular Attraction affected by Distance.—Little cubes of metal, with plane faces polished so smooth that they will adhere by slightly pressing them together, were made by Barton, and they are known as Barton's cubes.

A string of a dozen were made to cling together; they were supported by the mutual attraction between molecules of adjacent surfaces.

Experiment.—Press firmly together two small lead disks, giving to them a slight twisting motion. The surfaces in contact must be flat and bright. They will adhere quite firmly together. Two lead bullets, cut so as to present clean flat surfaces, may be used instead of the disks.

This experiment shows that molecular forces act only through insensible distances. They diminish rapidly as the distance between the molecules increases, and vanish at a range something like a twenty-thousandth of a millimetre.

19. Crystallization.—**Experiment.**—Dissolve 100 gm. of alum in a litre of hot water. Hang some strings in the solution and set aside in a quiet place for several hours. The strings will be found covered with beautiful transparent bodies of regular form and similar in shape.

Most bodies, when they pass slowly from the liquid to the solid state under conditions that allow freedom of motion to the molecules, assume regular geometrical forms called *crystals*. Such is the case when substances pass from the liquid state, either as a molten fluid or in solution, to that of a solid. In the former the crystallization is said to take place by the *dry way*; in the latter, by the *moist way*. Snowflakes are beautiful crystals of myriad forms, but all of them hexagonal in outline. Cast zinc when broken shows a crystalline fracture. Substances which exhibit no plan in the grouping of the molecules are said to be *amorphous*. Glass, glue, and paraffin are examples of amorphous bodies.

When crystalline bodies are broken apart, the fracture consists in separating crystals from the face of other

crystals. This separation is most easily effected along certain definite planes called *planes of cleavage*.

20. Tenacity. — *Tenacity* is the ability of a body to resist a force tending to tear it asunder. It is measured by the greatest force the body can bear per unit area of cross-section without breaking.

Wrought iron has greater tenacity than cast iron; the most tenacious metal is steel pianoforte wire. Lead has the least tenacity of all solid metals. Phosphor-bronze and aluminum-bronze have high tenacity, and they are therefore suitable for telegraph wires, where long spans are necessary.

21. Malleability. — *Malleability* is the property possessed by some substances of being beaten or rolled into sheets without breaking. Pure gold is more malleable than any other substance. By hammering between pieces of gold-beater's skin, it has been reduced to sheets so thin that 300,000 of them, placed one upon another, make but an inch in thickness. Platinum, silver, lead, and tin possess the same property, but to a less degree. Zinc is malleable when heated. It can then be hammered, rolled, or bent without fracture.

Articles of cast iron may be made somewhat malleable, or less brittle, by heating them for several days in the presence of a substance, like the black oxide of iron, which removes from them some of the carbon of the cast iron, and gives to them something of the properties of wrought iron.

22. Ductility. — *Ductility* is the property of a metal which permits of drawing it into wire through a draw-

plate. Gold, platinum, copper, and silver are highly ductile. Substances of great malleability are usually also highly ductile; lead is an exception.

Other substances become highly ductile when heated to a high temperature. Thus, glass has been spun into such fine threads that a mile would weigh only a third of a grain; and Professor Boys has made quartz fibres so fine, by attaching the white-hot quartz to an arrow shot from a bow, that they are invisible even under a microscope of high power. They are invisible because their diameter is less than a wave length of light (§ 294).

23. Hardness. — *Hardness* is the relative resistance which a body offers to scratching or abrasion by other bodies. The relative hardness of two bodies is ascertained by finding which of them will scratch the other. For example, glass is harder than copper because it scratches copper.

Most substances, if suddenly cooled from a high temperature, become very hard. A few, like copper and bronze, are softened by sudden cooling. The process of giving to a body a suitable degree of hardness is called *tempering*; that of making it as soft as possible at ordinary temperatures is called *annealing*. Both processes consist in raising the temperature of the body and then cooling, either suddenly or slowly, according to the result desired.



Fig. 5.

24. Absorption. — *Experiment.* — Fill a wide test-tube with dried ammonia gas by displacement over mercury. The gas may be obtained by boiling strong ammonia water in a flask and passing the gas through a tube filled with small lumps of unslacked lime.

Insert beneath the mouth of the test-tube a piece of freshly heated charcoal; the mercury will immediately begin to rise in the tube, because the charcoal absorbs the ammonia (Fig. 5).

Gases are condensed to a greater or less degree on the surface of all solids; and since porous bodies, like charcoal, present an immense surface in their pores, they may absorb large quantities of gas. One cm^3 . of boxwood charcoal will absorb 35 cm^3 . of carbonic acid gas and 90 cm^3 . of ammonia. It should be noted that gases which are most readily liquefied are, as a rule, absorbed by porous bodies most greedily. Finely divided platinum, in the form called platinum sponge, absorbs hydrogen from a small jet so rapidly that the heat generated by the condensation raises the sponge to a red heat and ignites the hydrogen. It will also ignite illuminating gas.

Air condensed on the surface of glass adheres to it with great persistence. In filling a barometer tube (§ 148), it is necessary to boil the mercury in order to expel the air. The last trace of air can be removed only by repeated boiling.

25. Diffusion. — Experiment. — Fill a large test-tube on foot two-thirds full of water, colored blue with litmus. Introduce a few drops of sulphuric acid into the liquid at the bottom of the tube by means of a thistle-tube (Fig. 6). A reddish color will appear at the bottom, and, if the liquids are not disturbed for several hours, this change of color will move slowly toward the top.

Experiment. — Fill two glass cylindrical jars with oxygen and hydrogen respectively, and cover each with a glass plate. Invert the jar of hydrogen and place it over the jar of oxygen. Remove the glass plates and let the jars stand for about 20 minutes. Then apply a lighted taper to each jar; an explosion will follow, showing that the gases have mixed.



Fig. 6.

In both of these experiments the heavier fluid moves up through the lighter, and the lighter down into the heavier. This intermingling of fluids in contact is called *diffusion*. Liquids diffuse very slowly, but gases more rapidly. Each gas expands and fills both vessels as if the other gas were not present. In time, therefore, both vessels contain a uniform mixture. The presence of a second gas only increases the time required for a uniform distribution.



Fig. 7.

26. Effusion. — **Experiment.** — Cement a small porous battery cup to a funnel tube. Connect the latter to a flask of water provided with a jet-tube, as shown in Fig. 7. Over the porous cup invert a large glass beaker, into which passes a stream of hydrogen through a glass tube. If all the joints are tight, water will issue from the jet-tube as a small fountain. The hydrogen passes freely through the walls of the porous cup and produces gas pressure in the flask.

When the pores of a solid are exceedingly fine, the passage of a gas through them is by a process called *effusion*. The rate of effusion of different gases is very nearly inversely proportional to the square root of their relative weights. Hydrogen, for example, which is one-sixteenth as heavy as oxygen, passes through very small openings nearly four times as fast as oxygen.

Gases may be separated from one another more or less completely by taking advantage of the different rates at which they pass, by effusion, through fine porous earthenware or unglazed porcelain.

27. Osmosis. — Experiment. — Remove the bottom from a small bottle, and tie over it a dampened piece of parchment paper. Through a stopper fitted to the mouth of the bottle, pass a glass tube terminating in a small funnel at its upper end. A scale of equal parts may be attached to the tube (Fig. 8). Pour into the tube a concentrated solution of copper sulphate. Support the apparatus so that the bottle dips into a jar of water just far enough to bring the two liquids to the same level. If the apparatus be left standing, we shall find that the liquid within the tube is slowly rising, while the water outside is acquiring a bluish tint. The two liquids pass through the membrane, but the greater flow is toward the denser liquid.

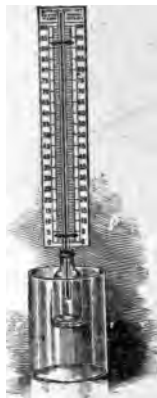


Fig. 8.

The passage of liquids at different rates through a porous membrane is called *osmosis*, and the pressure in the tube containing the salt solution is known as *osmotic pressure*. The osmotic pressures, due to solutions of different substances at several concentrations, have been measured by the use of a membrane permeable to water but not permeable to the dissolved substance. Such a membrane is called a semi-permeable membrane. The pressure required to maintain the liquid in the tube at the same level as the water outside the tube is the measure of the osmotic pressure exerted by the dissolved substance.

28. States of Matter. — The three states in which matter exists may be illustrated by water, which may assume either the solid, the liquid, or the gaseous state.

Ice is a solid. A *solid* has a definite shape, and resists any stress tending to change its shape or size.

Water is a liquid. A *liquid* has no shape of its own, but is mobile and conforms to the shape of the containing

vessel. It offers but slight resistance to a stress producing relative movement of its parts, so long as the volume remains unchanged.

Steam or water vapor is a gas. A *gas* offers no resistance to stress tending to change its shape. It has neither shape nor size of its own, but completely fills any vessel containing it.

In brief: —

Solids have a definite mass and both size and shape.

Liquids have a definite mass and size, but not shape.

Gases have a definite mass, but neither size nor shape.

Problems.

The student may solve the numerical exercises either by the use of the relations given in § 8 and § 10 and the tables of Arithmetic, or by means of the Conversion Tables in the Appendix.

1. Why is a person who leaps to the ground from a rapidly moving carriage likely to fall in the direction in which the carriage is going?

2. Why can the head of a hammer be driven on the handle better by striking the end of the handle against a large stone than by striking the head against the stone?

3. A man standing on a moving flat-bottomed car jumps vertically upward. Where does he come down? Explain.

4. Imperfect shot are separated from the perfect ones by letting them roll down an inclined plane, across which near the bottom there is a narrow opening. Into this opening the defective shot fall, while the round ones jump over it. Explain.

5. Carbon dioxide is heavier than air. Why does it not collect at the floor of inhabited rooms?

6. It is said that powdered charcoal sifted thickly on semi-decomposed organic matter will in a short time remove the offensive smell. Explain.

7. How many metres wide is a 4-rod street?

8. If the average barometer reading at sea-level is 30 in., what will it be on the centimetre scale?

9. Express in kilometres per hour a speed of 40 mi. per hour.

10. Calculate the number of cubic centimetres in a pint. [How would you find the number of cm^3 in a cubic inch?]

11. Calculate the number of litres in a gallon.

12. Express the difference in cubic inches between a litre and a quart.

13. Calculate the number of pounds avoirdupois in a kilogramme.

14. If an article sells for 50 ct. per pound, at what price should it sell per kilogramme?

15. How many pounds in 100 kgm.?

16. How many litres will a 10-qt. pail hold?

17. What price per metre is equivalent to 10 ct. per yard?

18. At 20 ct. per gallon what will a litre cost?

19. What will be the loss or gain in selling at 40 ct. per kilogramme an article that cost 20 ct. per pound?

20. If a cubic centimetre of water weighs one gramme, what will be the weight in kilogrammes of a cubic foot of water?

21. What error is made in considering a kilometre as equivalent to five-eighths of a mile?

CHAPTER II.

MECHANICS OF SOLIDS.

I. MOTION AND VELOCITY.

29. Mechanics.—The term *Mechanics* is applied to that portion of Physics which deals with the effects of force on matter. It includes the principles applied in the construction of machines.

By *force* we may understand muscular exertion, or whatever else produces the same effects. The making of a muscular effort to overcome resistance gives us our primitive idea of force. Whenever any inanimate agency produces effects precisely similar to those due to muscular exertion, it is said to exert force. Thus, a steam engine exerts force in pulling a train, turning a dynamo, or driving a mill; air compressed in a rubber balloon exerts force against the resistance of the stretched rubber; the gases due to the explosion of gunpowder exert force on the ball while it is passing from the breech to the muzzle of a rifle.

The most obvious effects of force on matter are (1) to produce change of motion, and (2) change of size or shape.

30. Motion.—A body moves when it is in different positions at different times. *Motion* is the change in the relative position of a body with respect to some point or place of reference. It involves, therefore, both time and space. The path of a moving body must be continuous,

that is, the body must pass in succession through every point of its path.

All rest and motion are relative, since there are no fixed points in space to which absolute motion may be referred. When a passenger walks on the deck of a ship, his motion is relative to the vessel; the motion of the ship across the ocean is relative to the earth's surface; the diurnal motion of the earth's surface is relative to its axis of revolution; while the motion of the earth's centre is relative to the sun.

When a body moves along a straight line, its motion is said to be *rectilinear*; when it moves along a curved line, its motion is *curvilinear*. In the latter case, its direction of motion at any point of its path is that of the straight line drawn tangent to the curve at the point.

- ✓ 31. **Velocity.** — When a body moves over equal spaces in successive equal times, its motion is said to be *uniform*; if it traverses unequal spaces in successive equal times, its motion is *variable*. For example, the tip of the minute hand of a watch has a uniform motion, though the direction of its motion is constantly changing. So also the apparent motion of a star across the field of a fixed telescope is an instance of uniform motion. On the other hand, the motion of a falling body is variable, for it moves faster and faster as it descends.

Velocity is the time-rate of motion of a body. By the time-rate of any change is meant the whole change taking place in a given time divided by that time. If the motion is *uniform*, the velocity is *constant*. Constant velocity is measured by the distance a body travels in a unit of time. The velocity of a railway train may be constant for a considerable distance. It may, for example, move 88 ft. for each second of an entire minute (equal to a mile a minute).

When the motion is variable, the velocity at any instant is the distance the body would move in the next unit of time if at that instant its motion were to become uniform.

For example: the velocity of a shell on leaving the mouth of a gun, called the muzzle velocity, is the space it would pass over in the next second if it should continue to move undisturbed at the same rate; the velocity of a falling stone at any moment is the distance it would fall during the following second, if the attraction of the earth and the resistance of the air could both be withdrawn.

32. Formulæ for Uniform Motion. — Let v be the constant velocity of a body with uniform motion. Then in t units of time the space s passed over will be t times the velocity v , or

$$s = vt. \quad (1)$$

From the same relation we have $v = \frac{s}{t}$ and $t = \frac{s}{v}$.

Even though the motion were not uniform, if s be the space passed over in time t , then the *mean* or *average velocity* for the whole time would be $v = \frac{s}{t}$. If both the space and the time be reduced to indefinitely small quantities, then this mean velocity becomes the actual velocity at the instant.

33. Acceleration. — *Acceleration is the time-rate of change of velocity.* If the change of velocity be the same from second to second, the motion is *uniformly accelerated*. If the velocity increases, the acceleration is positive; if it decreases, the acceleration is negative. A falling body has a positive acceleration; the acceleration of a body thrown upward is negative. When a heavy body falls, its gain in *velocity per second* is 9.8 m. *for every second*

of time. Its acceleration is, therefore, 9.8 m. per second per second; in other words, an increase in velocity of 9.8 m. per second is acquired in a second of time. This is equivalent to an increase of velocity of 588 m. *per minute* acquired in a *second* of time.

If a railway train starts from rest and increases its speed one foot a second for a whole minute, its velocity at the end of the minute is 60 feet a second. Since it acquires in one second a velocity of one foot a second, and in one minute a velocity of 60 feet a second, its acceleration is either one foot per second per second, or 60 feet per second per minute.

34. Formulæ for Uniformly Accelerated Motion. — Let a be the acceleration, or the gain in velocity per second acquired in a second of time. (Unless otherwise stated, the unit of time used will be the second of common or mean solar time, of which there are 86,400 in a mean solar day.) Then in t seconds the velocity acquired will be

$$v = at. \quad (2)$$

Since the gain in velocity is uniform, if the body starts from rest, the average velocity for t seconds is $\frac{1}{2}(0 + at)$, or $\frac{1}{2}at$. The distance s passed over in t seconds is then by (1) $\frac{1}{2}at \times t$, or $\frac{1}{2}at^2$. Hence

$$s = \frac{1}{2}at^2. \quad (3)$$

From (3)

$$a = \frac{2s}{t^2},$$

and

$$t = \sqrt{\frac{2s}{a}}$$

Substitute this value of t in (2), and

$$v = a \sqrt{\frac{2s}{a}} = \sqrt{2as}. \quad (4)$$

Problems.

1. If a railroad train has a speed of 60 mi. an hour, what is its speed in feet per second? *88 ft per sec*

2. How long will it require a man to walk 25 km., if he walks uniformly at the rate of 15 km. in 3 hr.? *5 hr*

3. A body moving uniformly in a circular path of 10 m. radius makes 10 complete revolutions in 5 sec. Find the speed per second. *125.6 m*

4. If the distance from Chicago to New York is 976 mi. and an express train makes the run in 24 hr., what will be the average speed per hour? Per minute? Allow 3 hr. for stops.

5. If the radius of the earth be taken at 4000 mi., what is the velocity per second of a point on the equator due to the earth's rotation on its axis? *3491.5 ft per sec* *153*

6. A railroad train 430 m. long, passes over a bridge 200 m. long at the rate of 45 km. per hour. How long does the train take to pass completely over the bridge? *50.4 sec*

7. A body starting from rest acquires a velocity of 10 m. per second in one second; 20 m. per second in two seconds, etc. What is the acceleration? *10 m per sec*

8. A body starting from rest has a uniform acceleration of 9.8 m. per second per second. What will be its velocity at the end of the 10th sec.? How far will it go in that time? *98 m per sec* *490 m*

9. A body starting from rest and moving with a uniform acceleration acquires a velocity of 100 ft. per second in 5 sec. What is the acceleration? How far will it go in that time? *250 ft* *200 ft*

10. A body starting from rest passes over 256 ft. in 4 sec. What is its acceleration? *10 ft per sec*

11. A train running at 60 mi. an hour is stopped with uniform retardation in 44 sec. by the application of the brakes. What is the retardation per second per second? *2 ft per sec*

12. If a body has a uniformly accelerated motion and starting from rest has a velocity of 100 cm. per second at the moment it completes a distance of 1 km., what is the acceleration? *0.5 cm*

13. How far will a ball roll before coming to rest, if its motion is uniformly retarded, its initial speed being 50 ft. per second and duration of motion 10 sec.? *250 ft*

14. An electric car has a uniform acceleration of 5 m. per second per second. What will be its acceleration per second per minute? Also per minute per second?

[Acceleration per second per minute means the increase in velocity per second acquired in one minute (§ 33).]

15. What acceleration per minute per minute must a body have to acquire in 20 min. a speed of 20 mi. an hour?

16. What acceleration per minute per minute must a body have to start from rest and pass over 10 mi. in 10 min.?

II. NEWTON'S LAWS OF MOTION.

35. **Momentum.** — We have so far considered motion in the abstract, that is, without reference to the **amount** of matter moving, and without reference to the **force** producing the motion or change of motion.

It is plain that in any case of actual motion, there must be a definite mass of matter moving, and the effect of a force in producing this motion will include the mass moved, as well as the speed imparted to it. It will therefore accord with the ordinary use of language, to speak of the "quantity of motion," and to consider it as proportional, first, to the speed of the body, and second, to its mass (§ 9). The name given to the "quantity of motion" is *momentum*. It is the product of the mass and the velocity of a moving body.

$$\text{Momentum} = \text{mass} \times \text{velocity}, \text{ or } M = mv. \quad (5)$$

In the centimetre-gramme-second (C.G.S.) system, the unit of momentum is the momentum of a mass of 1 gm. moving at the rate of 1 cm. a second.

36. **Impulse.** — Suppose a ball of 10 gm. mass to be fired from a rifle, with a velocity of 50,000 cm. a second. Its momentum would be 500,000 units.

If a truck weighing 500 kgm. moves at the rate of 1 cm. a second, its momentum is also 500,000 units. But the ball has acquired its momentum in a fraction of a second, while a minute or more may have been consumed in communicating the same momentum to the truck. In some sense, the propulsion required to set the ball in motion is the same as that required to give the equivalent motion to the truck, because the momenta of the two are equal. This equality is expressed by saying that the *impulse*, or the effect of the force in producing the same quantity of motion, is the same in the two cases. Since the effect produced is doubled if the value of the force is doubled, or if the time during which it acts is doubled, it follows that *impulse* is the product of the *force* and the *time* during which it acts.

In estimating the effect of a force, the time element and the magnitude of the force are equally important. The term *impulse* takes both into account.

37. Laws of Motion. — The relations of motions and changes of motion to the forces producing them are expressed in Newton's *laws of motion*. They are to be regarded as physical axioms, and are incapable of rigorous experimental proof. They rest on convictions drawn from observation and experiment, and the results derived from their application in the higher fields of mechanics and astronomy are found to be invariably true.

I. *Every body continues in its state of rest or of uniform motion in a straight line, except in so far as it may be compelled, by impressed force, to change that state.*

II. *Change of motion is proportional to the impressed force, and takes place in the direction in which the force acts.*

III. *To every action there is always an equal and contrary reaction; or the mutual actions of two bodies are always equal and in opposite directions.*

By "*change of motion*" we must understand *change of momentum*, and by "*impressed force*," *impulse*.

38. Discussion of the First Law. — Matter has no capacity in itself to change its condition of rest or of motion. Not only this, but a body offers resistance to any such change in proportion to the *mass* of matter contained in it. This property of matter is expressed by the term *inertia*, and the first law of motion is often called *the law of inertia*.

When no force acts on a body, it persists in *its state* of rest or of uniform motion relative to other bodies. If at rest and left wholly to itself, it will remain at rest; if at one moment it be at rest, and afterward be found in motion, then it has been acted on by some force; or if it be in motion and its motion change either in rate or direction, then a force must have acted on it.

From this law we derive a definition of force, for the law asserts that *force is the sole cause of change of motion*.

39. Discussion of the Second Law. — The first law asserts that a change of momentum is due to force. The second law shows, first, how force may be measured. Maxwell has restated it so as to read as follows: "*The change of momentum of a body is numerically equal to the impulse which produces it, and is in the same direction.*" By a proper choice of units, impulse may be placed equal to the change of momentum which it produces, instead of proportional to it, or

$$Ft = mv. \quad (6)$$

Hence

$$F = \frac{mv}{t} = m \frac{v}{t}.$$

The velocity of the mass m before the force acted is here supposed to be zero, and v is the final velocity. *Force is therefore measured by the rate of change of momentum.* Further, $\frac{v}{t}$ is the rate of change of velocity, or the *acceleration* a . We may then write

$$F = ma. \quad (7)$$

It follows that *force may also be measured by the product of the mass moved and the acceleration due to the force*; and acceleration is equal to the force producing it when the mass is unity.

This law teaches, further, that the change of momentum is always in the direction in which the force acts. Hence, when two or more forces act together, each produces its change of momentum independently of the others. This fact lies at the foundation of the method of finding the resultant effect of two forces acting on a body in different directions (§ 44).

40. Units of Force. — Two systems of measuring force are in common use, viz., the *gravitational* and the *absolute*. The latter is usually in the C.G.S. system. The gravitational unit of force is the *weight* of a standard mass, as the *pound of force*, or the *kilogramme of force*. These gravitational units are not strictly constant, but vary with the place on the earth's surface (§ 54). They are convenient for the work of the engineer, but are not suitable for precise measurements.

The absolute unit of force in the C.G.S. system is the *dyne*. *It is the force which, acting on a gramme mass for*

one second, produces a change of velocity of one centimetre a second. This unit is invariable in value. The earth's attraction for a gramme in New York is 980 dynes, since gravity will impart to a gramme in a second at that place a velocity of 980 cm. per second. A dyne is therefore $\frac{1}{980}$ of the gramme of force, and the numerical value of any force expressed in dynes is 980 times as great as in grammes of force (§ 66). Conversely, to convert dynes into grammes of force, divide by the acceleration of gravity, 980.

Problems.

1. What is the momentum of a 10-lb. ball moving at the rate of 20 ft. per second?
2. How many times greater is the momentum of a 20-lb. cannon ball moving with a velocity of 2000 ft. per second than that of a truck weighing 1 ton (2000 lb.) and moving at the rate of 7.5 mi. an hour?
3. Equal momenta are imparted to two bodies; the first has a mass of 10 gm. and a velocity of 1200 m. per minute; the second has a velocity of 100 m. per second. Find the mass of the second.
4. A gun whose mass is 10 kgm. is loaded with a ball whose mass is 50 gm. When the gun is fired the ball has a velocity of 500 m. per second. Calculate the velocity of the gun's recoil.
5. What is the momentum of a mass of 15 kgm, moving with a velocity of 6 m. per minute?
6. Two balls whose masses are 100 gm. and 600 gm. respectively are thrown apart by releasing a coiled spring placed between them. Calculate the velocity of the smaller ball, if the velocity of the larger one is 20 m. per second.
7. An unbalanced force acts for 10 sec. on a body whose mass is 50 gm. and imparts to it a velocity of 40 cm. per second. Compute the magnitude of the force. [Consult equation (6). § 39.]
8. A mass of 25 gm. acquires a velocity of 120 cm. per minute due to an unbalanced force acting on it for 5 sec. Find the magnitude of the force.

9. An unbalanced force of 20 dynes acts for 30 sec. on a body and imparts to it a velocity of 40 cm. per second. Find the mass of the body. *15 gm*

10. A body whose mass is 1 kgm. is acted on by an unbalanced force of 100 dynes for 5 min. What velocity will be imparted to it? *30 cm per sec*

11. An iron ball whose mass is 50 gm. is acted on for 10 sec. by an unbalanced force of 200 dynes. What will be the resulting acceleration and the final velocity? [Consult relations (6), (7), and (2).] *40 cm per sec. acc 4 cm per sec.*

12. If an unbalanced force of 50 dynes acts for 10 min. on any mass, what momentum will be generated? [To calculate momentum we must know either mass and velocity, or force and time. Consult equation (6), § 39.] *30000 units 49.5 F m A*

13. A mass of 25 gm., acted on by an unbalanced force, moves with an acceleration of 980 cm. per second per second. Calculate the value of the force. *24,500 dynes or 25 gm. 25*

14. How long must an unbalanced force of 1000 dynes act on a mass of 100 gm. to impart to it a velocity of 250 cm. per second? *25 sec 250*

15. What unbalanced force will produce an acceleration of 980 cm. per second per second in a stone whose mass is 500 gm.? What velocity will it impart to the stone in 5 min.? [Equations (7) and (2).] *490000 dynes 2.94 km per sec*

16. A mass of 490 gm. is acted on by an unbalanced force for 15 sec. and acquires a velocity of 30 cm. per second. Calculate the magnitude of the force. *980 dynes or 1 gm. of force*

17. An unbalanced force of 50 gm. acts on a mass of 50 gm. for 5 sec. What will be the resulting velocity? [How many dynes in 1 gm. of force?] *4900 cm per sec*

18. An unbalanced force of 100 gm. acts for 5 min. on a mass of 200 gm. What will be the velocity imparted? *1 km per min*

19. What unbalanced force will in 10 min. give to a mass of 100 gm. a velocity of 0.99 km. a minute? *295 dynes*

20. If any mass be pushed by an unbalanced force of 5 dynes for 5 min., how much will the momentum be increased? [Equation (6), § 39.] *1500 units (69) Ft = (m) v*

41. Graphic Representation of a Force.—A force is completely specified by its three elements: (*a*) *its point of application*; (*b*) *its direction*; and (*c*) *its magnitude*. These three particulars may be represented by a straight line drawn through the point where the force acts, in the direction in which the force changes the momentum of the body, and as many units in length as there are units of force. If a line 1 cm. long stands for a force of 1 dyne, a force of 4 dynes, in the direction *AB* (Fig. 9),

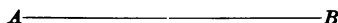


Fig. 9.

will be represented by a line 4 cm. long. Any point on this line, as *A*, may represent the point at which the force is applied.

Further, if it is desired to represent graphically the fact that two forces act at the same time on a body, for example, one a force of 3 lb. horizontally, and the other a force of 2 lb. vertically, we have only to draw two lines from the point of application of the forces *A* (Fig. 10), one 3 units long toward the right, and the other 2 units long on the same scale toward the top of the page. The two lines *AB* and *AC* represent the point of application, the direction, and the magnitude of the two forces in question.

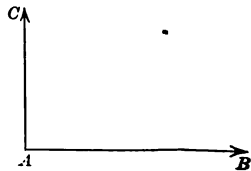


Fig. 10.

42. How a Force is Measured.—The simplest device for measuring a force is the spring balance. It consists of a coiled spring, to the free end of which is attached a pointer, adapted to move in front of a graduated scale (Fig. 11). According to Hooke's law (§ 15) the division

of the scale is in equal parts; it may be made to read in pounds, grammes, or dynes. The common drawscale is a spring balance graduated in pounds and fractions of a pound. If the spring be stretched by a force in *any direction* so that the pointer stands at 15 lb., for example, then the pull on the spring is the same as the pull of gravity on a mass of 15 lb.; that is, 15 lb. of force.



Fig. II.

43. The Resultant of Two or More Forces in the Same Line. — The single force which will produce the same effect on the motion of a body as two or more forces acting on it together is called the *resultant*. (Note exception in the case of a *couple*, § 47.) The process of finding the resultant of two or more forces is called the *composition of the forces*.

When two forces act on a body along the same line and in the same direction, the resultant force is simply their sum, and it acts in the same direction as the forces. The same is true when there are more than two forces acting in the same direction.

When two forces act on a body along the same line but in opposite directions, their resultant equals their difference and it acts in the direction of the greater force.

For example, if one man pulls on a truck with a force of 40 lb. and another pushes at the back with a force of 50 lb., the resultant force applied to move the truck is 90 lb. But if one pulls with a force of 100 lb. in one direction, and the other with a force of 75 lb. in the other direction, the resultant force tending to move the truck forward is obviously only 25 lb.

When the sum of the forces acting in one direction is equal to the sum of those acting along the same line in the opposite direction, the resultant is zero and the body acted on is in *equilibrium*; that is, the condition of the body as regards either rest or motion is entirely unaffected by the application of the forces whose resultant is zero.

44. The Resultant of Two Forces Acting at an Angle.—

When two forces act together on a body at an angle, the resultant is in a direction between the two. For example, let a force of 10 lb. act on a body at *A* (Fig. 12) toward the right, and an equal force upward. The two are represented by the lines *AB* and *AC* of equal length. Then obviously the *direction* of the resultant is *AD* midway between *AB* and *AC*; for no reason can be assigned why the resultant should lie nearer the one force than the other. The magnitude of the resultant is the length of the diagonal *AD* of the square *ABDC*.

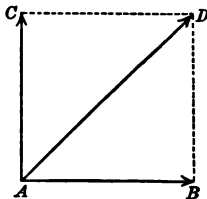


Fig. 12.

When the two forces to be combined are not equal and at right angles to each other, the case is not quite so simple. The resultant then is in a direction nearer to the greater force, and its direction and magnitude are found by the application of the principle known as *the parallelogram of forces*. This principle includes that of the square for equal forces at right angles; it may be verified as follows:—

Experiment.—Tie together three stout cords at *D* (Fig. 13) and fasten the free ends to the hooks of three drawscopes, *A*, *B*, *C*, respectively. The drawscopes may be graduated to read in grammes. Pass

their rings over wire nails set in a drawing board at such distances apart that the drawscopes will be stretched. Record the readings on the scales and mark on the board the position of the knot D and the points of attachment of the cords to the hooks of the drawscopes. Then remove the drawscopes and draw the lines marking the directions of the cords.

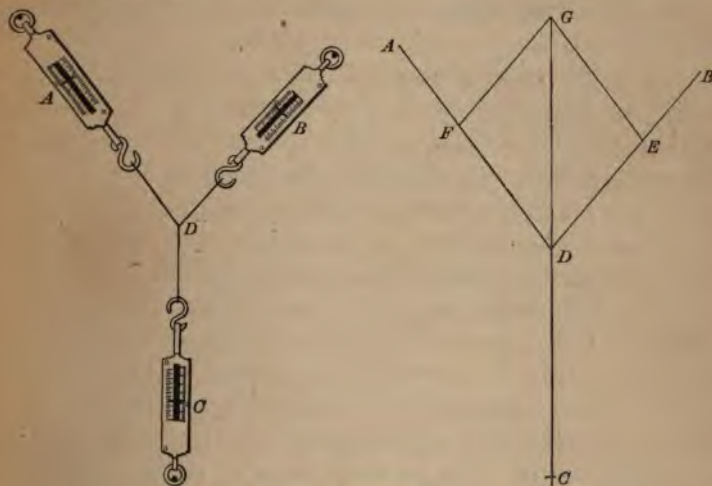


Fig. 13.

Lay off on DA , DB , and DC the readings of the drawscopes A , B , and C , respectively, on some convenient scale. Complete the parallelogram $DFGE$, draw the diagonal DG , and find its value on the scale used in laying off the sides. If the work be carefully done, it will be found that DG and DC are in the same straight line and are of equal length. The single force DG is the resultant of the forces DF and DE , since it is in equilibrium with DC . *Equilibrium may always be produced by applying a force equal and opposite to the resultant of all the other acting forces.*

The principle illustrated may be expressed as follows:

If two forces are represented in direction and magnitude by the adjacent sides (DF and DE , Fig. 13) of a parallel-

ogram, then their resultant will be represented in both direction and magnitude by the diagonal of the parallelogram (DG) drawn through the intersection of the two sides (DF and DE) representing the two forces.

45. The Composition of Velocities. — The composition of uniform velocities is effected by the same methods as the composition of forces. In fact, the same principles apply to the composition of all quantities having both magnitude and direction. Such quantities are known as *vectors* to distinguish them from those which have magnitude only. The latter are known as *scalars*.

When several motions are given to a body at the same time, its actual motion is a compromise between them, and the compromise path is the resultant. At the Paris Exposition in 1900 a continuous moving sidewalk carried visitors around the grounds. A person walking on this platform had a velocity with respect to the ground made up of the velocity of the sidewalk relative to the ground and the velocity of the person relative to the moving walk. The several velocities entering into the resultant are the *component velocities*.

For example, a boat, which would have in still water a velocity of 5 mi. an hour, will have what velocity when there is a current of 3 mi. an hour?

The resultant velocity will be $5 - 3 = 2$ mi. an hour up stream. Down stream it would be $5 + 3 = 8$ mi. an hour.

Again, if a boat can be rowed in still water at the uniform rate of 5 mi. an hour, what will be the actual velocity if it be rowed at right angles to a current running 3 mi. an hour?

Let the line AB (Fig. 14) represent in length and direction the velocity of 5 mi. an hour across the stream, and

the line AC , at right angles to AB , the velocity of the current, 3 mi. an hour, both on a scale of 8 mm. to the

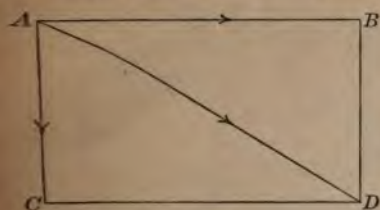


Fig. 14.

mile. Complete the parallelogram $ABDC$, and draw the diagonal AD through the point A common to the two component velocities. AD will represent the actual velocity; for while the boat is rowed 5 mi. in the direc-

tion AB , it is carried by the current 3 mi. in the direction AC . Hence at the end of an hour, instead of arriving at B , it will be at D , and will have travelled along the line AD with uniform motion. The length of the line AD on the same scale as the other lines will be 5.83. The resultant velocity is therefore 5.83 miles an hour in the direction AD .

When the angle between the components is a right angle, as in the present case, the diagonal AD is the hypotenuse of the right-angled triangle ABD . Its square is therefore the sum of the squares of 5 and 3, or

$$AD = \sqrt{5^2 + 3^2} = \sqrt{34} = 5.83.$$

When the angle A is not a right angle, the resultant must be found by a graphic process of measurement or by the principles of plane trigonometry.

46. The Resultant of Parallel Forces.—The resultant of parallel forces, that is, the effect on a body in producing motion of translation, in which all parts of the body move in parallel straight lines, is the same as when the forces act along the same straight line (§ 43). It is in both cases their *algebraic* sum; that is, if two parallel forces

act in the same direction, the resultant is their sum ; if in opposite directions, it is their difference.

Experiment. — Suspend two drawscopes *A* and *B* (Fig. 15) by cords from suitable supports in such a way that the two suspending cords shall be parallel. Let the graduated bar, from which the known weight *W* is suspended, be supported by the drawscopes in a horizontal position, and all the supporting cords vertical. Read the two drawscopes and the distances *CE* and *ED*. Change the position of the point *E*, adjust and read again. Then for each set of observations the weight *W* should equal the sum of the readings on the drawscopes, *A* and *B*, for it is equal and opposite to the resultant of the parallel forces, *A* and *B*. Moreover, in each case we should have

$$A : B :: ED : CE.$$

Hence the following principle :

The resultant of two parallel forces in the same direction is equal to their sum ; and the point of application of the resultant divides the line joining those of the two forces into parts inversely as the forces themselves.

When the two parallel forces act in opposite directions, their resultant is their difference, and it acts in the direction of the larger force.

47. A Couple. — When two parallel forces are not only in opposite directions, but are also equal, their resultant

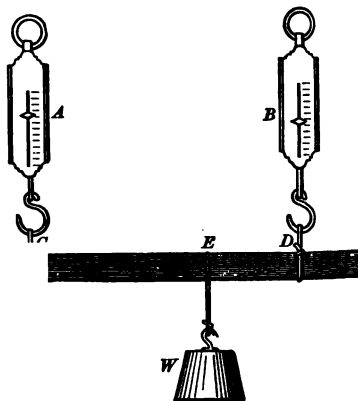


Fig. 15.

is zero. Such a pair of forces is known as a *couple*. A couple has no effect in producing motion of translation, for its resultant is zero. It produces motion of rotation only, in which all the particles of the body to which the couple is applied rotate in circles about a common axis.

A couple has no resultant, that is, a single force cannot be substituted for it and produce the same effect. It follows that equilibrium can be brought about only by applying another couple tending to produce rotation in the opposite direction.

A magnetized sewing needle floated on water is acted on by a couple when it is displaced from a north-and-south position. One end of the needle is attracted toward the north, and the other end toward the south with equal and parallel forces. The effect is to rotate the needle about a vertical axis till it returns to its north-and-south position.

47a. The Moment of a Force.—Under certain conditions, a single force may produce rotation. Such is the case when the body to which the force is applied can move only about a fixed axis. An example is the flywheel of an engine. It is obvious that an increase in the rotatory effect on the flywheel may be secured, either by increasing the force, or by lengthening the crank. The effectiveness of a force in producing rotation depends then on two quantities: (1) the magnitude of the force, and (2) the shortest distance of the axis from the line of action of the force.

The measure of this effectiveness is *the moment of the force*. It is the product of the force and the perpendicular distance from the axis of rotation to the line of action of the force. The *moment of a couple* is the product of one of the forces and the perpendicular distance between the parallel lines representing the two forces.

One couple is in equilibrium with an opposing couple when the moment of the one is equal to the moment of the other. Two forces are in equilibrium about an axis when their moments with reference to this axis are equal, and they tend to produce rotation in opposite directions. In general, any number of forces produce no rotation when the sum of the moments for rotation in one direction equals the sum of the moments in the other direction.

48. Resolution of a Force. — A force, or a velocity, may be resolved into components in any given directions. The component of a force in a given direction is its effective value in that direction. The most common case is the resolution of a force into two components along directions at right angles to each other.

To illustrate: Let it be required to resolve a force equal to the weight of 32 lb. into two rectangular components, one of them to be 12 lb. The problem is to construct a rectangle with a diagonal of 32 and one side 12, and to find the other side.

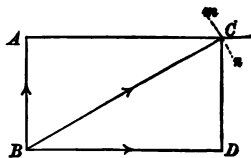


Fig. 16.

Draw AB (Fig. 16) 12 units in length, and at A draw AC perpendicular to AB . With B as a centre, and with a radius of 32 units, draw the arc mn cutting AC at C . Complete the rectangle at $ABDC$. Then BA and BD are the two components of BC , and BD is the one required. Its value may be found by a scale and a pair of dividers, or it may be calculated from the right triangle BDC .

$$BD = \sqrt{32^2 - 12^2} = 29.66.$$

As a second illustration, let a body W , whose weight is 20 lb., be supported on the smooth inclined surface AB

(Fig. 17). What force parallel to the plane will be necessary to maintain it at rest? Draw WH perpendicular to AB , and let the vertical line WF represent the weight of 20 lb. Draw WE parallel to AB , and complete the rectangle $WHFE$. Then WH and WE are components of WF . WH represents the pressure on AB , and WE the force urging the weight down the inclined plane. To maintain W at rest, it will be necessary to apply to it a force WK equal and opposite to WE .

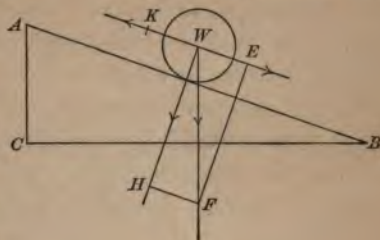


Fig. 17.

49. Discussion of the Third Law.—The meaning of this law of *reaction* is that force is always due to the *mutual action* between two bodies or parts of bodies. This mutual action is called a *stress*, and stress is always a two-sided phenomenon. It includes both *action* and *reaction*, and is called the one or the other according as the attention is directed to one aspect of it or the other; just as trade includes both sale and purchase, and is designated by the one term or the other according to the side of the trade which one considers.

The stress in a stretched elastic cord *pulls* the two things to which it is attached equally in opposite directions; the stress in a piece of compressed india rubber exerts an equal *push* both ways. The former is called a *tension* and the latter a *pressure*. Now, every force is one of a pair of equal and opposite forces composing a stress. Imagine a rope supporting a weight. The tension in the rope is a stress tending to part it by pulling adjacent

portions in opposite directions. The same is true if two men are pulling at the two ends of the rope.

The same conditions of action and reaction exist when the medium is invisible. A stone attracts the earth with the same force that the earth exerts on the stone. The action between a magnet and a piece of iron is a mutual one, the magnet attracting the iron and the iron the magnet with precisely the same force.

Suspend two elastic ivory or celluloid balls by long threads so that the balls just touch. Draw one aside in the plane of the threads and let it fall so as to strike the other one. It will be brought to rest, while the second ball takes the motion. The impact is a stress. The *action* of the first ball on the second sets the latter in motion, while the *reaction* of the second on the first brings the first to rest.

The third law of motion expresses the fact that the two phases of a stress, the action and the reaction, are always equal to each other and in opposite directions.

Problems.

The following problems should be solved graphically and the results checked by calculation, wherever possible, without using Trigonometry.

1. Two forces, 30 and 40 gm. of force, act at an angle of 90° . Find the magnitude of the resultant.
2. Two forces, 8 and 10 lb. of force, act at an angle of 60° . Find the magnitude of the resultant.
3. A ball while moving eastward with a velocity of 20 m. per second is given a velocity of 30 m. per second northward. What is the resultant velocity?
4. A boat is towed along a stream by means of two horses, one on each bank of the stream. The ropes by which the forces are applied

make an angle of 90° . If each horse exerts a force of 100 lb., what is the magnitude of the resultant force? If longer ropes were used so as to make the angle 60° , what would be the resultant?

✓5. The wind was blowing from the north with a velocity of 10 mi. an hour. What would seem to be its velocity to a man riding eastward on a bicycle at the rate of 15 mi. an hour? 14.1

[The eastward motion of the rider produces an apparent wind from the east of 15 mi. an hour.]

6. Find the force that will exactly balance two other forces of the magnitudes 20 and 25 dynes respectively, acting at a common point, the angle between their directions being 90° . 32 dynes

7. From an electric car running at the rate of 15 mi. an hour, a ball is thrown at right angles to the motion of the car and with a velocity of 20 ft. per second. What is the actual velocity of the ball at the beginning of its flight? 25.7 mi. per hr.

8. Resolve a force of 25 dynes into two components at right angles, one of the forces to be 10 dynes. Find the other. 22.9 dynes

9. A weight of 50 lb. is suspended by two cords which make an angle of 90° with each other. One cord makes an angle of 60° with the vertical. Find the tension in each cord. 25 lb. & 43.3 lb.

✓10. The wind is blowing from the southwest with a velocity of 12 mi. an hour. If a man should ride a wheel toward the west with a velocity of 10 mi. an hour, with what velocity would the wind strike his face?

[The velocity will be the eastward component of the 12 mi. increased by 10 mi.]

11. Two parallel forces of 5 and 15 gm. act in the same direction, with their points of application 36 cm. apart. Find the magnitude of the resultant and the distance of its point of application from the greater force. $R = 20$ gm. $x = 9$ cm.

[If x represent the distance of the resultant from the force of 15 gm., what would represent the distance from the force of 5 gm.? Consult § 46 for the statement.]

12. Two men carry a weight of 40 kgm. on a pole 3 m. long. Where must the weight be placed so that one shall carry three times as much of the weight as the other? 75 cm. from man with less weight

13. Two parallel forces of 30 and 50 dynes respectively have the point of application of their resultant 30 cm. from the larger force. Find the distance between the points of application of the forces.

14. Resolve a force of 60 dynes into two parallel components whose points of application are distant 10 and 20 cm. respectively from that of the given force.

[Represent the forces by x and $60 - x$.]

15. A man and a boy are to carry on a pole 12 ft. long a weight of 300 lb. placed at the middle of the pole. How far from one end must the man take hold, so that the boy by taking hold at the other end shall carry one-third of the weight?

III. GRAVITATION.

50. **The Fall of Bodies.** — The early philosophers thought that light bodies fall more slowly than heavy ones. Galileo was the first to find out the truth experimentally by dropping various bodies together from the top of the leaning tower of Pisa. He found that they fell to the ground in nearly the same time, whatever their size or weight. The lighter bodies fell slightly slower than the heavier ones, and the difference he rightly ascribed to the resistance of the air.

The "guinea and feather tube," devised since the invention of the air-pump, shows that all bodies fall toward the earth with the same acceleration.



Fig. 18.

Experiment. — Place a coin and a feather, or a pith ball and a shot, in a long tube (Fig. 18) closed at one end and fitted with a stopcock at the other. Hold the tube in a vertical position and suddenly invert it; the coin or

the shot falls to the bottom first. Now exhaust the air as thoroughly

as possible. When the tube is then suddenly inverted the two objects fall to the lower end of the tube in the same time.

The inference from this experiment is that all bodies would fall from rest through the same height in the same time if the resistance of the air were wholly removed. In air, the body which has the larger surface in proportion to its mass falls the more slowly because it meets with more resistance in falling.

The resistance offered by the air to bodies falling through it is illustrated by its effect on a small stream of water flowing over a high precipice. It breaks up the stream into a fine spray as it descends. In a vacuum water falls like a solid. The *water hammer* (Fig. 19) is an instrument devised to illustrate this fact. It consists of a heavy glass tube half filled with water, the air having been expelled by boiling the water for some time just before sealing the upper end. When it is suddenly inverted, the water falls like a solid, with a metallic ring.



Fig. 19.

51. Weight. — Since all bodies fall in a vacuum with the same acceleration, and so traverse the same distance in the same time when starting from rest, the forces acting on them, due to the earth's attraction and called *gravity*, are proportional to their masses. This force of gravity is called *weight*. The proportionality of mass and weight was first demonstrated by Sir Isaac Newton. We have already had the relation $F = ma$ in equation (7). Plainly, if the acceleration a is the same for all masses, the force F

is proportional to the mass m . In the case of gravity the particular force is the *weight*, denoted by W , and the particular acceleration is the *acceleration of gravity*, denoted by g . Making these substitutions in equation (7), we have weight expressed as a force:

$$W = mg. \quad (8)$$

52. Direction of Gravity. — The path described by a falling body is a *vertical line*. A line or plane perpendicular to it is said to be *horizontal*. The direction of a vertical line at any point may be determined by suspending a weight by a cord passing through the point. The weight and cord are called a *plumb-line*. The direction of the plumb-line is perpendicular to the surface of still water. Vertical lines drawn through neighboring points may be considered parallel without sensible error, for vertical lines 100 ft. apart make an angle with each other of one second of arc, and this is the angle subtended by a pinhead at the distance of about a quarter of a mile. At the poles of the earth and at the equator, the direction of gravity is that of the plumb-line; elsewhere there is a slight variation on account of the rotation of the earth on its axis (§ 66).

53. Law of Universal Gravitation. — Toward the end of the seventeenth century Sir Isaac Newton discovered the law of *universal gravitation*. He derived this great generalization from a study of the results obtained by two eminent astronomers, Copernicus and Kepler. The law may be expressed as follows: —

Every particle of matter in the physical universe attracts every other particle with a force whose direction is that of the line joining the two particles, and whose magnitude is directly as the product of the two masses, and inversely as the square of the distance between them.

This law expressed in symbols is

$$F = G \frac{mm'}{d^2} \quad (9)$$

where m and m' are the masses of the particles, d is the distance between them, and G is a proportionality factor or constant of gravitation to be determined by experiment. When applied to bodies like the earth, whose dimensions are large compared with that of any body on its surface, Newton proved that the distance involved in the law of gravitation is the distance from the body to the earth's centre; for any spherical mass like the earth or the sun attracts another body exactly as if all the matter it contains were concentrated at its centre.

54. Law of Weight.—Since the earth is flattened at the poles, it follows from the law of gravitation that the intensity of gravity, and therefore the weight of a body, increase, in going from the equator toward the poles. If the earth were a stationary sphere, the value of g would be the same all over its surface. It would then vary only on ascending above the surface or descending below it; as the inverse square of the distance on ascending, and simply as the direct distance on descending, assuming the density (§ 140) uniform. The value of g at the equator in C.G.S. units is 978.1 and at the poles 983.1. At New York it is a little over 980 cm. per second per second, or 32.15 ft. per second per second.

55. Centre of Gravity.—A solid body is composed of particles all acted on by gravity with a force equal to the product of their mass and the intensity of gravity g (§ 51). For bodies of ordinary size these forces are all parallel.

Hence the weight of a book or of a quoit disk, for example, is the resultant of an infinite number of parallel forces of gravity; and the point of application of the resultant of all these forces, *however the body be turned about*, is called the *centre of gravity* of the body. So long as the forces of gravity on the parts are strictly parallel and proportional to the masses of these parts, the centre of gravity coincides with what is known as the *centre of mass* or the *centre of inertia*. It is the point in a body about which the mass is evenly disposed. In a uniform sphere it is its centre. In a uniform ring, it is also its centre. In a uniform rod, it is its middle point.

56. Equilibrium under Gravity.—The conditions for the equilibrium of a body are, (1) the resultant of all the forces acting on it is zero; (2) the sum of their moments is zero. There will then be nothing to change either the linear or the angular velocity of the body. Equilibrium does not mean that the velocity is necessarily zero, but that the acceleration is zero. Rest means zero velocity; equilibrium zero acceleration.

In the case of a body free to turn about a horizontal axis, it can be in equilibrium only when the vertical line through its centre of gravity passes through this axis. Let a body, whose centre of gravity is at G , be supported by an axis through B (Fig. 20). Represent its weight by GE through the centre of gravity. When the direction GE does not pass through B , the moment of the weight about the axis through B is the product of GE and BC (§ 47a), and the body rotates clockwise. As

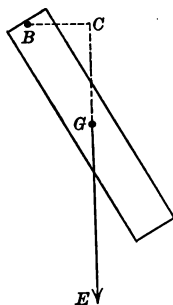


Fig. 20.

the body swings, shortening the line BC , the point G approaches the vertical through B ; and when the point C coincides with B , the moment becomes zero. The body is then in equilibrium. The weight downward through G and the equal and opposite reaction of the axis upward are then along the same vertical line.

When a body rests in equilibrium on a plane, the vertical line through its centre of gravity falls within its base of support. If this vertical line falls outside the base, the body will overturn, for the supporting force, consisting of the reaction of the plane, does not act opposite to the weight, but parallel to it, forming a couple. If a body, like a chair, is supported on legs, the base is the polygon formed by lines connecting the points of support.

57. Three Kinds of Equilibrium.—**Experiment.**—Fill a round-bottomed Florence flask one-quarter full of shot, crowding paper into the remaining space to keep the shot in place (Fig. 21). Tip the flask over; after a few oscillations it will return to the upright position. If the experiment be repeated with a similar flask empty, it will be impossible to find any other position of equilibrium for the flask than on its side. We may then roll it about, and it will remain in any position in which the top and bottom rest on the supporting plane.



Fig. 21.

This experiment illustrates the three kinds of equilibrium of position. (1) The centre of gravity of the loaded flask is nearer the supporting plane than that of the empty one. (2) In overturning the loaded flask, its

centre of gravity is raised, and at the same time the vertical line through it is thrown outside the point of support, so that the reaction of the plane upward, and the weight of the flask and contents downward, form a couple which returns the flask to its upright position. In overturning the empty flask, its centre of gravity is lowered, and the vertical line through it falls between the two points of contact with the plane. (3) When the empty flask is rolled about on its side, its centre of gravity is neither raised nor lowered. We have thus three kinds of equilibrium of position: *stable*, for any displacement which causes the centre of gravity to rise; *unstable*, for any displacement which causes the centre of gravity to fall; and *neutral* for any displacement which does not change the height of the centre of gravity of the body.

58. Illustrations. — The rocking-horse and the rocking-chair are familiar examples of stable equilibrium. The half of a split ball, or any segment of a sphere, will rock in stable equilibrium on its rounded side. An egg lying on its side has neutral equilibrium for rolling, and stable equilibrium for rocking; it is unstable in every direction when balanced on either end. In the case of a body pivoted at a point, it is stable when the centre of gravity is below the point, and unstable when it is above. With the balls in the position shown in Fig. 22, the movable system is in stable equilibrium; but if the balls are raised above the level of the pivot, the equilibrium



Fig. 22.

becomes unstable. The nearer the centre of gravity of a beam balance is to the point of support, the smaller is its stability, and the greater its sensitiveness; but its centre of gravity must be slightly below its support, or it will be unstable and topple over (§ 59).

59. Stability. — The most useful measure of *stability* is the work required to upset a body (§ 78); that is, it is the product of its weight and the difference between the

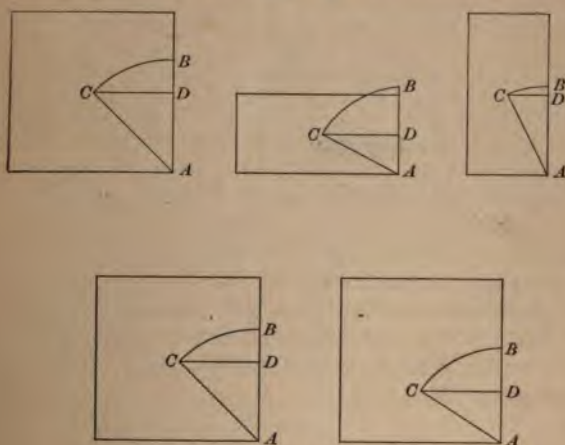


Fig. 23.

distances AC and AD in Fig. 23. An inspection of these diagrams shows that the stability is increased by lowering the centre of gravity, and by enlarging the base. C denotes the centre of gravity or centre of mass, A the point about which the body is turned, and BD the height through which the weight is lifted, to bring the body to the position of unstable equilibrium. If the masses are equal, BD is a measure of the stability.

A brick has less stability when standing on end or a

table, than when lying on edge; and it has less stability on edge than when lying on its broad side. In the first position, the centre of gravity is the highest, and the base the smallest; in the last position, the centre of gravity is the lowest, and the base the largest.

Experiment. — Make a disk or short cylinder of wood, and load it with lead on one side, near the circumference (Fig. 24). Find by trial the position G of the centre of gravity on one end of the cyl-

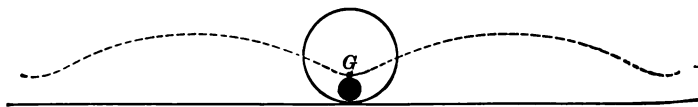


Fig. 24.

inder, and mark it. This cylinder may be placed on a slightly inclined plane, with the centre of gravity near the highest point, in such a position that the cylinder will roll up the plane into a position of stable equilibrium. Roll the cylinder along a horizontal plane, and watch the path described by its centre of gravity. It will be a curve, with crests and hollows, similar to the one in the figure; every hollow corresponds to a position of stable equilibrium, with the centre of gravity in its lowest position, and every crest is a position of unstable equilibrium. If the path were a straight line, the equilibrium would be neutral.

Problems.

1. Explain why a ball rolls down hill, but a cubical-shaped block does not, unless the hill is very steep. (§ 56.)
2. Why does a person lean forward in climbing a hill?
3. Why is a pyramidal-shaped structure more stable than a prismatic one of equal base and altitude?
4. Why is the base of a student's lamp filled with iron or lead?
5. If the attraction of an apple for the earth is equal to that of the earth for the apple, why does not the earth meet the falling apple half-way?

6. Why is it difficult to walk a stretched rope? Why is it easier if the performer carries a long pole loaded at the ends?

✓ 7. What will a 10-lb. ball weigh 1000 mi. below the earth's surface, the earth's radius being considered as 4000 mi.?

✓ 8. What will a 100-lb. ball weigh 1000 mi. above the earth's surface, the earth's radius being considered as 4000 mi.?

9. Assume the earth's radius to be 4000 mi. and the distance to the moon 240,000 mi. Then if g is 32 ft. per second per second at the earth's surface, what will it be at the orbit of the moon?

✓ 10. If a body weigh 20 kgm. at the equator, where g is 978 cm. per second per second, what will it weigh at the pole where g is 5 cm. greater? [Consult § 51.]

11. Assuming the diameter of the planet Venus to be the same as that of the earth, and its mass three-quarters the mass of the earth, what is the acceleration of gravity at the surface of Venus?

12. If the mass of the moon were doubled and that of the earth trebled, how would these changes affect their attraction for each other at the same mean distance?

✓ 13. The sun's mass is 332,000 times that of the earth, and its radius is 110 times that of the earth. If a body weighs 200 lb. on the earth's surface, what would it weigh on the surface of the sun?

14. If the acceleration due to gravity on the earth's surface is 980 cm. per second per second, what is it on the surface of Mars, the diameter of Mars being assumed as half that of the earth and its mass one-ninth? [Consult § 53.]

IV. LAWS OF FALLING BODIES.

60. Uniform Acceleration applied to Falling Bodies. —

Since the acceleration g is sensibly constant for small distances above the earth's surface, the formulæ already obtained for uniformly accelerated motion may be directly applied to falling bodies. The relations between velocity, time, space, and acceleration are expressed by the

equations, $v = at$ and $s = \frac{1}{2}at^2$ (§ 34). Substituting g for a , we have

$$v = gt, \quad (10)$$

and
$$s = \frac{1}{2}gt^2. \quad (11)$$

If in equation (11) t is made one second, then $s = \frac{1}{2}g$; or, the space described in the first second, when the body falls from rest, is half the value of the acceleration of gravity. A body falls 490 cm. the first second; the velocity attained in one second, and the acceleration, are 980 cm. per second.

To find the space passed over in any one second, find the space described in t seconds and in $(t - 1)$ seconds, and subtract the latter from the former. Denoting the distance sought by s' ,

$$s' = \frac{1}{2}gt^2 - \frac{1}{2}g(t - 1)^2 = \frac{1}{2}g(2t - 1). \quad (12)$$

The distance passed over in any second is equal to half the product of g and one less than double the number of the second. By combining equations (10) and (11) we have

$$v^2 = 2gs. \quad (13)$$

61. Laws. — The laws embodied in the preceding formulæ may be expressed as follows: —

I. *The velocity attained by a falling body is proportional to the time of falling.*

II. *The space described is proportional to the square of the time.*

III. *The acceleration is twice the space through which a body falls in the first second.*

62. Experimental Proof. — The laws of falling bodies were first verified experimentally by Galileo. His method

ed in rolling a ball down a smooth inclined plane,
 reducing the acceleration
 taking a part only of the
 of gravity effective in
 ing motion (§ 48). It
 becomes comparatively
 o measure the distances
 ed in successive seconds.
 he Hawkes-Atwood ma-
 (Fig. 25) a different
 l is employed. Two un-
 weights are suspended by
 ole ribbon of tissue paper
 very light wheel. The
 shown on a larger scale
 . 26, has light aluminum
 nd spokes. It must be
 ly balanced, and must
 nearly without friction
 sible on agate bearings.
 e top of the supporting
 is an electromagnet,
 ng a locking device and
 linked camel's-hair brush
 marker. The current
 h this magnet is con-
 by a seconds pendulum,
 closes the electric circuit
 eeping through a drop of
 ry on a small metallic
 t the bottom of its swing.
 e first swing of the pen-
 a mark is made on the



Fig. 25.

ribbon, the wheel is unlocked, and it starts to rotate because of the excess weight on one side. The paper ribbon is made endless, so that the excess weight shall remain constant. An electric brake enables the operator to stop the wheel at any desired point.



Fig. 26.

If the two suspended masses are a and b grammes, then the force effective in producing motion of the system is the force of gravity on $(a - b)$ grammes, and this force must overcome the inertia of a mass of $(a + b)$ grammes, neglecting the wheel and paper. Hence the effective acceleration is reduced from g to $\frac{a - b}{a + b}g$.

The distances described in 1, 2, 3, 4, etc., seconds may be found by measuring the distances from the first mark on the ribbon to the others in succession. They will be found to be as the numbers 1, 4, 9, 16, etc. The distances described in successive seconds, the first, second, third, etc., are the distances between successive marks on the recording ribbon. They are as the odd numbers 1, 3, 5, 7, etc. It is not necessary that the pendulum beat seconds. Any other interval will serve as well for the verification of the laws of falling bodies. The distances on the ribbon may be measured without removing it from the wheel, by using the slider and scale on the column.

In the table, the second column gives the actual distances s read from the ribbon; s' stands for the successive differences of s in the second column; v , the quantities under

s' increased by 2.91; and a , the successive differences of v , or the acceleration. From the formula

$$s' = \frac{1}{2} a(2t - 1) = at - \frac{1}{2} a,$$

equation (12), it will be evident that column four denotes the accumulated velocity at at the end of the successive time intervals.

t	s	s'	v	a	s -Ratios	s' -Ratios	a -Ratios
1	2.91	2.91	5.82	5.81	1.000	1.000	1.000
2	11.63	8.72	11.63	5.83	3.997	2.997	1.003
3	26.18	14.55	17.46	5.83	8.997	5.000	1.003
4	46.56	20.38	23.29	5.82	16.000	7.003	1.002
5	72.76	26.20	29.11	5.82	25.004	9.004	1.002
6	104.78	32.02	34.93	5.81	36.007	11.003	1.000
7	142.61	37.83	40.74	5.82	49.007	13.000	1.002

63. Projection Upward. — When a heavy body is thrown vertically upward, the acceleration is negative, and its velocity is diminished each second by g units (980 cm. or 32.15 ft.). Hence, the time of ascent to the highest point will be the time taken to bring the body to rest. If the velocity of projection upward is v , then we have, from equation (10), neglecting atmospheric resistance,

$$t = \frac{v}{g} \quad (14)$$

If the velocity lost is g units a second, the time required to lose v units of velocity will be the quotient of v divided by g . If, for example, the velocity of projection upward is 1470 cm. a second, the time of ascent, if there were no resistance of the air, would be $\frac{1470}{980}$, or 1.5 sec. The time of ascent is, therefore, the time of descent again to

the starting-point ; and the body will return to the starting-point with a velocity equal to its velocity of projection in the opposite direction.

V. CURVILINEAR MOTION.

64. Uniform Circular Motion. — Hitherto we have dealt only with the change in the *magnitude* of the velocity of a body. But force may also change its *direction*. The motion will then be *curvilinear*. Let us consider the simplest case in which a force acts to produce only *curvature* in the path of a body without affecting the speed or rate of motion.

Imagine a body moving round and round in a circle with constant speed. A heavy ball attached to a string and whirled around the hand has such a motion. The velocity along the circle is constant, but its direction is constantly changing, and there is a constant pull on the string. If this pull should cease, the body would instantly move on in the direction of the tangent line, by the first law of motion. The constant force which incessantly deflects the body from a rectilinear path, and compels it to move in a circle, is called the *centripetal force*. It is always applied at right angles to the motion of the body, and therefore cannot change its velocity along the circle.

65. Centripetal and Centrifugal Force. — The tension in the string which restrains the whirling stone is of course a stress. One aspect of it is the *action* of the hand or other central body on the revolving ball ; the other is the *reaction* of the ball on the hand or central body. The former is the *centripetal force*, or force toward the centre ; the latter is the *centrifugal force*, or force away from the

centre. The two are necessarily equal. Centrifugal force is the resistance which the body offers, *on account of its inertia*, to deflection from a straight path.

66. Centripetal Acceleration and Centripetal Force.— Since the rate of deflection of the revolving body from a straight line in uniform circular motion is constant, the acceleration is constant; and it is directed toward the centre of the circle at every point because there is no change in speed along the circle. If the acceleration were not toward the centre, it could be resolved into two components, one toward the centre and the other along the tangent to the circle; the latter would mean a change of velocity in the circle. But the velocity is uniform, and therefore there is no tangential component; the acceleration is wholly toward the centre. Uniform circular motion, then, is compounded of a uniform motion around the circle and a uniformly accelerated motion along the radius. If v is the uniform velocity around the circle whose radius is r , then the centripetal *acceleration* is

$$a = \frac{v^2}{r},^1 \quad (15)$$

¹ Let ABC (Fig. 27) be the circle in which the body revolves, and AB the minute portion of the circular path described in a very small interval of time t . Denote the length of the arc AB by s . Then, since the motion along the arc is uniform, $s = vt$. AB is the diagonal of a very small parallelogram with sides AD and AE . The latter is the distance through which the revolving body is deflected toward the centre while traversing the *very small arc* AB . Since the acceleration is constant, $AE = \frac{1}{2}at^2$. The two triangles ABE and ABC are similar. Hence $AB^2 = AE \cdot AC$. Calling the radius of the circle r and substituting for AB , AE , and AC their values, $v^2t^2 = \frac{1}{2}at^2 \times 2r = at^2r$. Then $a = \frac{v^2}{r}$.

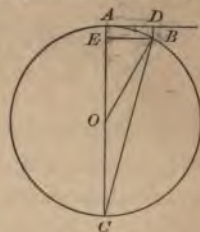


Fig. 27.

or the centripetal *acceleration* is equal to the square of the velocity along the circumference divided by the radius of the circle.

Since $F = ma$, equation (7), we have for either the centripetal or the centrifugal *force*,

$$F = \frac{mv^2}{r}. \quad (16)$$

If m is in grammes, v in centimetres per second, and r in centimetres, F is expressed in dynes. If it is desired to express F in gravitational units, divide the value obtained for F by g (§ 40). The result will be in grammes of force or pounds of force, according to the units employed.

To illustrate: If a mass of 20 gm. is attached to a string 1 m. long and is made to revolve with a velocity of 3 m. per second, the tension in the string may be found by applying equation (16). We have then

$$F = \frac{20 \times 300^2}{100} = 18,000 \text{ dynes; and } \frac{18,000}{980} = 18.37 \text{ gm.}$$

of force.

Again, if a body having a mass of 10 lb. move in a circle of 5 ft. radius with a velocity of 20 ft. per second, then,

$$F = \frac{10 \times 20^2}{5 \times 32.15} = 24.88 \text{ lb. of force.}$$

67. Illustrations of Centrifugal Force. — The occasional bursting of grindstones and flywheels when run at high speed illustrates the increase of centrifugal force with velocity. When the centripetal force becomes insufficient to hold the body to the centre, it flies off along a tangent line. A stone is thrown by whirling in a sling and finally releasing one of the strings. The overturning

of a carriage in rounding a corner at too high speed is an instance of centrifugal force. Drying machines are now made on the centrifugal principle. Instead of drying clothes in a laundry by first squeezing them in a wringer and then letting the moisture evaporate in the air, the modern way is to place the wet clothes in a large cylinder with openings in the sides, and then to whirl it rapidly. Centrifugal force causes the water to pass out through the holes in the cylinder.

In the process of refining sugar, the sugar crystals are separated from the syrup by centrifugal machines. Honey is extracted from the comb in a similar way. New milk is an emulsion of fat and a heavier liquid. If allowed to stand, the little fat globules rise slowly to the top and form the cream. But if the fresh milk be whirled in a dairy separator, the cream and milk will form distinct layers, and they may then be collected in separate chambers. In a Watt's steam engine governor, the balls open out by centrifugal force when the speed increases, and this motion is made to control the supply of steam.

The figure of the earth is an oblate spheroid, flattened at the poles. This flattening was doubtless caused by the centrifugal force of rotation when the earth was still in a liquid or at least a plastic condition, before cooling down to its present state.

68. The Simple Pendulum.—A small lead ball, suspended by a long silk string without sensible mass, represents a *simple pendulum*. The length of the pendulum may be taken as the distance from the point of support to the centre of the ball. When at rest the string hangs vertically like a plumb-line; but if the ball be pulled to one side and let go, it will swing to and fro, or oscillate,

about its position of rest, each swing being "damped" a little by the air. Its excursions on either side become gradually smaller; but if the time of the swing be carefully noted, it will be found to remain unchanged, if the arc described be small. This feature of pendular motion first attracted the attention of Galileo, who observed it in the oscillations of a "lamp" or bronze chandelier, suspended by a long rope from the roof of the cathedral in Pisa. This "lamp" may still be seen in the same place. Galileo noticed the even time of the oscillations as the path of the swinging chandelier became shorter and shorter. Such a motion, which continues to repeat itself in equal time-intervals, is said to be *periodic*.

69. Pendular Motion Explained. — *A* (Fig. 28) is the point of suspension, and *AN* is the length of the simple pendulum. Let the ball be drawn aside to the position



Fig. 28.

B, and released. Let *BG* represent the *weight* of the ball. Resolve this force into *BD* in the direction of the string, and *BC* at right angles to it, or tangent to the arc *BNE*. Only the latter component is effective in producing motion of the ball toward *N*. As the ball approaches *N*, the component *BC* becomes smaller and vanishes at *N*. (The student should convince himself of this by drawing a figure with *B* nearer *N*.)

In falling from *B* to *N*, the ball moves in the arc of a circle under the influence of a force which is greatest at *B* and vanishes at *N*. The motion is,

therefore, accelerated all the way from B to N , but the acceleration is not uniform. The velocity increases continuously from B to N , but at a decreasing rate. The ball passes through N with its greatest velocity, and continues on toward E . From N to E the component of the force of gravity along the tangent, which is always directed toward N , is opposite to the motion, or the acceleration is negative. Hence the pendulum is brought to rest again at E . It then retraces its path, and continues to oscillate with a periodic or pendular motion.

70. Definition of Terms. — A *single vibration* is the motion from N to either B or E , and back again to N ; a *complete* or *double vibration* is the motion from N to B , across to E , and then back again to N , the motion at the end of the complete vibration being in the same direction as at its beginning. The *period* of a complete oscillation is the time consumed in making a complete vibration, or the time-interval between two successive passages of the pendulum through N in the same direction. The *period* of a *single vibration* is half that of a double vibration. The *amplitude* is the arc BN .

71. Laws of the Pendulum. — When the amplitude does not much exceed three degrees, the period of the vibration depends very approximately on the length of the pendulum and the acceleration of gravity. For a single vibration the formula is

$$t = \pi \sqrt{\frac{l}{g}} \quad (17)$$

In this formula, l is the length of the pendulum and π is the ratio of the circumference of a circle to the diameter, 3.1416. This constant is evidently the "proportionality

factor" of the equation. The following are the laws of a simple pendulum:—

I. *The period of vibration is independent of the amplitude, if the latter is small.*

II. *The period of vibration is proportional to the square root of the length.*

III. *The period of vibration is inversely proportional to the square root of the acceleration of gravity.*

72. Experimental Illustrations.—The laws of the pendulum may be illustrated in the following manner:—

Experiment.—Suspend three small lead balls by fine silk threads as shown in Fig. 29. Make the lengths of the pendulums, to the centre of the ball in each case, 1 m., $\frac{1}{2}$ m., and $\frac{1}{4}$ m., respectively. Find the period of a single vibration for each pendulum by counting the number made in, say, 20 sec. These periods will be 1 sec., $\frac{1}{2}$ sec., and $\frac{1}{4}$ sec. nearly, showing that they are directly proportional to the square root of the lengths.



Fig. 29.

Experiment.—Again, make a pendulum by suspending an iron ball over a strong permanent horseshoe magnet, so that the ball will just fail to touch the poles of the magnet as it swings over them. Determine the period of vibration, first, with the magnet in position, and second, after removing it. The period with the magnet in place under the ball will be perceptibly shorter than without it. The attraction of the magnet is equivalent to an increase in the intensity of gravity. The relative intensity of gravity may therefore be measured by observing the period of vibration of the same pendulum at different places.

A pendulum will oscillate more slowly on the top of a high mountain than at sea level, and more slowly at the equator than at the poles.

73. A Seconds Pendulum is one making a single vibration in one second. Its length for the latitude of New York

be computed by making t equal to unity, and g , 980.19 in the formula $t = \pi\sqrt{\frac{l}{g}}$, and solving for l . (To be done by pupil.) Since g increases from the equator toward the pole, it follows that the length l of a pendulum to seconds increases in the same ratio as g .

The Compound Pendulum. — Any body suspended so as to oscillate about a horizontal axis is a *compound or physical pendulum*. Every actual pendulum is really a compound pendulum. A simple pendulum is an ideal one about which length there is no ambiguity. The equivalent length of a compound pendulum, to be used in the formula for the period, is not obvious.

Centres of Suspension and Oscillation. — Let AB (Fig. 30) be a bar suspended so as to have freedom of motion about a horizontal line through C . Then C is the *centre of suspension*. Let the centre of the mass be at G . This compound pendulum has a period of oscillation equal to that of some ideal simple pendulum, oscillating about the same axis, and with the whole mass collected at a point. Suppose that point to be at D . Then the distance CD is the *equivalent length* l of the simple pendulum which will oscillate in the same time as the physical bar; and point D on the line CG produced, is called the *centre of oscillation*. The length l of the compound pendulum is therefore the distance between the centres of suspension and oscillation.

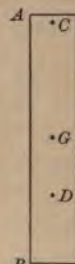


Fig. 30.

The centre of oscillation is also called the *centre of percussion*, because, if the suspended body be struck at this point in a direction at right angles to the axis of suspension, it will be set swinging without any jar. A base-

ball club, or a cricket bat, has a centre of percussion, and it should strike the ball at this point to avoid breaking the bat and jarring the hands.

Experiment. — Hold a thin wood strip a metre long by the thumb and forefinger near one end. Strike the flat side with a soft mallet at different points. One point may be found where the blow will not throw the wood strip into shivers, but will only set it swinging like a pendulum.

76. The Two Centres Interchangeable. — Huyghens, a celebrated Dutch physicist, discovered that the centres of suspension and oscillation are interchangeable, that is, that the period of vibration is the same whether the pendulum swings about the one as an axis or the other. This discovery led to the invention by Captain Kater of a pendulum with two parallel axes of suspension, and with adjustable weights which can be moved till the pendulum will swing in the same time about the two axes. The distance between them is then the length l .

77. Utility of the Pendulum. — The discovery of Galileo suggested a most obvious use of the pendulum as a time-keeper. The common clock is an instrument in which the to-and-fro motion of the pendulum regulates the rotary motion of the hands. The train of wheels is kept in motion by a weight or a spring, and the regulation is effected through an escapement (Fig. 31). One tooth of the escapement wheel escapes from the pallet with every double vibration, but the movement of the wheel round its axis is about equally divided between the two halves of the double vibration. The pendulum controls the escapement, and it receives in turn small impulses which keep it swinging against friction and opposition of the

air. The pendulum itself is supported by a thin flexible spring at the top.

The vibration period is affected by any change in the length of the pendulum. To secure a uniform rate this length must be kept invariable; and corrections must be made for changes in length due to changes of temperature. The common clock, with an uncompensated pendulum, loses time in hot weather and gains in cold. A correction may be made by raising or lowering the bob by means of a running nut shown at the bottom of the figure.



Fig. 31.

Astronomical clocks and clocks for precise physical measurements have compensated pendulums, which adjust themselves automatically when the temperature changes. The mercurial pendulum, commonly used for this purpose, has in it a mass of mercury, which expands upward while the pendulum rod expands downward. Compensation is thus effected. In one form the mercury in glass tubes forms the pendulum bob; in another the bob is lens-shaped, and the mercury partly fills a steel tube carrying the bob, similar to the one shown in the figure. For perfect compensation, the height of the mercury in the tube is adjusted for the latitude of the place and the height above sea level.

Problems.

In the following problems, unless otherwise stated in the problem, g is to be taken as 980 cm. or 32 ft. per second per second.

1. If a building were 100 ft. high, with what velocity would a stone dropped from the top strike the ground?

80 ft per sec

2. If a stone thrown upward returns to the ground in 4 sec., how high does it ascend?

3. A rifle ball is shot vertically upward with a velocity of 1600 ft. per second. In what time will it reach the ground, neglecting the resistance of the air?

4. How far must a ball fall in order to acquire a velocity of 98 m. per second?

5. A cannon ball is fired horizontally from the top of a cliff 200 m. high. In how many seconds will it strike the plain at the foot of the cliff?

6. A stone dropped from a bridge strikes the water in 3 sec. What is the height of the bridge?

7. What vertical velocity must be given to a stone that it may just pass over a flag-pole 50 m. high?

8. A brick falls from the top of a chimney 200 ft. high. In what time will it reach the ground?

9. A body sliding down a frictionless inclined plane passes over 10 ft. the first second. What is its acceleration, and how far will it travel in 3 sec.?

10. An iron ball is let fall from the top of the leaning tower of Pisa, 288 ft. high. One second later a second iron ball is dropped. How far will they be apart when the first ball reaches the ground?

[Find the time it takes the first ball to reach the ground. Then find how far the second ball has fallen when the first one has reached the ground.]

11. A mass of 100 gm. is revolving uniformly, once in 2 seconds, in a circle whose radius is 1 m. Find the centripetal force in dynes.

12. A mass of 50 gm. is connected to a fixed point by a string 2 m. long, and is whirled around in a circle once in 3 sec. Find the tension in the string in dynes; also in grammes of force.

13. A mass of 3 kgm. suspended by a string swings in a circle making the string describe a cone. The radius of the circular path is 1 m., and the ball makes 20 complete revolutions in 30 sec. Calculate the centrifugal force in dynes; also in grammes of force.

14. A locomotive whose mass is 24 tons passes around a circular curve whose radius is half a mile, with a velocity of 30 mi. an hour. What is the centrifugal force? *1150 lb of force 35200 ft*

15. If a pendulum is 98 cm. long, what will be the period of a vibration when $g = 980$ cm. per second per second? *abt 1 sec 0.99*

16. What is the length of a seconds pendulum at Ann Arbor, Mich., the value of g being 980.3 cm. per second per second? *99.3 cm*

17. If a pendulum makes 35 vibrations in 42 sec., and is 143 cm. long, what is the value of g ? *980-1000*

18. If a seconds pendulum be taken as 39 in. long, what would be the length of a pendulum that would make 70 vibrations per minute? *21*

19. If g is 980 cm. per second per second, how long must a pendulum be to swing once in 5 sec.? *2482 cm*

20. The length of a seconds pendulum at the equator is 99.1 cm. What is the acceleration due to gravity? *999.1 cm*

21. The length of a seconds pendulum at Washington is 99.304 cm. and at Greenwich it is 99.416 cm. What is the ratio of the acceleration due to gravity at these places? *.999*

22. The force of gravity on Jupiter is 2.65 times greater than on the earth. Will the Jovian seconds pendulum be longer or shorter than the earth's, and how many times? *2.65 times*

23. Foucault used a pendulum 220 ft. long in his experiment at Paris to show the rotation of the earth on its axis. Calculate the time of a vibration, the value of g being 32.2 ft. per second per second. *8.2*

24. A metre stick of uniform cross-section is pivoted at one end so as to swing as a pendulum. The centre of oscillation of such a bar is two-thirds its length from the pivoted end. Calculate its time of vibration, when g is 980 cm. per second per second. *.82 sec*

VI. WORK AND ENERGY.

78. **Work.**—Whenever an agent exerting a force produces any effect on a body, and the point of application moves in the direction of the force, then the agent exerts

ing the force is said to *do mechanical work*. For example, steam exerts pressure on the piston in the cylinder of an engine, causes it to move, and does work. Gravity does work on the weight of a pile driver, causing it to descend; a horse does work in pulling a wagon up an inclined roadway; the electric current, by means of a motor, does work when it drives a pump and forces water up into the tank of a water tower.

Unless the point of application moves in the direction of the force, no work is done, however great the force may be; the pillars supporting a pediment over a portico *do* no work, though manifestly they support a great weight and exert force.

Work is the production of an effect in bodies by means of a force whose point of application moves in the direction of its own line of action, and it is measured by the product of the force and the distance moved, or

$$W = Fs. \quad (18)$$

79. Units of Work. — The three units of work in common use are: —

1. The *foot-pound*, or the work done by a force of one pound working through a distance of one foot. This unit is the one still used by English-speaking engineers. It is open to the objection that it is variable, on account of the variation in the weight of a pound with the latitude (§ 54).

2. The *kilogramme-metre*, or the work done by a force of one kilogramme working through a distance of one metre. This is the gravitational unit of work in the metric system; it is open to the same objection as the foot-pound.

3. The *erg*, or the work done by a force of one dyne working through a distance of one centimetre. The erg is the absolute unit in the C.G.S. system and is invariable.

Gravity gives to a gramme in a second a velocity of about 980 cm. a second. It is therefore equal to 980 dynes. If, then, a gramme be lifted vertically one centimeter, the work done against gravity is one gramme-centimetre, or 980 ergs.

A silver dollar weighs about 26.73 gm., and the height of an ordinary table is about 76 cm. The work done in lifting a silver dollar from the floor to the top of a table is then the continued product of 26.73, 76, and 980, or 2,000,000 ergs nearly.

The erg is, therefore, an excessively small unit, and it is more convenient to use a multiple for practical measurements. The multiple commonly employed is the *joule*, which is equal to 10^7 , or 10,000,000, ergs. Expressed in this larger unit the work done in lifting the silver dollar is 0.2 joule.

80. Time not an Element in Work. — It necessarily takes time to do work, but the *amount of work done* has nothing whatever to do with the time taken to do it. To lift the silver dollar from the floor to the table top requires the expenditure of 2 million ergs of work, whatever the time consumed in lifting it. If a man weighing 150 lb. walks up the nine hundred steps leading to the highest attainable level in the Washington monument, 500 ft. high, he does work against gravity equal to 75,000 foot-pounds, irrespective of the time taken in the ascent.

81. Power. — It is frequently necessary to take into account the time an agent takes to do a certain quantity of work. Then the work done in a given time, divided by the time, is called *power* or *activity*. *Power is the time-rate of doing work.*

In the English gravitational system, the unit of power is the horse-power (H.P.); it is the rate of doing work equal to 33,000 foot-pounds a minute, or 550 foot-pounds a second.

In the C.G.S. system the unit of power is the *watt*. It equals work done at the rate of one joule (10^7 ergs) a second. One horse-power is equivalent to 746 watts. A kilowatt (K.W.) is 1000 watts. It is therefore very nearly $\frac{3}{4}$ horse-power. To convert kilowatts into horse-power, add one-third; to convert horse-power into kilowatts, subtract one-fourth. For example, 60 K.W. equals 80 H.P., and 100 H.P. equals 75 K.W.

82. Energy. — In general, a body upon which work has been done is found to have an increased power of doing work itself. It is then said to possess more *energy* than before. The increase of energy acquired by the body is the most essential part of the effect produced when work is done on it. When work is done on a quantity of water by lifting it to a high level, its energy is increased because it is capable of doing work by flowing down again through a water motor. The winding of an eight-day clock lifts its weight against gravity, and the clock thereby acquires enough energy to keep it running against resistance for an entire week. The rate of putting energy into it is very large compared to the rate at which the clock does it out.

Energy is, then, the capacity for doing work.

83. Potential Energy. — The energy or capacity to do work, possessed by a lifted weight or by a coiled spring, or in general the energy which a body has by virtue of its position relative to some other body, or the relative position of its parts, is called *potential energy*. In bending a

bow work is done in distorting it, or placing it under stress; and it then possesses potential energy due to its altered shape, for it can do work on an arrow and give it rapid motion. Potential energy is often called *energy of position* or *energy of stress*.

The potential energy of the weight of a pile-driver is the work done against gravity in lifting it. The measure of this energy is then

$$E = Fh, \quad (19)$$

where F is the lifting force and h the height through which the weight is lifted.

84. Kinetic Energy. — A moving body has the capacity of giving motion to another body. It then possesses energy. The energy which a body has by virtue of its motion is called *kinetic energy*. Work is done on a cannon ball by the agency of the gases due to the explosion of the powder. The ball acquires high speed and something more than that; for it then has the capacity of overcoming resistance. This moving mass may imbed itself in earthworks, demolish fortifications, or pierce the nickel-steel armor of a battleship. The energy which the ball acquires from the explosion is *kinetic energy*, or energy of motion. Motion is the essential fact in a body possessing kinetic energy.

85. Measure of Kinetic Energy. — Not only is energy a measurable quantity, but it is measured in terms of the same units as those used in measuring work.

Let a body of mass m , moving with a velocity v , be acted on by a constant force F in a direction opposing the motion; and let it be brought to rest after it has passed over the distance s . Then the work done by the moving

body against the force F , before it is brought to rest, is Fs (equation 18). But $F = ma$ (equation 7), and $s = \frac{v^2}{2a}$ (§ 34), since the body loses velocity v in a space s .

$$\text{Therefore} \quad E = ma \times \frac{v^2}{2a} = \frac{1}{2} mv^2. \quad (20)$$

The measure of kinetic energy in terms of the mass and velocity of the moving body is therefore half the product of the mass and the square of the velocity. If m is expressed in grammes and v in centimetres per second, the kinetic energy is in ergs. In the English gravitational system

$$E = \frac{mv^2}{2g} = \frac{mv^2}{64.3}, \quad (21)$$

where, if m is expressed in pounds, and v in feet per second, E is in foot-pounds.

In the latitude of New York ergs can be reduced to kilogramme-metres by dividing by 98,000,000 (§ 79).

86. Transformations of Energy.—If a ball be thrown vertically upward, it gradually loses its motion and its kinetic energy, but it gains energy of position. When it reaches the highest point its energy is all potential. It then descends, and again acquires energy of motion at the expense of energy of position. The one form of energy is therefore convertible into the other.

The pendulum illustrates the same principle. While it is moving from the lowest point of its path toward either extremity, its kinetic energy is converted into potential energy; and the reverse transformation sets in when the pendulum reverses its motion. All physical processes involve energy changes, and such changes are in ceaseless progress. A machine is only an instrument or device

for the transformation of energy and the turning of it to useful account. A watch when wound has a small store of potential energy which it expends very slowly in the work of turning the train of wheels against friction and the resistance of the air, and producing the sound of ticking. In a week a watch distributes the energy of winding it seven times among over 3,000,000 ticks.

Potential energy is the highly available or useful form. It always tends to revert to the kinetic form, but in such a way that only a portion of the kinetic energy is available to effect useful changes in nature or art. The remainder goes into useless heat. The energy of the solar system is therefore becoming all the time less and less available. Strictly, the capacity which a body possesses for doing work does not depend on the total quantity of energy which it may possess, but only on that portion which is *available*, or is capable of being transferred to other bodies. We have to deal chiefly with the variations of energy in a body, and not with its total value. In subsequent sections we shall have occasion to consider many other forms of energy than those already mentioned, and we shall find that they are all mutually convertible the one into the other.

87. Conservation of Energy. — The question arises, when work has been done on a body and energy communicated to it, has the energy been made out of nothing, or has it been merely transferred? The answer of science is that the latter is the truth. Innumerable facts and experiments show that it is as impossible to create *energy* as to create *matter*.

Whenever energy appears as the result of work done on a body or system, it is always at the expense of energy in some other body or system.

The agent, or body which does work, always loses energy; the body which has the work done on it gains the same amount. On the whole, there is neither gain nor loss of energy, but only transference from one body to the other.

The law of *Conservation of Energy* means that no energy is created or destroyed by the action of forces that we know anything about.

88. Matter and Energy. — "All that we know about matter relates to the series of phenomena in which energy is transferred from one portion of matter to another till in some part of the series our bodies are affected, and we become conscious of a sensation. We are acquainted with matter only as that which may have energy communicated to it from other matter. Energy, on the other hand, we know only as that which in all natural phenomena is continually passing from one portion of matter to another. It cannot exist except in connection with matter."¹

Problems.

1. A man weighing 150 lb. climbs to the top of the Eiffel Tower, height 984 ft. How many foot-pounds of work does he do? *147,600*
[The man lifts himself against gravity and hence exerts a force numerically equal to his weight.]

2. A stone weighing 50 kgm. was lifted to the top of a building 20 m. high. Calculate the amount of work done. *1000 kg m.*

3. How many ergs of work are done in raising a mass of 20 kgm. vertically through 10 m.? *196 x 10⁸ ergs*

4. An unbalanced force of 10 kgm. moves a mass of 100 kgm. through the distance 100 m. How much work is done? If the same force moves a mass of 200 kgm. a distance of 100 m., how much work is done? *1000 kg m.*

¹ Maxwell's *Matter and Motion*, p. 163.

5. By means of a force whose magnitude is 1000 dynes, a mass is moved through a distance of 250 m. How many joules of work are done? *2.5 joules*

6. A man was employed to carry 2 tons of coal to the third floor of a building, the height being 30 ft. By accomplishing the task in 4 hr., how much work does he do per minute? *500 ft lbs*

7. How much work is done against gravity in hauling a load of one-half of a ton to the top of a hill 200 ft. high? The hill is found to be 2000 ft. long. What force against gravity is necessary to pull the load up the hill? *100 lbs force*

8. At what rate is an engine working which raises 1000 tons of coal in 10 hr. from a mine 300 ft. deep? *30.3 HP*

9. A steam pump can fill a tank with water in 4 hr. The capacity of the tank is 5000 gal. and the elevation is 40 ft. If a gallon of water weighs 8 lb., what is the horse-power of the pump? *202 HP*

10. How much coal can a 20 H.P. engine raise in 10 hr. to the mouth of a mine 400 ft. deep? *495 tons*

11. A tank whose capacity is 10,000 gal. is placed on a platform 66 ft. above the level of the water of a near-by lake. How long will it take a 2 H.P. pump to fill it with water? Assume a gallon of water to weigh 8 lb. *50 min*

12. An electric motor raises an elevator cage whose unbalanced weight is 2000 kgm. through a height of 40 m. in 10 sec. How much work, expressed in joules, does the motor do? What power, expressed in kilowatts, does the motor exhibit? *784 $\times 10^3$ joules*

13. Express in kilowatts the activity of an agent that can raise 1500 kgm. to a height of 20 m. in 15 sec. *19.6 Kw work*

14. What is the horse-power transmitted by a rope passing over a wheel 30 ft. in circumference which makes one revolution per second, the tension in the rope being 100 lb.? *5.454 HP*

15. The area of the piston of an engine is 10 sq. in., and the mean pressure on the piston is 100 lb. of force per square inch. What is the total force moving the piston? If the length of the stroke is 20 in., and 240 revolutions are made by the fly-wheel per minute, through how many feet per minute does the piston travel? How much work per minute is done on the piston by the force? What is the horse-power of the engine?

16. A resistance of 1000 lb. of force is overcome by an engine which is moving a train at the rate of 30 mi. an hour. How much work is done per second? What H.P. has the engine? $4 \sim 576 \text{ F?}$
 50 W?

17. A stone weighing 2000 lb. rests on the edge of a vertical cliff 150 ft. high. How much energy is stored in it? What would be the energy of the blow with which it would strike the base of the cliff? How much work would an agent do in replacing the stone on the top of the cliff?

18. A ball whose mass is 20 gm. is thrown with a velocity of 100 cm. per second. How much energy in ergs is imparted to it?

19. What is the energy in ergs of a cannon ball whose mass is 10 kgm. when moving with a velocity of 500 m. per second? $25 \sim 10$

20. Compare the penetrating power of two similar balls, one having a mass of 10 kgm. and a velocity of 500 m. per second, the other having a mass of 5 kgm. and a velocity of 60 m. per minute. Also compare their momenta. 300 11

21. A force of 1 kgm. acts for 10 sec. on a free mass of 10 kgm. How much kinetic energy will the body possess? $10 \text{ 10} \text{ 10}$
[Express the force in dynes; then find velocity by equation (6).]

22. How much work is done in stopping a 5-kgm. ball moving with a velocity of 20 m. per second? Express the result in joules. 100 10

23. A bicyclist weighing 175 lb., mounted on a wheel weighing 25 lb., while riding at the rate of 15 mi. an hour collides with a tree. What is the energy of the blow? [Consult § 85.] 10 10

24. The Great Pyramid is 450 ft. high. How much energy was expended in placing at the top of this pyramid a stone weighing 30 tons?

25. A party of boys are coasting on a hill 100 ft. high. The sled and the load represent a mass of 600 lb. Neglecting all loss by friction, what will be the energy of the moving load on reaching the foot of the hill?

[Find the potential energy of the load.]

26. What is the energy in foot-pounds of a locomotive weighing 100 tons (of 2000 lb.) moving with a velocity of ten-elevenths of a mile a minute? [Consult § 85. Take $g = 32 \text{ ft. per sec. per sec.}$] 10 10

27. A mass of 100 gm. moving with a velocity of 50 cm. per second meets with a constant resistance of 500 dynes. How far will it move before coming to rest? *250 cm*

28. A train of 150 metric tons is running at the rate of 36 km. an hour. How much energy does it possess? A metric ton is 1000 kgm. *75 × 10*

29. A 2-kgm. cannon ball is discharged with a velocity of 500 m. per second. How much work does the force of the explosion do on the ball? If the barrel of the gun is 2 m. long, what is the explosive force of the powder in kilogrammes of force? *25 × 10⁵ dynes = 12760*

[Note the distance through which the force acts and the energy which it must impart to the ball.]

VII. MACHINES.

89. **A Machine** is a device designed to transform or transfer energy, and to do useful work. An electric lighting and power plant illustrates both features of a useful machine or collection of machines. The heat energy of the steam actuates the moving parts of the engine, and is thence transferred mechanically to the armature of the dynamo, where it is converted into the energy of an electric current.

Simple machines, or mechanical powers, are restricted to devices for merely transferring energy. They are six in number, the *lever, pulley, wheel and axle, inclined plane, wedge, and screw*. Since it will appear from what follows that the pulley and the wheel and axle are only modified levers, and the screw and wedge are modified inclined planes, it is therefore possible to reduce the six simple machines to two, the lever and the inclined plane. All complex machines are mechanically only combinations of two or more simple machines.

90. **Mechanical Advantage.** — The effective force exerted by the agent losing energy, and the force exerted by the

body receiving energy in a simple machine, may be denoted by two terms introduced by Rankine, namely, *effort* and *resistance*. The problem in simple machines consists in finding the ratio of the resistance to the effort, and this ratio is known as the *mechanical advantage*. In elementary discussions it is customary to neglect friction and to assume that the parts of a machine are rigid and without weight.

91. General Law of Machines.—Every machine must conform to the principle of the conservation of energy, or *the work done by the effort must equal the work done in overcoming the resistance*, except that some energy may be dissipated as heat or may not appear in a mechanical form. A machine can never produce an increase in the quantity of energy.

Denote the effort by F and the resistance by R , and let d and D denote the distances through which they act, respectively. Then we have, from the law of conservation,

$$Fd = RD, \quad (22)$$

or the effort multiplied by the distance through which it acts is equal to the resistance multiplied by the distance moved against it.

92. Efficiency.—If a machine could be made that would waste no energy, that is, one in which the resistance is all *useful* and not *wasteful*, the machine would be perfect and its efficiency would be unity. But in practice there is always some wasteful resistance due to friction (§ 107), rigidity of cords, etc. The work done is therefore always partly *useful* and partly *wasteful*. The efficiency of a

ine is the ratio of the useful work done by it to the work done on it. Efficiency is always, therefore, a fraction, and it is expressed as a percentage. An efficiency of 90 per cent means that the energy recovered is 90 per cent of the energy put into the machine. A machine which will do either useful or useless work continuously, without a supply of energy from without, is clearly impossible.

A **Lever** is a rigid bar, straight or curved, turning about a fixed axis called the *fulcrum*. The perpendicular distances between the fulcrum and the lines of action of the effort and the resistance are called the *arms* of the lever. A *straight* lever has the fulcrum and the arms in the same straight line.

If the fulcrum is between the points of application of the effort and the resistance, the lever is of the *first class* (Fig. 32); if the resistance is between the effort

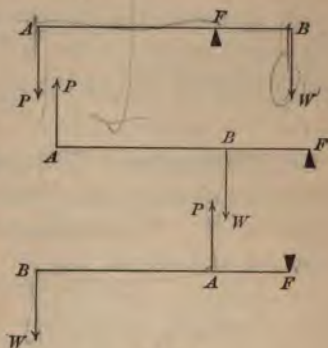


Fig. 32.

the fulcrum, the lever is of the *second class*; if the effort is between the resistance and the fulcrum, the lever is of the *third class*.

Mechanical Advantage of the Lever.—In the three classes of Fig. 32, the arms of the straight lever are AF and BF . Let P be the effort and W the resistance, the weight lifted. Then if the lever is weightless and without friction, the moment of the effort about the

fulcrum F is equal to that of the resistance or weight about the same point, as the condition for equilibrium (§ 47 *a*). The two forces tend to rotate the lever in opposite directions. Hence, $P \times AF = W \times BF$, or

$$\frac{W}{P} = \frac{AF}{BF}$$

Therefore, *the mechanical advantage of the lever equals the inverse ratio of its arms.*

If it is desired to take into account the weight of the lever, then the moment of this weight, considered as acting at the centre of gravity of the lever, must be added to the moment of either the effort or the resistance, according as the weight of the lever acts to produce rotation in the same direction as the one or the other.

95. Illustrations. — The *common balance* (Fig. 33) is a lever of the first class with equal arms. P is therefore equal to W . For accuracy the two arms of the beam must be strictly of the same length; and for high sensibility, the friction must be small, the beam light, and the centre of gravity only slightly below the “knife-edge” forming the fulcrum. If the arms are not of equal length, the true weight of a body may be found by weighing it first in one scale pan and then in the other, and taking the square root of the product of these apparent weights.



Fig. 33.

The *steelyard* (Fig. 34) is a lever of the first class with unequal arms. Scissors are double levers of the first class. A crowbar, used to lift a weight with one end on the ground, is a lever of the second class. Nutcrackers are



Fig. 34.

double levers of the same class. When a weight is held in the palm of the hand, the forearm acts as a lever of the third class; for the fulcrum is at the elbow and the effort is applied through the tension of the muscles which are attached between the elbow and the hand.

96. The Wheel and Axle (Fig. 35) consists of a cylinder and a wheel of larger diameter turning together on the same axle. In the figure the axle passes through C , the radius of the cylinder is BC and that of the wheel is AC . If the weights P and W are suspended by ropes wrapped around the circumferences of the two wheels, the moment of P in one direction about C is equal to that of W in the other, or

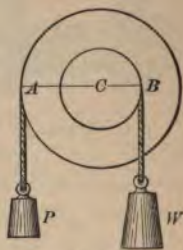


Fig. 35.

$$P \times AC = W \times BC. \quad \text{Then } \frac{W}{P} = \frac{AC}{BC} = \frac{R}{r},$$

where R and r are the radii of the wheel and the axle respectively. The weight P may represent the effort applied at the circumference of the wheel, and the weight W the resistance at the circumference of the axle. Hence, *the mechanical advantage is the ratio of the radius of the wheel to the radius of the axle.*

97. Applications.—The *derrick* is a form of wheel and axle much used for raising heavy weights. The essential parts are shown in Fig. 36. This may be looked upon as a double wheel and axle. The axle of the first system works upon the wheel of the second by means of the spur

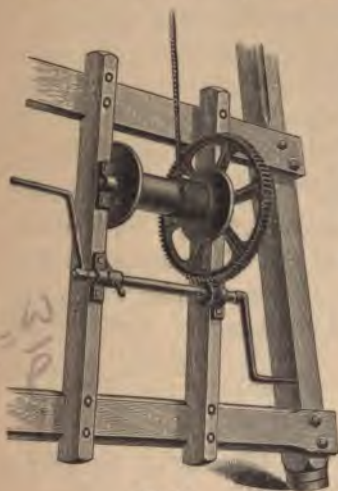


Fig. 36.

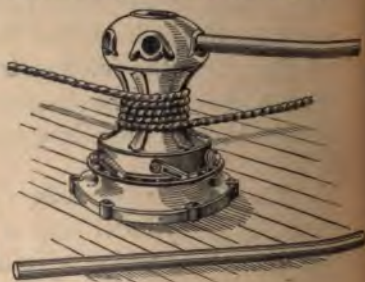


Fig. 37.

gears. The cranks or handles of the first system answer the same purpose as a wheel. The mechanical advantage in this case is the ratio of the product of the radii of the wheels to the product of the radii of the axles.

In the *capstan* (Fig. 37) handspikes inserted in the holes at the top are used instead of a wheel; while the rope, on which the work is done, is wrapped around the body of the capstan as an axle.

98. The Pulley is a wheel, called the *sheaf*, free to turn out an axle in a framework called a *block*. The effort and the resistance or weight are attached to a rope which lies in a groove cut in the circumference of the wheel. A simple fixed pulley is used to change the direction of the motion produced by a force, and the effort and resistance are equal to each other; for, neglecting friction and the stiffness of the rope, the tension throughout the rope is the same.



Fig. 38.

In the movable pulley (Fig. 38) a weight may apparently be supported by a force half as great as itself; but the other half of the force is supplied by the fixed



Fig. 39.

hook to which the cord is attached. If the weight is lifted it rises only half as fast as the free end of the cord travels. The mechanical advantage of a simple fixed pulley is thus one, and of a simple movable pulley two.

99. Systems of Fixed and Movable Pulleys.

—The most useful combination of pulleys consists of two blocks, each with several sheafs which usually turn on the same axle. One of these blocks is attached to a fixed point, while to the other is attached the resistance or weight (Fig. 39). This is the common “block and tackle.”

100. Mechanical Advantage of the Pulley.

—The principle involved in determining

the mechanical advantage of the pulley is the transmission of the same tension to all parts of the cord or rope. The only purposes served by the wheels of the pulley are to diminish friction and to change the direction of the effort.

When the cord passes in succession around each pulley, as in Fig. 39, it is obvious that the weight is sustained by the several parts of the cord, the tension in each part being P , the effort applied at the free end. If there are n parts to the cord, the total tension supporting the weight W will be nP ; that is, $W = nP$ and $\frac{W}{P} = n$.

The mechanical advantage of a system of pulleys, when a single cord is used, is therefore equal to the number of times the cord passes to and from the movable block.

101. The Inclined Plane. — Suppose a body rests on an inclined plane without friction. The weight of the body acts vertically downward, while the reaction of the inclined plane is perpendicular to its surface; so that to maintain the body in equilibrium on the incline, a third force must be applied. Two principal cases occur; first, when the force applied is parallel to the *face* of the plane; second, when it is parallel to the *base* of the plane.

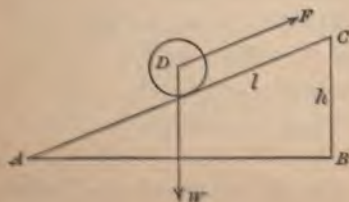


Fig. 40.

102. Mechanical Advantage of the Inclined Plane. —

Case I: When the force is applied parallel to the face of the plane. The most convenient method of obtaining the relation between

the force F (Fig. 40) and the weight W is to apply the principle of work. Suppose D to move under the influence of the force F from A to C . Then the work done by F is $F \times AC$. The work done on the body D , of weight W , is at the same time $W \times BC$, since W is lifted through a vertical distance, BC . Therefore, $F \times AC = W \times BC$, or

$$\frac{W}{F} = \frac{AC}{BC} = \frac{l}{h},$$

or the mechanical advantage, when the force is applied parallel to the face of the plane, is the ratio of the length of the plane to its height.

Case II: When the force is applied parallel to the base of the plane. If, in this case, we estimate the work done by the force F in moving the body up the plane from A to C (Fig. 41), we must take the distance moved in the direction of the force. Now the displacement of the body in the direction of the force in going from A to C is not AC , but the base of the inclined plane AB . Therefore, the work done is $F \times AB$. The work done on the weight W is the same as in the first case. Hence, $F \times AB = W \times BC$, or

$$\frac{W}{F} = \frac{AB}{BC} = \frac{b}{h}.$$

The mechanical advantage, when the force is applied parallel to the base of the plane, is the ratio of the base of the plane to its height.

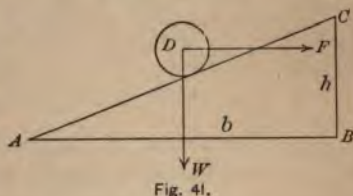


Fig. 41.

103. The Wedge (Fig. 42) is a double inclined plane, with the effort applied parallel to the base, so as to enlarge an opening or lift a weight. The effort is generally ap-

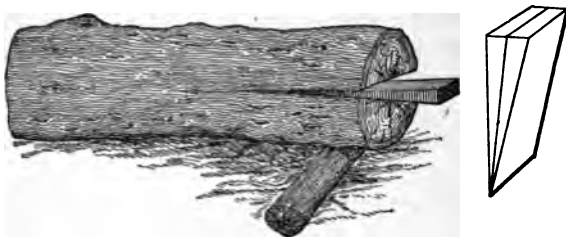


Fig. 42.

plied by a blow with a heavy body. Although the principle of the wedge is the same as that of an inclined plane, yet no exact statement of the mechanical advantage is possible, since the resistance has no definite relation to the faces of the plane, and friction cannot be neglected.

104. The Screw is a cylinder, on the outer surface of which is a uniform spiral projection called the *thread*. The faces of this thread are inclined planes, as may be

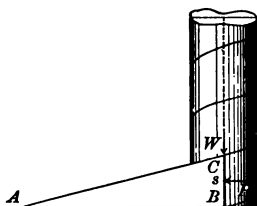


Fig. 43.

seen by wrapping a long triangular strip of paper around a rod like a pencil (Fig. 43). The base of the triangle is perpendicular to the axis of the cylinder, and the hypotenuse traces a spiral like the thread of a screw. The screw works in a block called the *nut*, on the inner surface of which is a groove. This groove is the exact counterpart of the thread (Fig. 44). The effort

is applied at the end of a lever, fitted either to the screw or to the nut. When either makes a complete turn, the screw or nut moves through a distance equal to that between two contiguous threads, measured parallel to the axis of the screw cylinder. This distance (s in Fig. 43) is called the *pitch* of the screw.



Fig. 44.

105. Mechanical Advantage of the Screw. — Since a screw is usually combined with the lever, the simplest method of finding the mechanical advantage is to apply the principle of work expressed in the general law of machines (§ 91). If the pitch be denoted by s , and the lever arm by l , then when P makes a complete revolution, the work done is $P \times 2\pi l = Ws$, or $\frac{W}{P} = \frac{2\pi l}{s}$; that is, *the mechanical advantage of the screw equals the ratio of the distance traversed by the effort in one revolution to the pitch of the screw.*



Fig. 45.

106. Applications. — Familiar uses of the screw are illustrated by the lifting jack (Fig. 45), copying and book presses, cotton and hay presses, the screw propeller of ships, and air fans. In most cases it depends for its efficiency on friction, as in holding together the parts of machinery and woodwork.

The screw is used also for the purpose of measuring small dimensions, as in the *wire micrometer*

(Fig. 46). An accurate screw, *C*, has its head *D* divided into a number of equal parts, so as to register any portion

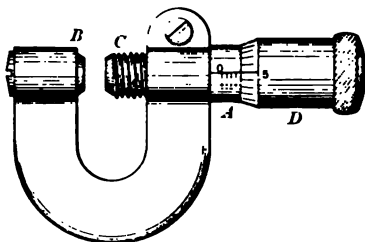


Fig. 46.

of a complete revolution. If, for example, the pitch of the screw is $\frac{1}{2}$ mm., and the head is divided into 25 equal parts, then for each revolution the end of the screw advances $\frac{1}{2}$ mm.; and if the head of the screw be turned through one of the 25 divisions,

the end of the screw will advance $\frac{1}{25}$ of $\frac{1}{2}$, or $\frac{1}{50}$ mm.

107. Friction. — The *resistance* which opposes an effort to slide or roll one body over another is called *friction*. Friction is called into action only when a force is applied to make one surface move over another, and it always resists this motion. It arises from inequalities in the surfaces, and is diminished by polishing the surfaces in contact and by the use of a lubricant. The amount of friction is proportional to the pressure of the one surface on the other; it has a limiting value, which is its value just before motion takes place.

108. Limiting Angle. — A body may rest on an inclined plane without sliding, the force to maintain equilibrium being supplied by friction. For every pair of substances there is an angle of elevation, which is the greatest at which a plane made of one substance can be inclined to the horizontal before a body made of the other will start to slide on it. This angle is called the *limiting angle of friction*, or the *angle of repose*.

109. Sliding Friction.—Friction continues after the sliding motion has begun, and opposes the motion; but its magnitude in general is less than the friction of rest at the moment the slipping begins.

The friction is independent of the area of the surfaces in contact if the pressure remains the same. The fraction of the pressure which must be applied as a force to maintain uniform motion against friction is called the *coefficient of friction*.

110. Rolling Friction.—The friction of a round solid rolling on a smooth surface is less than when it slides. Advantage is taken of this fact to reduce the friction of bearings. A ball-bearing (Fig. 47) substitutes the rolling friction between balls and rings for the sliding friction between a shaft and its journal.



Fig. 47

111. Loss of Energy Due to Friction.—Friction acts in every case as a resistance opposing motion. Whenever any displacement actually takes place, work must be done against frictional resistance. The energy equivalent to this work is converted into heat, which is gradually diffused among neighboring bodies, and this energy is then no longer available. Friction, therefore, decreases the efficiency of machinery and wastes energy.

112. Uses of Friction.—Friction, like every other affliction, has its uses. Screws and nails hold entirely by friction. We are able to walk because the friction between the shoe and the pavement is less than the limiting friction. Shoes with nails in them are dangerous on cast iron

plates, because the limiting friction is small between smooth iron surfaces. The friction between leather and wood or iron permits of driving machinery by leather belts. They should not slip on the pulleys.

The friction between the driving wheels of a locomotive and the rails acts forward to prevent the wheels from slipping on the track. The effort that an engine can make to "pull a train" is limited to the friction of its driving wheels on the rails.

Problems.

1. Using a lever of the first class, a weight of 50 kgm. placed 20 cm. from the fulcrum, is to be balanced by a second weight placed 40 cm. from the fulcrum. Find the magnitude of this weight. 25

2. On a lever of the second class, a weight of 30 lb. is suspended a distance of 10 in. from the fulcrum; the effort is 5 lb. Find the length of the lever.

3. A lever of the first class is 8 m. long, with the fulcrum 2 m. from one end. What weight will balance an effort of 20 kgm. on the longer arm, the lever being a uniform bar weighing 2 kgm. to the metre?

[The weight of the lever is to be considered as a force applied at the centre of the bar. Calculate its moment, and add it to that of the effort.]

4. A uniform bar 6 ft. long has weights of 22 lb. and 44 lb. respectively suspended from the extremities. Where must the fulcrum be placed to produce equilibrium?

[Represent the distances of the fulcrum from the two weights by x and $6-x$.]

5. A false balance is one whose arms are unequal. A body was found to weigh 12 kgm. in one pan of such a balance and 18 kgm. in the other. What was the true weight?

6. In a wheel and axle the diameter of the axle is 40 cm., to which is attached a weight of 200 kgm. The axle is turned by a lever 1 m. long. Find the force necessary to secure equilibrium.

7. In the capstan of a ship, such as shown in Fig. 37, the diameter of the cylinder is 16 in., and the length of the handspike is 6 ft. What force is exerted in raising an anchor that weighs 1800 lb.? *200 lb*

8. In a derrick of the form shown in Fig. 36, a force of 50 lb. is applied to one of the cranks. The crank is 20 in. long, the small wheel, or pinion, has 20 teeth, and the large wheel has 100 teeth. The barrel on which the rope winds has a diameter of 10 in. What will be the tension in the rope? *1000 lb.*

[The circumferences are proportional to the number of teeth.]

9. Seven pulleys, four fixed and three movable, are connected by a single cord. What weight will a force of 20 lb. raise, neglecting friction and the weight of the pulleys? *140 lb*

10. In a system of pulleys, as shown in Fig. 39, there are three sheaves in each block. What force will be necessary to balance 240 kgm.? *40 kgm*

11. A number of pulleys are equally distributed between two blocks and connected by one cord, one of the blocks to be fixed. How many must there be in each block so that a force of 20 lb. can just balance a weight of 160 lb.? *4 sheaves in each block*

12. A force of 25 kgm. applied parallel to the surface of an inclined plane, 100 m. long and 5 m. high, just supports a frictionless weight. Find the magnitude of this weight. *500 kgm*

13. An inclined plane is 200 ft. long at the base and its perpendicular height is 10 ft. What force acting parallel to the base can just support a 500-lb. ball on the plane?

14. A force of 20 lb. applied parallel to a smooth inclined plane supports a weight of 80 lb. What is the pressure on the plane? *77.46*

15. A cart weighing 210 kgm. is to be pushed up an inclined plane by a force of 15 kgm. If the height of the plane is made 5 m., what must be the length? *70 m*

16. A screw of a copying press has five threads to the inch, the diameter of the wheel is 14 in., and the force applied is 25 lb. Calculate the pressure on the plate.

17. A weight of half a ton is raised by a jackscrew. What force must be applied, in addition to that required to overcome friction, if the lever is 1 ft. long and the screw threads are 6 to the inch?

- ✓ 18. The wheel at the top of a faucet is 3 in. in diameter and the screw has 8 threads to the inch. If a force of 3 lb. is applied to the circumference of the wheel, what will be the pressure on the valve?
- ✓ 19. What is the efficiency of a machine by which a force of 25 lb. moving 40 ft. raises a weight of 200 lb. through 4 ft.? 80%
- ✓ 20. The radii of a wheel and axle are 5 ft. and 6 in. respectively. On trial it is found that a force of 100 lb. can lift a weight of only 900 lb. In a machine without friction what weight would be lifted? What is the efficiency of this machine? 75%

CHAPTER III.

MECHANICS OF FLUIDS.

I. MOLECULAR PHENOMENA IN LIQUIDS.

3. **Characteristics of a Fluid.** — A solid has rigidity or elasticity of form (§ 14), but a fluid cannot resist a stress as it is supported on all sides. The molecules of a fluid are displaced by the application of the slightest force. A fluid has therefore only elasticity of volume. Every fluid, however, offers some resistance to change of shape on account of internal friction. This property is called *viscosity*. Viscosity varies through wide limits, being large, for example, in tar and very small in hydrogen gas. A *perfect fluid* would be one entirely without rigidity and viscosity.

4. **Liquids and Gases.** — Fluids are divided into liquids and gases by means of two distinguishing properties:— *first, liquids*, such as water and mercury, are but slightly compressible, while *gases*, such as air and hydrogen, are highly compressible. A *liquid* offers great resistance to forces tending to diminish its volume, while a *gas* offers relatively small resistance to reduction of its volume. Liquids have perfect elasticity of volume, but the measure of this elasticity differs widely. Water is reduced 0.00005 of its volume by a pressure of one atmosphere (§ 146), while air is reduced 0.5 of its volume by the same pressure.

Second, gases are distinguished from *liquids* by the fact that any mass of a gas introduced into a closed vessel always completely fills it, whatever its volume. A liquid has a bulk of its own, but a gas has not. This second characteristic may be regarded as a corollary of the first, since a gas expands indefinitely as the pressure on it decreases.

115. Cohesion in Liquids. — If a clean glass rod be dipped into water and then withdrawn, a drop will adhere to the end of the rod. If enough water runs down the vertical rod to enlarge the drop sufficiently, its weight will tear it away from the rod, and it will fall as a little sphere of water.

If by means of a pipette a large globule of oil be placed in a mixture of alcohol and water, the mixture having the same mass per unit volume as the oil, the globule of oil will assume a spherical form, because the influence of gravity on it is eliminated, and it will float anywhere in the mixture.

In both cases the spherical form is due to the *cohesion between the molecules of the liquid*. *Cohesion in a liquid is the attraction existing among its molecules* (§ 17).

116. Surface Conditions of a Liquid. — **Experiment.** — Place a sewing needle on the surface of clean water. If carefully done, the needle will float. On close examination it will be seen that the surface of the water round the needle is depressed, the latter resting in a little hollow large enough to hold perhaps four such needles. If the needle is forced below the surface, it will at once sink.

Experiment. — Float two wooden toothpicks on water, placing them parallel and separated by a few millimetres. Let a drop of alcohol fall on the water between them, and they will suddenly fly apart.

The needle indents the surface of the water as if the surface were a tense membrane or skin, and tough enough to support the needle. The second experiment indicates that this membrane is stretched, the effect of the alcohol being to weaken it between the toothpicks, thus permitting the parts to separate through the superior tension of the portions outside.

Experiment. — Spread a thin film of water over a very clean glass plate, and touch it with a drop of colored alcohol on a glass rod. The alcohol makes a weak spot in the film. It breaks, and the tension around it draws the water away, leaving a dry area about the alcohol.

117. Surface Tension. — The surface of a liquid is physically different from the interior. The molecules composing the surface are not under the same conditions of equilibrium as those within the liquid. The latter are attracted equally in all directions by the surrounding molecules, while those composing the surface layer are attracted downward and laterally, but not upward. The result is that this surface layer is compressed and tends to contract. The contraction means that the surface acts like a stretched membrane, and, hence, when it is curved it exerts a pressure toward the centre of curvature.

By reason of this surface tension the surface always contracts so that it shall be as small as possible. Liquids in small masses always tend, therefore, to become spherical, since the surface is then the smallest that will enclose the given volume. Tears, dewdrops, and drops of rain are for this reason spherical. Surface tension rounds the end of a glass rod or a stick of sealing-wax when softened in a flame. It also breaks up a small stream of molten lead, and molds the detached masses into spheres, which cool as they descend, and form shot. This is only another way of saying that the spherical form is due to cohesion.

118. Further Illustrations of Surface Tension. — Experiment. —

Make a ring 8 or 10 cm. in diameter, of stout iron wire, with a supporting handle. Tie to this a loop of thread, so that the loop may hang near the middle of the ring



Fig. 48.

hang near the middle of the ring (Fig. 48). If now the ring is dipped into a soap solution, and a film is formed across it, the loop of thread will spring out into a circle when the film inside the loop is carefully broken by thrusting a hot wire through it. The tension in the film pulls the thread

outward in all directions equally. If the ring be tilted, the circle will float about on the film.

Experiment. — Blow a soap bubble on the wide end of a thistle tube. Hold the other end of the tube close to the flame of a candle. The bubble, which has both an outer and an inner surface film, will contract and drive a current of air through the tube with perhaps sufficient force to extinguish the flame.

Experiment. — Place some fragments of camphor gum on *clean* water. The camphor dissolves unequally at different points and produces an unequal weakening of the film. This causes the particles of camphor to move about in the most erratic manner.

An interesting modification of the above experiment is to make a miniature tin or wooden boat, having a notch cut in the stern, in which rests a bit of camphor gum (Fig. 49). The camphor weakens the tension astern, and the tension at the bow draws the boat onward.



Fig. 49.

on. — In all experiments on the surface tension of water, the care should be taken to keep the surface chemically clean. The slightest trace of oil, or a touch with a greasy finger, will often cause failure in the experiment.

Capillary Phenomena. — The first phenomenon of surface tension to be observed and studied was the rise of water in capillary tubes, or those with a fine bore.

Experiment. — Support a clean strip of glass in water. The water will be seen to reach up above its level on the glass, making the surface concave upward. If mercury be used instead of water, the surface is depressed and is convex.

Experiment. — Support vertically two clean plates of glass inclined at an angle (Fig. 50), with their lower edges in water. The distance to which the water rises between the plates is inversely proportional to the distance between the plates; and therefore the water line is a curve in mathematics as a hyperbola.

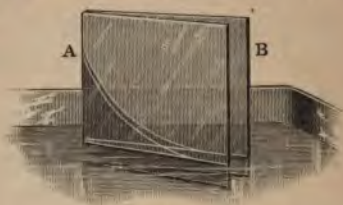


Fig. 50.

Experiment. — Support several clean glass tubes



Fig. 51.

(Fig. 51) of small internal diameter in a vessel of pure water. The water will be seen to rise in these tubes, highest in the one of smallest diameter and least in the one of the greatest. If mercury be used, instead of water, it will be depressed, the most in the smallest tube. On examining the surface of the liquid within the tubes, it is found to be concave upward when the liquid rises and convex when it is depressed.

120. Laws of Capillary Action. — The following laws have been established by experiment : —

I. *Liquids ascend in tubes when they wet them, that is, when the surface is concave; and they are depressed when they do not wet them, that is, when the surface is convex.*

II. *The elevation or the depression is inversely as the diameter of the tube.*

III. *The elevation or the depression decreases as the temperature increases.*

121. Familiar Illustrations of capillary action are numerous. Blotting paper absorbs ink because it is porous, and oil rises in a wick by capillarity. A sponge absorbs water for the same reason. The spread of water through a lump of sugar is explained in a similar manner. Small objects float together on water or cling to the sides of the vessel because of capillary action. Water rises around a fine wire dipping into it and interferes with its free rotation.

122. Explanation of Capillary Action. — It has already been shown that the attraction of glass for water is greater

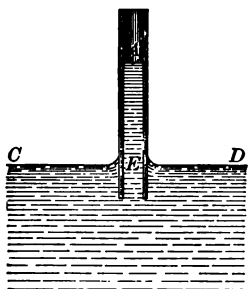


Fig. 52.

than the attraction of water for itself (§ 17). When a solid thus attracts a liquid, the liquid wets it and rises with a concave surface upward. The surface tension in a curved film produces a normal pressure toward its centre, as shown in the case of the soap-bubble. When, therefore, a liquid rises in a glass tube (Fig. 52) this normal force produced by surface tension in the concave film at *A* is upward. Where the surface is level at *C* and *D* there is no curvature, and the surface tension has no effect on the level of the liquid. The liquid therefore rises to

such a level in the tube that the pressure of the liquid column AE downward just equals the resultant normal force of the film upward. When the liquid does not wet the tube, the normal pressure of the film is downward, and for equilibrium this must be counterbalanced by pressure of the liquid on the outside.

II. PRESSURE IN FLUIDS.

123. Laws of Fluids. — There are three fundamental principles of pressure in fluids which may be called the *laws of fluids*: —

I. *Fluid pressure is normal to any surface on which it acts.*

II. *Fluid pressure at a point in a fluid at rest is of the same intensity in all directions.*

III. *Pressure applied to a fluid from without is transmitted undiminished in all directions.*

Fluid pressure is measured by the force exerted per unit area.

The pressure at a point is estimated by supposing the pressure equal to it to be exerted uniformly over a unit of area. The force on unit area is called the pressure at the point.

124. Pascal's Principle. — The first of these laws is a consequence of the mobility of a fluid. It exhibits no friction at rest, and therefore yields under any force not normal to its surface. If at rest, therefore, the resultant force on it at any point is normal to the surface.

The other two laws are included in Pascal's principle of the equal transmission of pressure in all directions. A solid transmits pressure only in the direction in which the force acts; but a fluid transmits pressure in every direction. Hence Pascal's law: —

Pressure applied to any area of an enclosed fluid is transmitted in all directions and without diminution to every part of the fluid and of the interior of the containing vessel.



Fig. 53.

This is the fundamental law of the mechanics of fluids, and it applies to both liquids and gases. It was first enunciated by Pascal in 1653.

125. Illustrations. — Experiment. — Fit accurately to the mouth of a thin-walled pint bottle a close-grained cork (Fig. 53). Fill the bottle full of water, and then force in the cork by pressure, using a lever if necessary. The bottle will probably break. Explain. How could the bursting force be estimated?

Experiment. — Glass-blowers make a form of syringe which is attached to a hollow sphere provided with several small openings distributed over its surface (Fig. 54). Fill the apparatus with water, and force the piston into the cylinder. The water will escape in a series of jets of apparently equal velocities, though only one of these jets is in direct line with the piston.

Experiment. — Fit a glass tube to a toy-balloon. Blow air into the tube; the balloon will swell out equally in all directions, showing equality of pressure.

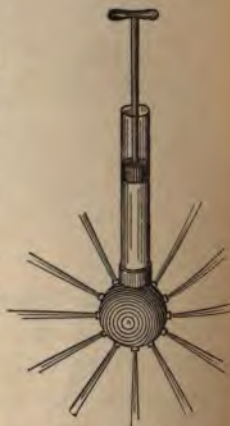


Fig. 54.

126. The Hydraulic Press. — An important application of Pascal's principle is the *hydraulic press*, a machine employed for exerting great pressure, as in baling hay and cotton, making lead pipe, and lifting heavy masses in Bessemer steel mills, locomotive works, and on warships. It was invented by Bramah in 1795, and is

shown in section in Fig. 55. Two heavy metal cylinders connected by a strong tube K . A cast-iron piston is water-tight through the collar n of the large cylinder, while in the smaller cylinder, the piston p is worked up and down as a force pump (§ 160), and pumps water into a reservoir at the bottom and forces it through the tube K into the cylinder B . When the plunger p of the

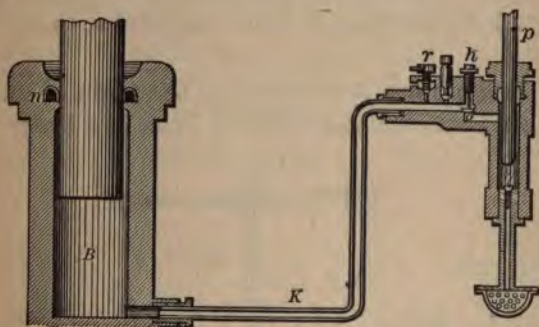


Fig. 55.

When the plunger p is forced down, the liquid in the machine transmits the pressure to the base of the large piston or ram, which is forced up with its load. If the cross-sectional area of the plunger of the pump is a and the downward force on it is P , then the pressure on the water is $\frac{P}{a}$. This pressure is transmitted to the cylinder B , and a pressure $\frac{P}{a}$ acts on each unit of surface of the base of the ram. If the area of the base is A , the total upward force, W , exerted on it is $\frac{P}{a} \times A$. Hence, the mechanical advantage is

$$\frac{W}{P} = \frac{A}{a} = \frac{D^2}{d^2},$$

if D is the diameter of the ram and d that of the plunger. If, for example, the diameter of the larger piston is 25 cm. and that of the smaller one 5 cm., then the force P applied is multiplied $\frac{25^2}{5^2} = 25$ times.

This machine conforms to the principle of work, for it is evident that the small piston moves as many times farther than the large one as the force exerted by the large one is greater than the effort applied to the small one.

127. Pressure due to Gravity.—The weight of each layer of a liquid is transmitted to every layer at a lower level.



Fig. 56.

Experiment.—Grind to a true plane one end of a cylindrical lamp chimney till a metal disk closes it water-tight. Suspend the disk by a cord from one end of a scale beam and counterpoise it (Fig. 56). Now place, say, 200 gm. on the scale pan and pour water into the cylinder

until its pressure detaches the disk, and mark the depth. Repeat the experiment with 400 gm. on the pan. It will be found that the depth of the water when the disk is detached is twice as great as before.

Hence, *the downward pressure of a liquid is proportional to the depth.*

Experiment.—With the apparatus of the last experiment it is found that with a given weight on the scale pan a fixed depth of water is necessary to detach the disk. Use a heavier liquid, such as a saturated solution of common salt. Determine its density (§ 144). Find the weight in the scale pan at which the disk is detached for the same fixed depth of liquid as before. It will be greater than with water in the same ratio as the density is greater than one.

Hence, *the downward pressure of a liquid is proportional to its density.*

128. Pressure at a Point.—**Experiment.**
—Bend three glass tubes into the J-forms shown in Fig. 57. Each one has a long and a short arm. The short arms are of equal length and are bent so as to open in different directions—upward, downward, and sidewise. Place the same depth of mercury in each tube. Now lower them into a tall jar filled with water. When the openings of the short arms are all as nearly as possible at the same point, the change of mercury level is seen to be the same in each.

Hence, *the pressure at a point in a liquid is the same in all directions.*

It is immaterial whether this pressure is due to the weight of the liquid or is applied from without. The equality of pressure in all directions is a consequence of the equal

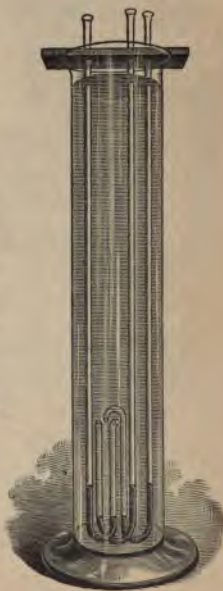


Fig. 57.

transmission of pressure in all directions. The absence of currents in a vessel containing a liquid of uniform temperature demonstrates this principle, since any unbalanced pressure would produce motion in the liquid.

129. Pressure Independent of Shape of Vessel.—Experiment.—Using successively Pascal's vases, vessels differing in shape but having equal bases (Fig. 58), it will be found that with a given weight in the scale pan, the disk will be detached when the depth of water is the same in each case, notwithstanding the difference in amount of water used.

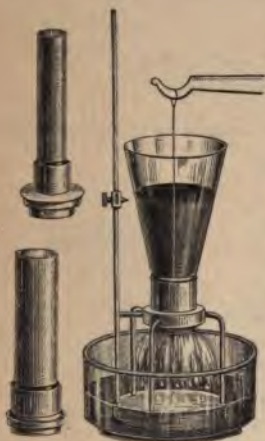


Fig. 58.

Therefore, *the pressure on the bottom of a vessel, or the downward pressure, is independent of the shape of the vessel.* The apparent contradiction of unequal masses of water producing equal pressures is often known as the *hydrostatic paradox*.

130. Total Pressure on any Surface.—Since liquid pressure depends on the depth and the density of the liquid, we may calculate the

pressure on any horizontal area as follows:—

Let A denote the area pressed upon, H its depth, and d the weight of the unit of volume. Then the whole pressure on this area will be

$$P = AHd. \quad (23)$$

In the metric system, d for water is 1 gm. per cubic centimetre. (For other liquids, consult Table of Densities in the Appendix.) In the English system, d is 62.4 lb. per

cubic foot for water. For any other liquid, multiply by the specific gravity (§ 141) of the liquid.

Hence, *the total pressure of a liquid on any horizontal surface is equal to the weight of a column of the liquid whose base is the area pressed upon, and whose height is the depth of this area below the surface of the liquid.*

The pressure on any immersed surface, whatever its inclination, is found by computing the pressures on all the elementary areas and adding them together. The result is expressed as follows:—

The total pressure of a liquid on any immersed surface is equal to the weight of a column of the liquid whose base is the area pressed upon, and whose height is the distance of the centre of figure, or the centre of gravity, of this area below the surface of the liquid.

131. Surface of a Liquid at Rest.—The free surface of a liquid under the influence of gravity alone, is horizontal. If it were not horizontal, then the weight BW (Fig. 59) of a particle of the liquid surface would have a component BC tangent to the surface of the liquid. Since the air pressure on the surface is everywhere the same, there is no hydrostatic pressure to resist this force; and as there is no friction of rest in a liquid, the particle B would move. When the surface is level, BC vanishes and there is no motion. Even very viscous liquids assume a horizontal surface in course of time.

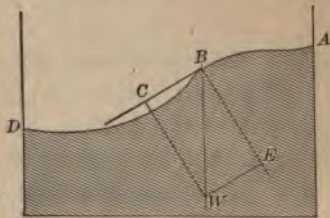


Fig. 59.

The sea or any large expanse of water is a part of the

spheroidal surface of the earth. When one looks with a field glass at a long straight stretch of the Suez Canal, the water and the retaining wall may be plainly seen to curve over in a vertical plane.

132. Liquid in Communicating Tubes.—In Fig. 60, tubes of various shapes open into a connecting horizontal arm attached to the jar on foot. If a colored liquid be poured into the jar, it will rise to the same level in all the tubes. There is then equilibrium, because the pressures on the opposite sides of any imaginary cross-section of the liquid in the connecting tube are equal, since they are all due to liquid columns of the same height. Liquids are therefore said to “find their own level.” The water supply of towns depends on this principle. When the water is pumped into a reservoir on a height, it is thence distributed everywhere by its own weight.



Fig. 60.

133. Superposed Liquids in Equilibrium.—If several liquids of different densities (§ 140) be placed in a vessel, with the heaviest at the bottom, and the others in order to the lightest at the top, they will show horizontal surfaces of separation, especially if no two in contact mix readily. Figure 61 shows mercury, water, oil, and alcohol with well-defined surfaces of separation between them. The contrast will be the more striking if the water and the alcohol are colored with an aniline dye. If a small iron ball be dropped into the



Fig. 61.

jar, it will settle down as far as the mercury and float there. In a tidal river the salt water flows up underneath the fresh water, which floats on top.

Experiment.—Place an egg in fresh water in a tall jar. It will sink to the bottom. Some strong brine may then be poured down a glass tube to the bottom of the jar. It will remain underneath the fresh water, and the egg will be seen suspended at the surface of separation at the top of the brine.

Problems.

1. When a soap bubble bursts what becomes of the film? Have you ever noticed the spray from the bubble?
2. Why is a dam across a stream made thicker at the bottom than at the top?
3. A cylindrical vessel full of water is provided with two faucets, one half-way down the side and the other at the bottom. Out of which will water flow with the greater velocity? Why?
4. Why is a water pipe of a city's water system more likely to burst in that part of the city where the pipes are lowest?
5. What is the distinguishing or defining characteristic of a fluid?
6. Why will a shallow dish whose bottom is made of a fine-meshed wire cloth float on water?
7. The diameters of the cylinders of a hydraulic press are 6 in. and 1 in. respectively. What is the pressure on the larger piston when a force of 100 lb. is applied to the smaller piston? *3600 lbs 3597*
8. A hydraulic lift carries a weight of 10,000 lb. If the piston supporting the lift be 10 in. in diameter, what pressure per square inch will be necessary to sustain it? *127.34*
9. A tank, 5 ft. deep and 10 ft. square, is filled with water. What will be the pressure on the bottom? What on one side?

bottom 31200 lbs
each side 7800

✓ 10. If the water is 20 ft. deep at a dam, what will be the horizontal pressure against a rectangular portion of the dam 10 ft. long? *1248 lb*

11. A piston, 1 sq. ft. in area, is pushed along by water-pressure due to a head of 200 ft. What will be the total pressure on the piston? ["Head" means "height."]

✓ 12. A cylindrical glass vessel is 5 cm. in diameter and 20 cm. high. Compute the lateral pressure when the vessel is filled with mercury.

[A cm³. of mercury weighs 13.6 gm.]

✓ 13. How high must the reservoir of a city's water system be above any point to produce a pressure of 50 lb. per square inch at that point? *115 ft*

✓ 14. A rectangular board 6 ft. long, 1 ft. wide, and 1 in. thick is submerged in water. Its position in the water is horizontal, face upward, and 10 ft. below the surface of the water. What is the upward pressure on the under surface of the board? *3775 lb*

✓ 15. A cubical vessel is filled with water. Show that the total pressure on all its faces is three times the weight of the water it contains.

✓ 16. A cubical block 10 cm. on one edge is submerged in water till the top face is 100 cm. below the surface. Calculate the total pressure on the six faces. *bottom 10,000 gm. Vertical face 10,000 gm. Total 60,000 gm*

✓ 17. A vertical tube has a cross-sectional area of 1 cm². If it is filled with mercury to a depth of two metres, what is the pressure per square centimetre at the bottom? *2720 gm per cm*

✓ 18. When a pressure gauge was applied to a water pipe in a building, it was found to register 52 lb. per square inch. How high a column of water would be required to produce this pressure? *120 ft*

✓ 19. A cylindrical tank 10 ft. high and 10 ft. in diameter is full of water. Compute the total pressure on the bottom and on the side. *49010 lb bottom 98020 lb side*

✓ 20. What will be the pressure per square foot at the depth of 5 mi. in the ocean, sea water being 2½ per cent heavier than fresh water? *304.5 tons*

III. DENSITY AND SPECIFIC GRAVITY.

134. Buoyancy. — A marble will sink in water, but will float on mercury. A piece of oak floats in water, but a piece of the dense wood known as “*lignum vitæ*” sinks. An egg will sink in fresh water and float in brine. When a swimmer wades up to his neck in sea water, he is nearly lifted off his feet by the water, which buoys him up.

Experiment. — Suspend a weight by a string from the hook of a spring balance and note the reading. Now submerge the weight in water. The index reading will be less. If salt water be used, the apparent loss of weight will be greater; if kerosene, it will be less.

Experiment. — Balance two half-kilogramme weights by a silk string over an easily running pulley, and bring a beaker of water under one of them. On lifting the beaker, the weight will not enter the water and remain immersed, but will rise to the surface.

These experiments show that the resultant pressure of a liquid on a body immersed in it is a vertical force upward, and it counterbalances a part or the whole of the body's weight. This resultant upward pressure of a liquid is known as its *buoyancy*.

135. The Principle of Archimedes. — The law of buoyancy was discovered by Archimedes about 240 B.C. while attempting to determine the composition of King Hiero's crown.¹ It is as follows: —

A body immersed in a liquid is buoyed up by a force equal to the weight of the liquid displaced by it.

¹ *Encyclopædia Britannica*, Vol. II, Art. “Archimedes.”

Let a cube be immersed in water (Fig. 62). The pressures on the vertical sides a and b are equal and in opposite directions. The same is true of the other pair of vertical faces. There is therefore no resultant horizontal pressure. On d there is a downward pressure equal to the weight of the column of water having the face d as a base and of a height dn . On c there is an upward pressure which is equal to the weight of

a column of water whose base is the area c , and whose height is cn . The upward pressure therefore exceeds the downward pressure by the weight of the prism of water whose base is the face c of the cube, and whose height is the difference between dn and cn , or cd ; and this is the weight of the volume of water displaced by the cube.

136. Experimental Proof.—Experiment.—A metallic cylinder 3.5 cm. long and 1.9 cm. in diameter has a volume of 10 cm³. nearly. Suspend it by a fine thread from one of the arms of a balance (Fig. 63), and counterpoise. Then place a vessel of water under it so that the cylinder is completely submerged. The equilibrium will be destroyed, and may be restored by placing a 10-gramme weight in the pan above the cylinder. Since the cylinder displaces 10 cm³. of water weighing 10 gm., and loses 10 gm. in weight when submerged, it follows that the body is buoyed up by a force equal to the weight of the water displaced. The resultant fluid pressure on a body of fixed volume immersed in a given liquid is the same whatever its substance. It depends on its volume only.



Fig. 62.



Fig. 63.

137. The Cartesian Diver. — Descartes illustrated the principle of Archimedes by means of a grotesque figure, since called a *Cartesian diver*, or a bottle imp. It is made of glass, is hollow, and the tail has a small opening at the end. The figure is partly filled with water so that it just floats in a jar of water (Fig. 64). When pressure is applied to the sheet of rubber tied over the top of the jar, it is transmitted to the water, more water enters the imp through the tail, and the air in it is compressed. The imp then displaces less water and sinks. When the pressure is withdrawn, the air in the diver expands and forces water out again. The displacement is then increased and the figure rises. The water in the diver may be so nicely adjusted that the little figure will sink in cold water, but will rise again when the water has reached the temperature of the room and the air in the figure has expanded. A good substitute for the diver is a small homœopathic vial in a flat 12 oz. prescription bottle with rubber stopper. Press on its flat sides.



Fig. 64.

138. Equilibrium of Floating Bodies. — When a body is immersed in a fluid, it may displace a weight of fluid *less* than, *equal* to, or *greater* than its own weight. In the first case, the upward pressure will be less than the weight of the body and the body will sink. In the second case, the upward pressure will equal the weight of the body and the body will be in equilibrium, remaining in the liquid wherever placed. In the third case, the upward pressure will exceed the weight of the body, and the body will rise till these forces become equal. In liquids the

buoyancy is practically independent of the depth so long as the body is wholly immersed, but will decrease as soon as it begins to emerge from the liquid. Hence,

When a body floats on a liquid it sinks to such a depth that the weight of the liquid displaced equals its own weight.

The weight of a body acts vertically downward, and the resultant pressure of the liquid acts vertically upward through the centre of gravity of the displaced liquid, which is called its *centre of buoyancy*. These two forces must be equal, and in the same vertical line for equilibrium.

139. Equilibrium of Floating Bodies Demonstrated.—**Experiment.**—Make a wooden bar 20 cm. long and exactly 1.5 cm. square. Bore a hole in one end and fill with enough shot to give the bar a vertical position when floating, with nearly its whole length in water. Fill the pores of the wood with hot paraffin. Graduate the bar in millimetres along one edge. Find the weight of the loaded bar in grammes, and then observe the length of the bar immersed when it floats in a tall jar of water. Calculate the immersed volume in cubic centimetres. This will also be the volume of water displaced; and since one cubic centimetre of water weighs 1 gm., we have the measure of the buoyancy. It will be found very nearly equal to the weight of the bar and shot. Hence, *a floating body displaces its own weight of the sustaining liquid.*

140. The Density of a body is the number of units of mass of it contained in a unit of volume. In the metric system it is the number of grammes per cubic centimetre. If m denotes mass, v volume, and d density, then

$$d = \frac{m}{v}, \quad v = \frac{m}{d}, \quad \text{and} \quad m = vd. \quad (24)$$

141. The Specific Gravity of a body is the ratio of the mass of any volume of it to the mass of the same volume of pure water at 4° C. Specific gravity is, therefore,

only the *relative density* as compared with water. It is also evident that the specific gravity of solids and liquids is numerically equal to the density when expressed in grammes per cubic centimetres, since the density of water is then unity.

Let m be the mass of a body and m' the mass of an equal volume of the standard, as water. Then the specific gravity $s = \frac{m}{m'}$ and $m = m's$. If the mass m' of water is expressed in pounds and its volume v in cubic feet, then $m' = v \times 62.4$, and $m = v \times 62.4 \times s$.

Since the density of water in the C.G.S. system is sensibly unity, there is no occasion to use the term specific gravity unless the mass and volume are given in some other system of measurement.

142. Density of a Solid.—To find the density of a body it is necessary to know its mass and volume. Its mass is ascertained by a balance. The most accurate and convenient method of obtaining the volume is furnished by Archimedes' principle. The buoyant effort of a liquid equals the difference between the weight of the body in air and its weight when immersed in the liquid. This difference is the weight of a volume of the liquid equal to that of the body. Hence, if this difference be divided by the density of the liquid, the quotient will be the volume of the liquid and also that of the body. The mass divided by this volume will be the density.

Water is the liquid generally used, and in the metric system its density is sensibly unity. If the solid is soluble in water, then a liquid of known density, in which the solid is not soluble, must be used.

In case the solid is lighter than the liquid, a sinker

sufficiently heavy to sink the body must be employed. By subtracting the buoyant effort on the sinker from the buoyant effort on both, the weight of the liquid displaced by the given body is obtained. Then by proceeding as in the first case, the density of the body can be computed.

143. Examples. — First, *for a body heavier than water.*

Weight of body in air . . .	10.5 gm.
Weight of body in water . . .	6.3 gm.
Weight of water displaced . . .	4.2 gm.

Since the density of water is 1 gm. per cubic centimetre, the volume of the water displaced is 4.2 cm³. This is also the volume of the body. Therefore, $10.5 \div 4.2 = 2.5$ gm. per cubic centimetre is the density. The specific gravity is the ratio 2.5.

Second, *for a body soluble in water.* Suppose it is insoluble in alcohol, the density of which is 0.8 gm. per cubic centimetre.

Weight of body in air . . .	4.8 gm.
Weight of body in alcohol . . .	3.2 gm.
Weight of alcohol displaced . . .	1.6 gm.

The volume of alcohol displaced is $1.6 \div 0.8 = 2$ cm³. This is also the volume of the body. Therefore, the density of the body is $4.8 \div 2 = 2.4$ gm. per cubic centimetre. Its specific gravity is 2.4.

Third, *for a body lighter than water.*

Weight of body in air . . .	4.8 gm.
Weight of sinker in water . . .	10.2 gm.
Weight of body and sinker in water . . .	8.4 gm.

The combined weight of the body in air and the sinker in water is then $4.8 + 10.2 = 15$ gm. But when the body is attached to the sinker, their apparent combined weight is only 8.4 gm. Therefore the buoyant effort on the body is $15 - 8.4 = 6.6$ gm., and this is the weight of the water displaced by the body, and hence its volume is 6.6 cm³. The density is then $4.8 \div 6.6 = 0.73$ gm. per cubic centimetre.

144. Density of Liquids.—*First, by the specific gravity bottle.* With liquids, as with solids, the chief feature of the problem is to ascertain the volume. The simplest method is by the use of the *specific gravity bottle*. This bottle is usually made to hold a definite amount of distilled water at a specified temperature, as 25, 50, 100, or 1000 gm. at 15° C. (Fig. 65). To use it, find the mass of the bottle when empty and when filled with the given liquid. The difference of these masses will be the mass of the liquid, which, divided by the volume of the bottle, will be the density.



Fig. 65.

To check the volume of the bottle, weigh it filled with ice-cold water and subtract its mass when empty and dry. The difference will be its volume in cubic centimetres. The stopper is a ground capillary tube for convenience in filling completely.

Second, by a glass sinker. Weigh a glass sinker in air and then in the liquid. The difference will be the mass of the liquid displaced by the sinker. (Why?) Then weigh the sinker in water; the loss divided by the density of water will be the volume of water displaced by the sinker, and hence the volume of the liquid whose mass has been found. Divide the mass of the liquid displaced by the volume displaced and the quotient will be the density.

Third, the hydrometer method. The common *hydrometer* is usually made of glass and consists of a cylindrical stem and a bulb weighted with mercury or shot to make it float vertically. Within the hollow glass stem is a scale graduated in some arbitrary manner or by trial, the zero being



Fig. 66.

the point to which it sinks in distilled water at either 4° C. or 60° F. The mark to which the instrument sinks in the liquid under test determines the density, either directly or by referring to an accompanying table. These instruments are often provided with a thermometer in the stem (Fig. 66), to give the temperature of the liquid at the time of taking the density.

Problems.

1. Which has the greatest buoyant force, water, olive oil, or alcohol? Why?
2. A bottle imp adjusted to float in water sinks if the temperature of the water falls. Explain. The addition of common salt to the water may cause the imp to rise. Why?
3. Why is the water line on a loaded vessel higher for an inland lake than for the ocean?
4. Why should a life preserver be attached to the body as near to the shoulders as possible?
5. If a pound of lead be counterbalanced on a beam balance by a quantity of feathers, are there equal masses in the two scale pans? Explain.
6. A body weighs 60 gm. in air and 40 gm. in water. Find its density. *3 gm/cm³*
7. A solid weighs 100 gm. in air and 60 gm. in a liquid whose density is 0.8 gm. per cm³. What is its density? What is its specific gravity? *2.5 gm/cm³*
8. An empty specific gravity bottle weighs 15 gm.; when filled with water it weighs 65 gm., and when filled with glycerine 78 gm. What is the density of the glycerine? *1.176 gm/cm³*

9. A glass stopper weighs 150 gm. in air, 90 gm. in water, and 42 gm. in sulphuric acid. Calculate the density of the acid. 1.8 gm/cm^3

10. If a cubic centimetre of ivory weighs 0.82 gm. under water, what does it weigh in air? What is its density? 1.82 gm/cm^3

11. A cork weighs 5 gm. in air; with a sinker attached and both under water the weight is 71 gm.; the sinker alone weighs in water 86 gm. Calculate the density of the cork. 1.25 gm/cm^3

12. A brass ball weighs 117.6 gm. in air. The density of brass is 8.4 gm. per cm^3 . When submerged in water what will be the buoyant force? 14 gm

13. A piece of zinc weighs 35 gm. in air and 30 gm. in water. What will it weigh in alcohol with a density of 0.8 gm. per cm^3 ? 31 gm

14. If the density of sea water is 1.025 gm. per cm^3 , and that of ice is 0.9 gm. per cm^3 , what fraction of an iceberg floating in sea water is under water?

[Find the weight of sea water displaced by 100 cm^3 of ice.]

15. A piece of stone weighs 10 oz. in air and 6 oz. in water. Calculate its specific gravity. 2.5

16. A silver spoon weighing 21.12 gm. is suspended by a cord from one scale pan of a balance. Find the tension in the cord when the spoon is submerged in water, the specific gravity of silver being 10.56. 19.12

17. Compute the weight of an iron ball (sp. gr., 7.7) 2 in. in diameter. [See § 141 and Appendix IV.]

18. A piece of walnut wood weighs 120 gm. in air and 215 gm. in water with a sinker attached. The sinker alone in water weighs 275 gm. Find the density of the wood. 1.667 gm/cm^3

19. Compute the weight of a copper bar (sp. gr., 8.8) 1 sq. in. in cross-section and 6 in. long. 1.91 lb

20. A hollow brass ball weighs 1 kgm. What must be its volume so that it will just float in water? 1000 cm^3

21. An iron cube 10 cm. on each edge (sp. gr., 7.7) is suspended from one arm of a balance so that three-fifths of its volume is under water. What weight on the other arm of the balance will just support the cube? 7100 gm

22. If a force of 750 gm. is required to support in water a man whose weight in air is 75 kgm., what is the man's specific gravity? 1.01

23. A litre flask, weighing 75 gm., is half filled with water and half with glycerine. The flask and liquids together weigh 1205 gm. Find the specific gravity of glycerine.

24. If a body floats half submerged in water, what part of its volume will be submerged in glycerine?

IV. PRESSURE OF THE ATMOSPHERE.

145. **Air has Weight.**—**Experiment.**—Attach a stopcock to a thin copper globe, such as plumbers use for a float. Suspend from the scale pan of a balance and counterpoise. Then exhaust the air from the globe (§ 151) and hang on the balance again. It will be lighter than before. Open the stopcock; the air will rush in and the equilibrium will be restored. Compress the air in it (§ 155) and weigh again. It will now be heavier than when the stopcock was open.

It has been determined that the mass of 1 litre (cubic decimetre) of air at 0° C. and 760 mm. pressure of mercury is 1.296 gm.

146. **The Torricellian Experiment.**—In the middle of the seventeenth century Galileo was called upon to explain why certain pumps erected by the Duke of Tuscany would not cause the water to rise more than about 30 feet. He suspected that the pressure of the air sustained a column of water of this height, but died without demonstrating it. Torricelli, a pupil of Galileo, first measured the pressure of the atmosphere in 1643 by the following method:—

Experiment.—Select a stout glass tube 80 or 90 cm. long and closed at one end. Fill with mercury, close the open end with the finger, and invert it in a cup of mercury (Fig. 67). When the finger is removed the mercury will settle in the tube a few centimetres, leaving a vacuum, called a *Torricellian vacuum*, above it. This column of mercury *AB* in the tube is supported by the pressure of the atmosphere on the mercury in the larger vessel at the bottom.

the demonstration was completed by Pascal, who found the height of the mercury was less on the top of a high mountain in Paris than on the plain; and that it fell nearly 30 centimetres when the apparatus was carried to the top of the Puy-de-Dôme, about 1500 metres high, showing the atmospheric pressure less at that height.

The height of the mercurial column supported by the atmosphere varies considerably from time to time. A standard value of 76 cm. has more recently been adopted to represent the mean pressure of the atmosphere at sea level.

The height of the column is independent of the cross-section of the tube. Suppose an internal cross-sectional area of a^2 . The volume of mercury supported by the atmospheric pressure on 1 cm^2 will then be 76 cm^3 . The density of mercury at 0° C. is 13.596. Hence 76 cm^3 weighs 76×13.596 , or 1033.3 gm. At sea level, then, the atmosphere exerts an average pressure of $1033.3 \text{ gm. per cm}^2$. This is equivalent to $14.7 \text{ lbs. per square inch.}$ Either is a pressure of one atmosphere.



Fig. 67.

Illustrations of Air Pressure. — Experiment. — Fill a tumbler of water, cover it with a sheet of paper, and invert (Fig. 68) so as to prevent the water from escaping. The air exerts a pressure on the bottom of the paper which is more than sufficient to support the weight of the water.

Experiment. — Select two test-tubes, one wider than the other. The narrower one should fit the larger one rather loosely. Fill the

larger one with water, insert the smaller one and quickly invert them. As the water escapes, the air-pressure will force the smaller tube upward into the larger one against gravity and hold it there.



Fig. 68.

Experiment. — Fasten a string to a round piece of leather. Wet the leather so as to make it pliable, and press it down evenly on a smooth flat stone. The stone, if not too heavy, can be lifted by the string, the pressure of the air keeping the leather pressed down on it.



Fig. 69.

148. The Mercurial Barometer. — The mercurial *barometer*, for measuring atmospheric pressure, in its simplest form consists of a Torricellian tube about 86 cm. (nearly 34 in.) long, attached to a supporting board. A scale, whose zero is at the surface of the mercury in the cistern, is fastened by the side of the tube, to give the height of the mercury column. Torricelli suggested that a J-shaped tube be used, the short open arm taking the place of a cistern. The form shown in Fig. 69 was designed by Gay-Lussac. The short arm has a small pin-hole near the top for the admission of air. The height of the mercury column is given by the difference of the readings of two pointers on the scales on the right; for example, if the upper pointer reads 78.45 cm., and the lower one 4.23 cm., the pressure is 74.22 cm. of mercury. Readings must be taken with the tube in a vertical position. (Why?) When

accuracy is required, corrections must be made for temperature, capillarity, and gravity.

A good barometer must contain clean mercury, and the mercury must be boiled in the glass tube to expel air and moisture.

149. Barometric Variations.—Since the mercury in the tube of the barometer is sustained by the pressure of the column of air resting on the mercury outside, any change in this pressure will produce a change in the barometric reading. Changes of this kind are going on continually at every place. Certain very slight changes are found to be periodic, but the greater changes follow no known laws. These irregular movements point to corresponding fluctuations in the pressure of the air, and consequently herald important atmospheric movements.

150. Uses of the Barometer.—The barometer is a faithful indicator of all changes in atmospheric pressure, and constant use is made of it by the Weather Bureau in forecasting changes of weather. Experience has shown that barometric changes are generally indicative of changes in the state of the weather, according to the following rules:—

I. *The rising of the barometer indicates the approach of fair weather.*

II. *The rapid fall of the barometer denotes the near approach of a storm.*

III. *A high, unchanging barometer, indicates continued settled weather.*

Since the pressure of the atmosphere diminishes with the elevation above the surface of the earth, the difference in the altitude of two stations may be computed from

barometric readings taken at the two places simultaneously. The various rules proposed to express the relation between the height of the barometer and the elevation above sea level are more or less arbitrary, but they are used for determining the heights of mountains or other places with considerable accuracy. A simple rule for places near the sea level is to allow 0.1 inch for every 90 feet of ascent.

Problems.

1. What is the atmospheric pressure per cm^2 when the barometer reading is 74 cm.? [See equation (23).] *1006 gm*

2. What is the atmospheric pressure per square inch when the barometer reading is 29 in.? *14.75*

[Find the weight of a cubic inch of mercury.]

3. Calculate the atmospheric pressure on the top of a table 1 m. square when the barometer reading is 75 cm. *10.2 Meters*

4. Compute the height in inches of the mercurial barometer when the atmospheric pressure per square inch is 15 lb. *10.2 Meters*

5. When the mercurial barometer reads 74 cm., what will be the reading of a glycerine (sp. gr., 1.26) barometer? *79.5 cm*

6. If a litre of air weighs 1.29 gm. when the barometer reading is 76 cm., calculate the buoyancy for a ball 10 cm. in diameter.

7. 200 cm^3 of cork are weighed in air (sp. gr., 0.0013) and then in a vacuum. Find the difference between the two weights. *0.24 gm*

8. Calculate in dynes per cm^2 the atmospheric pressure when the barometer reading is 73.5 cm. *999600 dynes*

9. The diameter of a pair of Magdeburg hemispheres (§ 152) is 3 in. and the barometer reading is 29 in. What force will be necessary to separate the pair if all the air is removed from within?

10. If at sea level the barometer stands at 76 cm. and in Denver at 60 cm., find the difference in pressure (in gm.) on an area of one square metre. *2176000 gm*

V. INSTRUMENTS DEPENDING ON PRESSURE OF THE AIR.

151. The Air-pump, as the name denotes, is a device for removing air or any gas from a vessel and depends for its action on the fact that gases are indefinitely expansible. The first pump was devised by Otto von Guericke about 1650.

Figure 70 represents the general appearance of one of the best forms made at the present time. Figure 71 shows the essential parts in section. A piston P , with a valve S in it, works in a cylindrical barrel, communicating with the outer air by a valve V at its upper end, and with the receiver on the pump table by a tube. The valve S' is carried by a rod which passes through the piston, fitting tightly enough to be lifted by the piston when the up stroke begins; but its ascent is almost immediately arrested by a stop near the upper end of the rod, and the piston slides on this rod during the remainder of the up stroke. This allows the air from the receiver to flow into the space below the piston. In the top plate of the cylinder is a lever, one end of which covers the valve rod. When the piston reaches the top of the cylinder it strikes this lever, and the lower valve, S' , is thus closed. In the down stroke of the piston the valve S opens automatically, and the

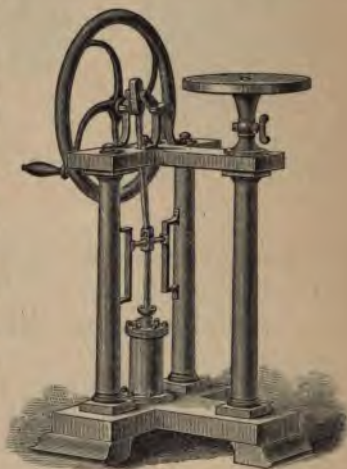


Fig. 70.

enclosed air passes through it into the upper part of the cylinder. The ascent of the piston again closes it; and

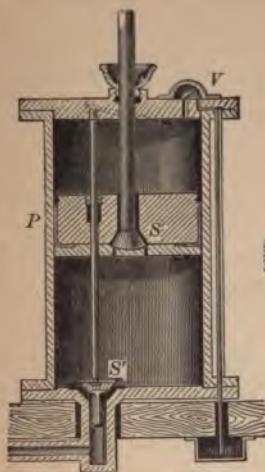


Fig. 71.



as soon as the air is sufficiently compressed, it opens the valve *V* and escapes. Each complete double stroke of the piston removes a cylinder full of air; but as the air grows rarer with each double stroke, the mass removed each time is less. On account of the tendency of a gas to fill the containing vessel, irrespective of quantity, the removal of all the air from the receiver is

not possible, although we might continually approach a vacuum were it not for the unavoidable mechanical defects of the pump, such as leakage of valves, untraversed space, etc.

152. Experiments with the Air-pump.—1. **Football.** Fill a small rubber football half full of air, and place it under a bell-jar on the air-pump table. Exhaust the air from the bell-jar and notice that the ball expands till it is free from all wrinkles. What property of air is illustrated?

2. **The Bladder Glass.** Over one end of a glass cylinder tie a piece of bladder or paste a piece of paper (Fig. 72). Place it on the air-pump table and exhaust the air. The membrane or paper will break with a loud report. Why?



Fig. 72.

Repeat the experiment with sheet-rubber tied over the glass (Fig. 73). The rubber will be pressed into the glass. If the glass be turned on its side the effect is the same. Explain.

3. **The Bacchus Experiment.** Select two bottles; fit to one of them a perforated stopper. Connect the two by a bent tube reaching nearly to the bottom of each (Fig. 74).



Fig. 74.

Fill the stoppered one nearly full of water and place them under a bell-jar on the air-pump table. Exhaust the air. Explain why the water flows out of the stoppered bottle and then flows back on admitting air into the bell-jar.

4. **The Vacuum Fountain.** A tall glass vessel is provided with a stopcock and jet-tube. (A bottle fitted with a rubber stopper can be used.) Having exhausted the air, place the mouth of the jet-tube in water and open the stopcock (Fig. 75). Why does the water rush into the vessel? Is it possible to determine how much air was not removed?



Fig. 76.



Fig. 73.



Fig. 75.

5. **The Magdeburg Hemispheres.** This famous historical apparatus was invented by Otto von Guericke, Burgomaster of Magdeburg. It consists of two accurately fitting hollow metallic hemispheres, provided with handles and a stopcock (Fig. 76). Attach the apparatus to the pump and exhaust the air. Close the stopcock, screw on the handles, and try to pull the hemispheres apart. How could it be shown that they are held

together by atmospheric pressure? In computing the pressure that holds them together, which is the surface to be considered, the spherical surface or the cross-sectional area? Why?

153. Buoyancy of the Air. — The principle of Archimedes applies to gases as well as to liquids. The resultant pressure of the atmosphere on bodies in the air is an upward



Fig. 77.

force equal to the weight of air displaced. A body therefore weighs more in a vacuum than in the air, unless the volume of air displaced by it is the same as that displaced by the weights.

The *baroscope* is an instrument designed to exhibit the upward pressure of the air. A thin hollow globe is slightly overbalanced by a lead or brass weight on a small pair of druggist's scales (Fig. 77). (A cork sphere may be used in place of the hollow sphere.) When the baroscope is placed under a large receiver and the air is exhausted, the hollow sphere or the cork sinks, showing that it is really heavier than the counterpoise, but in the air it is buoyed up more because its volume is greater. Why would this experiment fail if the globe were not air-tight?

154. Balloons. — The upward pressure of the air is utilized in balloons. A balloon must be filled with a gas lighter than air, so that the weight of the gas and of the balloon with its car and contents shall be less than that of the air displaced. A balloon is not quite filled with gas at first, but as it rises it expands as the pressure of the air decreases. Its buoyancy then decreases but little as it rises into a rarer atmosphere.

With hydrogen, the ascensional force is about one kilogramme per cubic metre of gas; with common illuminating gas it is about half as great, but the latter is much less expensive.

On September 30 and October 9, 1900, two long-distance balloon races were made from Paris in an easterly direction. One of the contestants, Count de la Vaulx, the winner in both races, reached Russian territory in both, having travelled, the first time, a distance of 766 mi. in 21 hr. and 34 min.; and the second time, a distance of 1193 mi. in 35 hr. and 45 min. The maximum altitude reached was 5700 m., or 18,700 ft.

The aeronauts testify that when the sun shone on the balloon and heated it, the expansion of the gas enlarged the balloon and increased its buoyancy, so that it shot up to higher altitudes. It became necessary, in consequence, to let out some gas to cause the balloon to descend again. In the night, when the temperature fell, the buoyancy, on the other hand, decreased. Ballast was then thrown out to lighten the balloon and prevent its descent. These alternate losses of gas and ballast at length exhausted the capacity of the balloon to keep afloat, and it finally descended to the ground.

155. The Air Compressor.—If the discharge pipe of an air-pump were connected to a suitable vessel, air would be forced into the vessel during the action of the pump. Such a device would be an *air compressor*. Since the valves of the ordinary air-pump will not stand high pressure, a pump designed as in Fig. 78 is more suitable for compressing a gas. The plunger is solid, and in the bottom of the cylinder are two valves, one opening inward and the other outward. When the piston moves upward, the gas is

admitted through the left-hand tube, the valve being lifted by the pressure of the gas below it. When the

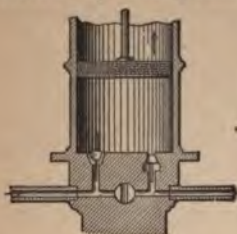


Fig. 78.

piston descends, this valve closes and the right-hand one opens, affording an exit for the confined gas.

This machine is evidently an air-pump when the left-hand tube is connected with a receiver, but it is not capable of producing very high exhaustion, on account of the heavy valves required to give it strength for compression purposes.

156. Applications.— Both the air-pump and the compressor are extensively used in the arts. Sugar refiners employ the air-pump to reduce the boiling point of the syrup (§ 337); manufacturers of soda water use a compressor to charge the water with carbon dioxide; in pneumatic despatch tubes, now extensively employed for rapidly transporting small packages, both pumps are used, the one to exhaust the air from the tubes in front of the closely fitting carriage, and the other to force compressed air into the tube behind it, so as to propel it with great velocity. The air compressor is also employed to improve the draft of furnaces, to facilitate the ventilation of buildings and mines, to operate pneumatic clocks, Westinghouse brakes on cars, and machinery in places difficult of access.

157. The Siphon, in its simplest form, is a U-shaped tube employed to convey liquids from one vessel to another at a lower level by means of atmospheric pressure (Fig. 79). To set it in action, the usual way is to fill the tube with

liquid, close the ends, place the shorter branch in the liquid, and open the ends. The flow will continue as long as the liquids in the two vessels are at different levels, and the shorter branch dips into the liquid. The siphon may also be started by suction; in the case of corrosive liquids, a delivery tube (Fig. 80) is attached in a manner to prevent contact of the mouth with the liquid. The vertical distance of the highest part of the tube above the surface of the liquid in the vessel being emptied equals the length of the *short arm* of

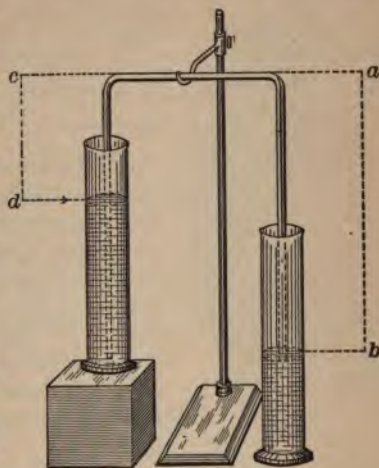


Fig. 79.

the siphon, as cd , Fig. 79; the vertical distance of this highest point above the outlet of the tube equals that of the *long arm*. When the outlet is within the liquid, the measurement must be made to the plane of the surface of the liquid, as ab , and not to the end of the tube.



Fig. 80.

158. Its Action Explained. — Experiment. — Connect a piece of rubber tube to the long arm of a siphon, so that the length of that arm may be varied by raising or lowering the end of this

When the siphon is set in operation, it will be found that the rate of flow will increase as the outer arm is lengthened, and will

decrease as this arm is shortened, the flow stopping entirely when the arms are of equal length.



Fig. 81.

Experiment. — Make a siphon of the form shown in Fig. 81, where the short arm is provided with a jet-tube opening within a bottle. If the length of the long arm is increased, the force of the fountain jet within the bottle will increase.

Experiment. — Make a glass siphon, the bore of the tube not to exceed 2 mm. in diameter. Set it in action under a bell-jar on the air-pump table, with mercury as the liquid. When the air is exhausted from the jar, the flow in the siphon will stop, but it will resume on admission of air. If the pump has a pressure gauge, it can be shown that the siphon stops when the pressure of the air in the bell-jar is not sufficient to support a mercury column as high as the top of the siphon.

The following explanation accords with these experiments:—

Let p represent the upward atmospheric pressure at the end of the tube d (Fig. 79). The pressure h of the liquid in that arm is downward. Hence the resultant pressure acting upward in the tube is $p-h$. Similarly if h' is the pressure of the liquid in the long arm, then the upward pressure in that arm is $p-h'$. The difference in resultant pressures at the two ends of the arms is therefore the difference between $p-h$ and $p-h'$, or $h'-h$, a force acting toward b . Hence the force causing the liquid to flow is measured by the pressure of a column of liquid whose height is the difference between the lengths of the arms. It follows also that the elevation over which a liquid can be siphoned cannot exceed the height of a column of that liquid which atmospheric pressure will support.

159. The Suction Pump.—In the *suction pump* a piston *c*, in which there is a valve opening upward, moves practically air-tight in a cylinder, at the lower part of which is an opening fitted with a valve *v*, also opening upward (Fig. 82). From this opening a pipe *s* leads down to a point below the surface of the water. When the piston is drawn upward, the valve in it closes by the pressure of the air above it, and a vacuum is formed in the cylinder below. The pressure of the air in the tube *s* opens the valve *v*, and the space between the piston and the water is filled with air under reduced pressure. Hence, the pressure of the atmosphere on the water in the well forces water up the tube to a height sufficient to produce equilibrium. When the piston descends, the lower valve *v* closes, while the valve *v'* in the piston opens and allows the air in the space below the piston to escape.

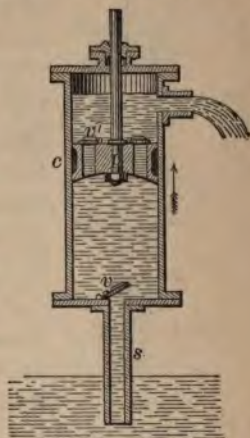


Fig. 82.

It thus appears that each double stroke of the piston removes some air from the cylinder, while the pressure of the atmosphere on the water in the well causes the water to rise higher and higher in the pipe. If the piston at the summit of its course is less than 34 ft. above the water in which the pipe *s* dips, the water under atmospheric pressure will at length follow the piston to its highest point. When the piston again descends, the valve *v'* will open and let the water through. The following up strokes will lift the water to the level of the spout.

160. The Force Pump. — In this pump (Fig. 83) the piston is solid, and the opening through which the water escapes is between it and the lower valve v , and is closed by a valve v' , opening outward from the cylinder. The explanation of the working of this pump is similar to that given for the suction pump. As in the latter, the piston p must be within 34 ft. of the water to be pumped. The height to which the water can be forced in the pipe d depends on the force applied to the piston.

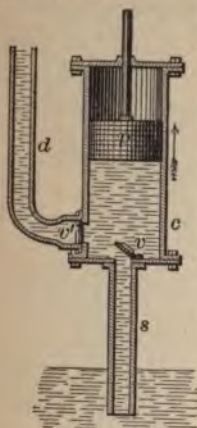


Fig. 83.

In powerful pumps the water usually passes into an air chamber called the *air dome*. Its object is to give steadiness of flow to the water from the delivery pipe. Fire engines and most pumps operated by steam are provided with an air dome.

161. Boyle's Law. — The simple relation existing between the volume of a gas and the pressure applied was first established by Robert Boyle and announced by him in 1662. It is known as *Boyle's Law* among English-speaking peoples, but the French call it Mariotte's Law. It is as follows:—

At a constant temperature the volume of a given mass of gas varies inversely as the pressure to which it is subjected.

If the volume v of a gas under a pressure p becomes v' on changing the pressure to p' , then $\frac{v}{v'} = \frac{p'}{p}$, or $p v = p' v'$; that is, the product of the volume of the gas by the corresponding pressure remains constant for the same temperature.

162. The Law Verified. — Experiment. — The apparatus (Fig. 84) consists of two glass tubes connected by a stout rubber hose and attached to a wooden support carrying a metric scale. The left-hand tube is closed at the top with an iron cap. Either tube can be fastened at any desired point on the supporting board. Clamp both tubes near the middle of the scale, unscrew the cap, and pour in mercury till the tube with the screw cap is half full. Now screw on the cap, and lower the open tube as far as possible. Note on the scale the position of the mercury in each tube, the position of the top of the capped tube, and also the reading of the barometer. Move the open tube upward a few centimetres, and repeat the readings. Continue in this way till the top of the scale is reached. Since the closed tube is of uniform bore, the volume of air will vary as the length of the column, and hence the length may be used instead of the volume. Find the length of the air column for each set of observations. The difference in the mercury readings, increased by the barometer reading, will give the pressure of the air in the tube in centimetres of mercury. If the temperature of the tube is kept constant, and the air in it is free from moisture, it will be found that the product of the length of each air column by the corresponding pressure is practically constant.

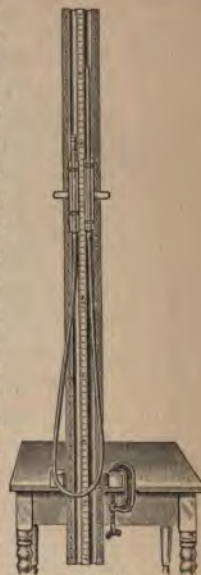


Fig 84.

The following record illustrates the foregoing:—

Top of air column 119.45 cm., Reading of barometer 74.18 cm.

MERCURY READINGS IN CENTIMETRES.		VOLUME OF AIR.	PRESSURE.	PRODUCT.
Air Column.	Pressure Column.	v	p	pv
90.50	67.20	28.95	50.88	1473
92.64	73.60	26.81	55.14	1478
94.60	79.90	24.85	59.48	1478
96.40	86.40	23.05	64.18	1479
98.03	92.80	21.42	68.95	1477
99.50	99.45	19.95	74.13	1479

Figure 85 will serve to explain the reductions of the observations in the table. *A* is the top of the mercury column in the air tube and *B* is the surface of the mercury exposed to atmospheric pressure. The volume *v* of enclosed air in terms of unit length of the tube is $119.45 - 90.5 = 28.95$. This is for the first reading in the table. The pressure on the surface *B* is 74.18 cm. of mercury as given by the barometer. The pressure at *A* is less than atmospheric pressure by the length of the column *AC*, which is $90.5 - 67.2 = 23.3$ cm. Hence the pressure *p* at *A* is $74.18 - 23.3 = 50.88$. Then $pv = 28.95 \times 50.88 = 1473$. The other reductions are made in the same way.

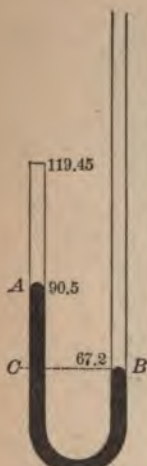


Fig. 85.

163. The Law Approximate. — Extended investigations have shown that Boyle's law is only approximately true even for air at moderate pressures. In general, gases are more compressible than Boyle's law requires. Gases like sulphur dioxide, chlorine, and carbon dioxide, which are easily liquefied by pressure, show the largest variations from the law. Near the point of liquefaction the product pv is much smaller than accords with the law.

Such gases as oxygen and nitrogen show a minimum value of pv ; beyond this minimum value an increase of pressure causes the product pv to increase. For hydrogen the value of the product is always higher than the law requires.

Within moderate limits of pressure, however, Boyle's law is extremely useful as a working relation.

Problems.

1. If a mass of gas under a barometric pressure of 72 cm. has a volume of 1900 cm³, what would be the volume at the same temperature if the pressure were 76 cm.? *1800 cm³*
2. When the barometer reading is 29 in., what will be the limiting height over which a siphon will carry water? *32.87 ft*
3. If a gas tank of 100 l. capacity contains oxygen gas under a pressure of 10 kgm. per cm², what will be the volume of the gas at the same temperature under a pressure of one atmosphere? *967.8 l*
4. If 100 cu. ft. of hydrogen gas at normal pressure are forced into a tank of 2 cu. ft. capacity, what will be the pressure per square inch in atmospheres at the same temperature? *50 atmos*
5. An open vessel is found to contain 200 gm. of air when the barometric pressure is 76 cm. How much does it contain at the same temperature when the barometric pressure is 72 cm.? *189.5 gm*
[The mass of air will vary directly as the pressure.]
6. If a litre of air at 0° C. and under a barometric pressure of 76 cm. weighs 1.29 gm., what will the same volume weigh at the same temperature if the barometric pressure is 74 cm.? *1.256 gm*
7. What pressure must be applied to a mass of gas whose volume is 1000 cm³. under a pressure of 1.033 kgm. per cm², to reduce it to 250 cm³, at the same temperature? *11.152 kgm per cm²*
8. In collecting hydrogen gas over mercury in a graduated cylinder, the volume of the gas was 25 cm³, the mercury standing 14 cm. high in the cylinder, the barometer reading 74 cm. What would be the volume of the gas, if it were under normal pressure?
[The mercury in the cylinder exerts a back pressure and reduces the direct pressure by that amount.]
9. The cylinder of an air pump has a volume of 10 cu. in., and it is connected to a receiver whose volume is 100 cu. in. How much of the original air will be left in the cylinder and receiver after the third double stroke, beginning at the top?
10. When the barometer reading is 76 cm., what is the greatest possible length for the short arm of a siphon when used for sulphuric acid (sp. gr., 1.8)? *574 cm*

CHAPTER IV.

SOUND.

I. WAVE MOTION.

164. Vibration.—**Experiment.**—Suspend a ball by a long thread and set it swinging to and fro like a common pendulum. Notice that the ball returns at regular intervals to the starting-point. Now set the ball moving in a circle, the string describing a conical surface. The ball again returns periodically to the point of departure.

A *vibrating* or *oscillating body* is one which repeats its limited motion at regular short intervals of time. A *complete vibration*, or simply a *vibration*, is the motion comprised between two successive passages of the object in the same direction through any position (§ 70).

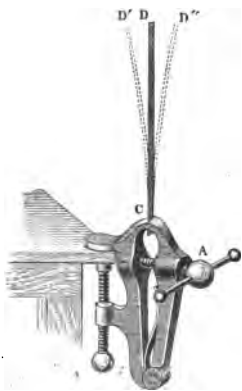


Fig. 86.

165. Vibrations Classified.—**Experiment.**—Clamp one end of a strip of brass or a lath in a vise (Fig. 86). Draw the free end aside and then release it. It moves to and fro like a common pendulum.

Vibrations of this character are called *transverse*, the motion being in a direction at right angles to the length of the vibrating body.

Experiment.—Fasten one end of a long spiral spring¹ to a hook in the wall and

¹Such a spring may be made by winding No. 18 iron or brass wire on a long rod.

hold the other end in the hand. Crowd together a few turns of the spiral and then release them. A vibratory movement will be started,



Fig. 87.

in which each coil swings to and fro in line with the length of the spiral. Fig. 87 shows the appearance of

a portion of the spiral after releasing the compressed coils.

Vibrations of this character are called *longitudinal*.

Experiment.—Twist the bob of the torsional pendulum (Fig. 3) part way around. When released it returns periodically to its initial position, as the wire twists and untwists.

Vibrations of this character are called *torsional*.

166. Simple Harmonic Motion.—

Experiment.—Suspend a ball by a long thread. Set it swinging in a circle (Fig. 88). The string describes the surface of a cone and, consequently, the pendulum is known as a conical pendulum. Place a white screen back of the pendulum and in front a lighted lamp, the light being in the plane of the circle. When the room is darkened the shadow of the pendulum bob will be seen to move to and fro across the screen in a straight line, slowly near the ends of the vibratory motion and rapidly near the middle. This shadow has nearly a *simple harmonic motion*.



Fig. 88.

Let the circle of Fig. 89 represent the path of the bob. Divide the circumference into, say twelve, equal parts, as *ab, bc, cd, etc.* Through the points of division draw perpendiculars

to the line AG . Then, the distances AB , BC , CD , etc., are the projections on the straight line of the equal arcs

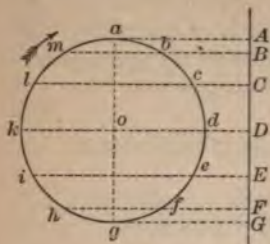


Fig. 89.

ab , bc , cd , etc., and represent the motion of the ball in successive equal periods of time as viewed from a distance. When a body vibrates to and fro in a straight line, as AG , in such a manner that its position at any moment is the same as the projection on that line of a point moving uniformly in a circle whose diameter

is the length of the straight line, it moves with what is known as *simple harmonic motion*. All pendular motions of small amplitude are simple harmonic. The name is due to the fact that musical sounds are caused by bodies vibrating in this manner. The length DA or DG is the *amplitude* of vibration, and the *period* of vibration is the time-interval between two successive passages of the body through any point in the same direction. For example, the time of the body's moving from C to G , back through C to A , and then to C is the period.

167. Waves.—Experiment.—Tie one end of a soft cotton clothesline to a rigid support. Grasp the other end and move it *up* and down quickly. Each point of the cord will be seen to *vibrate* transversely with a simple harmonic motion, and the disturbance started by the hand will move along the cord from one end to the other.

These curved forms traversing the cord are *waves*, which may, in general, be defined as the configuration of a medium caused by its parts vibrating and passing successively through corresponding positions.

168. The Harmonic Curve or Graphic Wave Form. — Experiment. — Make a pendulum, having for a bob a weighted funnel, with a very narrow outlet. Fill the funnel with fine sand, set the pendulum swinging in a plane through a small arc, and slide beneath it a board with constant velocity at right angles to the plane of the arc; the sand will be deposited in a wavy line.

This wave is the result of compounding a simple harmonic motion, with a uniform rectilinear motion at right angles to it. In Fig. 90, the vertical parallel lines *A, B,*

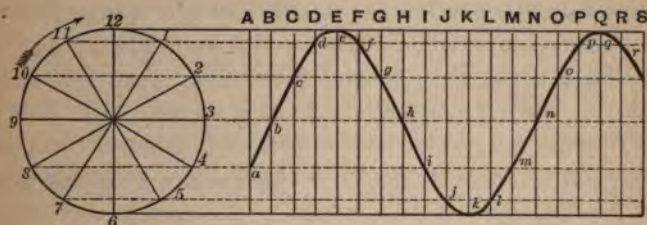


Fig. 90.

C, D, etc., represent the paths of a series of particles moving with simple harmonic motion. To find the position of these particles in their paths at equal intervals of time, draw a circle whose radius is the amplitude of vibration, divide it into any convenient number of equal parts, say twelve, and through these points draw horizontal lines cutting the vertical ones. These lines will mark off on the lines *A, B, C, D,* etc., spaces which the vibrating particles traverse in intervals of $\frac{1}{12}$ of a period. Now, if the particle in the line *A* has made any number of vibrations and one-third of an additional one, reckoned from the point 12, it will be at *a*; and if each particle in succession is $\frac{1}{12}$ of a period behind the preceding, they will be at *b, c, d,* etc., respectively. A smooth curve drawn through these points gives the wave form.

If each particle be advanced in its path $\frac{1}{2}$ of a period, and a new curve be drawn (the student should do this), the wave form will be seen to have moved to the right; and if the particle be advanced through a whole period, the wave will have gone through one complete change, the top of the wave or crest having moved from *E* to *Q*.

QUERY. — How would you diagram a wave of greater amplitude?

169. Wave Length. — The *length* of a wave is the distance from any particle in the wave to the next one in the same vibration stage, that is, in the *same phase*; as from *a* to *m*, *c* to *o*, *E* to *Q*, etc. (Fig. 90). Art. 168 shows that the wave form travels from *a* to *m*, *c* to *o*, *E* to *Q*, etc., during one period or complete vibration of the particle. Hence, *the wave length is the distance traversed by the wave during one vibration period.*

170. Kinds of Waves. — **Experiment.** — Drop a pebble into a large vessel of water. Circular waves will move outward from the disturbed point to the sides of the vessel. Float a small cork on the surface. It will rise and fall as the waves advance, but will not be carried along with them, showing that the motion of the water is up and down and not forward.



Fig 91.

Experiment. — Place a lighted candle at the contracted end of a long tin tube (Fig. 91). Over the other end tie a paper membrane. Strike two books together in front of the closed end; the flame is agitated. Tap the membrane with the finger or with a cork mallet; the flame will probably be extinguished. The effect on the candle is

due to a current of air, since the end of the pipe is closed by the membrane. If the tube is filled with smoke, it will not be driven away by a wind, but it will be agitated by the vibratory movement going through it.

These experiments illustrate two kinds of waves: the *gravitational*, or waves of *troughs* and *crests*; and the *longitudinal*, or waves of *condensation* and *rarefaction*.

1. Gravitational Waves.—The waves on the surface of water are due to the motion of its particles in closed

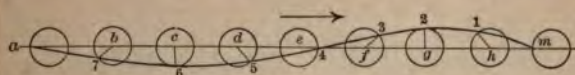


Fig. 92.

curves, which are circular when the amplitude is small. The motion which produces a wave form is shown in Fig. 92. Each circle is divided into eight equal parts; a particle is supposed to move in the circumference of each circle at the same rate, but in any two consecutive circles it is to be at points separated by $\frac{1}{8}$ of a period. Then when *a* has completed one revolution, *b* will be $\frac{1}{8}$ behind, *c* will be $\frac{2}{8}$ behind, etc. A smooth curve traced through the points found in this manner represents the form of the surface of the water. The figure shows that the crest and trough are not of equal size, the former being narrower. If the circles were larger, that is, if the amplitude were greater, the crests would be still narrower, and would be sharp or looped when the amplitude is very large. The surface of the large waves would then break into foam or white caps.

2. Compressional Waves.—In the second experiment of Fig. 170, the air in the tube next to the membrane is

compressed. The elasticity of the air forces these air particles apart again, and in turn compresses the air farther along the tube. The continued repetition of this process carries the disturbance through the tube to the flame. Each air particle vibrates longitudinally with a simple harmonic motion, the whole phenomenon being quite similar to the vibrating spiral. Fig. 93 shows the distribution

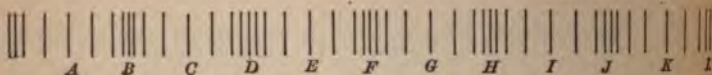


Fig. 93.

of the air particles when disturbed by compressional waves. *A, C, E, etc.*, are regions of *rarefaction*; *B, D, F, etc.*, are regions of *condensation*. Particles *A* and *C, B* and *D, etc.*, are in the *same phase*; hence the distance *AC* is a *wave length*, and *comprises one rarefaction and one condensation*.

173. Composition of Two Simple Harmonic Motions in the Same Direction. — Let the first two curves (Fig. 94) represent two wave motions in the same medium, having the same amplitude, but differing in wave length. Draw the vertical lines through *A'', B'', etc.* Lay off $A''a'' = A'a' + Aa$, $B''b'' = B'b' + Bb$, etc.; through the points *a'', b'', c'', etc.*, trace a smooth curve. It will represent the resultant wave form (§ 37). A study of the figure shows that sometimes the two waves act together, resulting in an increased amplitude, while at other times the motions impressed on the particles are opposite in direction, thus reducing the amplitude, and even at times destroying all motion. The figure illustrates the following principle: *If two waves pass simultaneously through the same medium,*

the actual motion of each particle is the resultant of the motions due to each system separately.

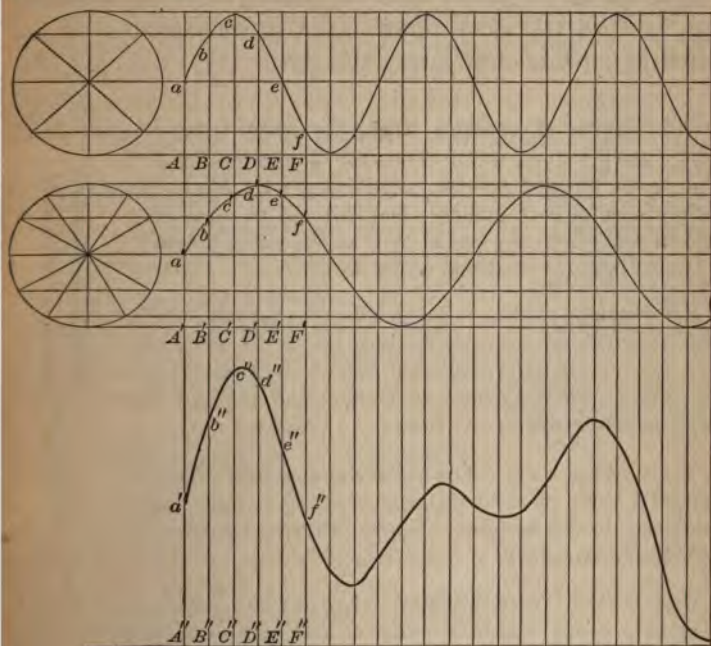


Fig. 94.

Problems.

1. Diagram two harmonic curves having equal wave lengths, the amplitudes being in the ratio of 2 to 3.
2. Diagram two harmonic curves having equal amplitudes, the wave lengths being as 1 to 2.
3. Diagram two harmonic curves of equal amplitudes and equal wave lengths, but differing in phase by one-fourth of a period.
4. Diagram two harmonic curves differing in both amplitude and wave length and then combine them.
5. Combine two harmonic curves of equal amplitudes and equal wave lengths, but differing in phase by one-fourth of a period.

II. SOUND AND ITS TRANSMISSION.

174. Sound, as distinguished from the sensation of hearing, is that vibratory disturbance in an elastic medium which is capable of affecting the ear.

175. Source of Sound a Vibrating Body.—**Experiment.**—Suspend a small ball by a thread so that it just touches the edge of an inverted bell-jar. Strike the edge of the jar with a felted or cork mallet. The ball will be repeatedly thrown away from the jar so long as the sound is heard. What must be the condition of the jar?

Experiment.—Stretch a piano wire over the table and a little above it. Draw a violin bow across the wire, and then touch it with the suspended ball of the previous experiment. So long as the wire emits sound, the ball will be repeatedly thrown away from it. Inference?

Experiment.—Tap one prong of a tuning-fork (Fig. 95) against a block of soft wood, and, while sounding, touch one prong to the surface of water. In what condition is the fork shown to be?



Fig. 95.

Experiment.—Insert a whistle in one end of a glass tube (Fig. 96). Distribute evenly within the tube a little cork dust, made by filing



Fig. 96.

cork. Close the other end of the tube, and blow the whistle, holding the tube in a hori-

zontal position. The cork dust will rise in parallel vertical layers, falling back into ridges transverse to the axis of the tube when the sound ceases. Does the behavior of the cork dust indicate a vibratory motion of the air in the tube, or a current?

These experiments prove that the sources of sound are bodies in a state of vibration, the energy of the motion being sufficient to affect the ear.

176. Air a Medium. — Experiment. — Suspend an electric bell in a receiver on the air-pump table (Fig. 97). Set the bell ringing and exhaust the air from the receiver. The bell is heard less and less distinctly as the exhaustion proceeds, and would become inaudible in a perfect vacuum were it not that the suspension wires conduct the sound (§ 178). Readmit the air and the sound is restored. If, after exhausting the air, hydrogen is admitted and then exhausted, the diminution of the sound of the bell will be more marked (§ 197).

The experiment shows that air transmits sound, and also that sound cannot traverse a vacuum. By filling the jar with any kind of gas it may be shown that any gas transmits sound.



Fig. 97.

177. Liquids as Media. — Experiment. — Fill a tumbler with water, or any other liquid, and set it on the table. Insert the stem of a tuning-fork in a thin disk of wood about 3 cm. in diameter. Set the fork in vibration and hold it with the wooden disk resting on the liquid in the tumbler. The fork, which could scarcely be heard when held in the hand, will now be heard distinctly, the sound seeming to come from the table.

The vibrating fork, through the agency of the wooden disk on its stem, throws the liquid into vibration. These vibrations are transmitted by the liquid to the table, and thence to the air of the room.

178. Solids as Media. — Experiment. — Hold one end of a long slender wooden bar against the door of the room. Rest the stem of a vibrating tuning-fork against the free end. The sound of the fork will appear to come from the door.

The wood, like the water of the previous experiment, transfers the energy of the fork's vibrations to the

door, and the door in turn to the air of the room. It is a familiar fact that, by placing the ear in contact with the metal rail of a railway track, two sounds can be heard, if the rail be struck at some distance away, one sound coming through the rail and the other through the air. The report of a cannon has been heard more than 250 miles by applying the ear to the ground. The great eruption of Cotopaxi in 1744 was heard distinctly 500 miles away, although several gigantic mountains and numerous deep valleys intervened.

The *acoustic telephone*, familiarly known as the *string telephone*, is a practical application of the sound-transmitting qualities of solids. It was invented in 1667 by Robert Hooke. It consists of a string or wire attached to the thin elastic bottoms of two small conical boxes. By speaking into either of these boxes, one listening at the other can hear distinctly, even for a considerable distance. The membrane vibrating transversely sets up longitudinal vibrations in the wire. These are transmitted to the membrane of the receiving instrument, and reproduce the sound actuating the transmitting instrument.

179. Sound Waves.—When a body, as a tuning-fork, is set in vibration, the disturbances produced in the air around it are known as *sound waves*. These waves consist of a series of condensations and rarefactions, succeeding each other at regular intervals, and forming concentric spherical shells of air of different densities. Each air particle vibrates harmonically and longitudinally in a short path along the radius of the expanding sphere. A *ray* of sound is the line which marks the direction of propagation; it is a radius of the spherical shell, and hence is a perpendicular to the wave front.

III. VELOCITY OF SOUND.

180. Velocity in Air.—In 1738 a commission of the French Academy, and again in 1822 a second scientific commission, experimented to determine the velocity of sound. The method of procedure was to divide into two parties, and by firing a cannon alternately at the two stations to determine the interval between the observed flash and the report. The mean of an even number of results eliminates very nearly the effect of the wind. The final result obtained was 331 m. per second at 0° C. The defect in this method is that the perception of sound and of light are not equally quick, and vary with different persons. Stone determined the velocity of sound in 1871 by stationing two observers three miles apart to give signals by electricity on hearing the report of a cannon. This method employs the sense of hearing only. After correcting as far as possible for all sources of error, the value obtained was 332.4 m. or 1090.5 ft. per second at 0° C. At 20° C. the velocity is about 1130 ft. per second.

181. Velocity in Gases.—It was shown by Newton that the velocity of propagation of a wave through any medium varies directly as the square root of the coefficient of elasticity of volume (§ 14), and inversely as the square root of the density ($v = \sqrt{\frac{e}{d}}$). Since the density of oxygen is sixteen times that of hydrogen, it follows that sound will travel in hydrogen four times as fast as in oxygen. Subjecting a gas to pressure increases its coefficient of elasticity and its density at the same rate, and hence does not affect the velocity of sound in it. Heating a gas, however, increases the coefficient of elasticity, and hence

increases the velocity of sound. Experiment and calculation agree in showing that the correction is 0.6 m., or nearly 2 ft. for 1° C. in air.

182. Velocity in Liquids.—In 1827 Colladon and Sturm, by a series of experiments in Lake Geneva, found that sound travels in water at the rate of 1435 m. per second at a mean temperature of 8.1° C. Subsequent experiments show that the velocity is affected by changes of temperature and by the intensity of the vibration producing the sound. The velocity of sound in liquids is greater than in gases, owing to the fact that their coefficient of elasticity in proportion to their density is much greater.

183. Velocity in Solids.—The velocity of sound in solids is generally greater than in liquids on account of their high coefficient of elasticity as compared with their density. The velocity in iron is 5127 m. per second; in glass 5026 m. per second; but in lead, on account of its low elasticity, it is only 1228 m. per second, the temperature in each case being 0° C.

Problems.

1. Calculate the velocity of sound in air when the temperature is 25° C.

2. How long will it take sound to travel 2 mi. in air, the temperature being 20° C.?

3. In an experiment to determine the velocity of sound, two stations were chosen, distant 5 km. from each other. It was found that the interval between seeing the flash of the gun and hearing the report was 15.5 sec. at one station and 14.5 sec. at the other. Calculate the velocity of sound and that of the wind.

4. On a day when the temperature was 24° C., the interval between seeing a flash of lightning and hearing the thunder was 5 sec. How far away was the lightning?

5. A shell fired at a target, distant half a mile, was heard to strike it 5 sec. after leaving the rifle. What was the average velocity of the bullet, the temperature of the air being $20^{\circ}\text{C}.$ ✓

[Let x represent the velocity of the bullet. How long was the bullet in going? How long was the sound in returning?]

6. To what temperature must the air be heated to increase the velocity of sound 10 per cent. ? 55.4°C ✓

7. If the velocity of sound in air at $20^{\circ}\text{C}.$ be taken as 1120 ft. per second, what would be the velocity in carbon dioxide at that temperature, carbon dioxide being assumed to be one and one-half times as heavy as air ? 918.5 ft ✓

8. The leaning tower of Pisa is 288 ft. high. For an observer at the top, what time will elapse between the dropping of an iron ball from the top of the tower and hearing it strike the pavement at the bottom, if the temperature of the air is $24^{\circ}\text{C}.$ ✓

9. If sound travels in air at the rate of 340 m. per sec., and in iron at the rate of 5130 m. per sec., what would be the interval between hearing a sound through an iron bar 1 km. long and through air ? 2.75 sec ✓

$4-6-8$ (5-7)

IV. REFLECTION AND REFRACTION OF SOUND.

184. Reflected Sound Waves. — Experiment. — Suspend a loud-ticking watch a little in front of the focus (§ 250) of a large concave reflector, as at W in Fig. 98. A place will be found at some distance in front where the watch can be heard with great distinctness, as at E ; but if the reflector be removed, the ticking is nearly, if not quite, inaudible.

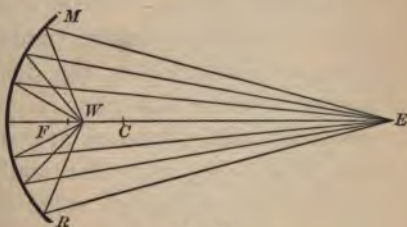


Fig. 98.

When sound waves strike against a smooth surface they are reflected in the same manner as when an elastic body strikes such a sur-

face. If the surface be concave, they are reflected to a point. In Fig. 98 the rays of sound are used instead of waves for simplicity. These, diverging from *W*, are focussed at *E*, just as in the case of light (§ 252).

The ear trumpet is an instrument whose action depends upon the reflection of sound. The sides of the bell-shaped mouth reflect the sound into the tube which conveys it to the ear. Sounding-boards are sometimes placed back of speakers in large halls to reflect those portions of the sound waves that pass back of the speaker.

185. Echoes.—An *echo* is the repetition of a sound caused by the reflection of sound waves from some distant surface, like that of a building, or from cliffs, clouds, trees, etc. The interval between the production of a sound and the perception of its echo is the time that sound takes to travel from the source to the reflecting body and back again. The sensation of sound lasts for about one-tenth of a second, and during that time the sound wave travels $\frac{1}{10} \times 1130$, or 113 ft. If, then, the reflecting surface be about 56 ft. distant, a short sound will be followed immediately by its echo, since the first sound wave will travel to the reflecting surface and back to the ear to renew the sensation just as the first one ceases. If the distance be much less than 56 ft., the reflected sound tends to strengthen the original one, as illustrated by the distinctness of sounds in an ordinary room. The poor acoustic properties of many large halls and churches are due to the confusion of echoes from the large flat walls. Rooms draped with bunting or hangings of some sort have the echo deadened, because of the diffused reflection from the folds of the drapings.

Multiple echoes are caused either by independent reflec-

from bodies at different distances, or by successive reflections, as in the case of parallel walls at a suitable distance apart. The roll of thunder is partly due to multiple reflection. Multiple echoes following one another rapidly produce *reverberations*.

6. **The Whispering Gallery.**—A curious effect of sound reflection is met with in the *whispering gallery*, where a sound produced at one point of a very large room is heard distinctly at some distant part, but is inaudible at places between; or where it is heard all round near the walls, but at no other place. In the first case, the walls are curved reflectors (§ 184), and concentrate the sound to a point. In the second case, the sound is reflected from point to point along the curved wall, traveling in a series of equal chords, making the sound audible all round the wall. This is the case with the circular gallery in the dome of St. Paul's Cathedral in London, where a whisper is perfectly audible when the speaker and listener are exactly opposite each other.

7. **Refraction.**—**Experiment.**

Suspend a large toy balloon with carbon tetrachloride and hang it up. Suspend a ticking watch near it (Fig. 99).

There will be a point on the opposite side, but farther away, at which the ticking can be heard very distinctly.

The balloon acts on the sound waves in the same manner that a convex lens acts on light waves, converging them to a focus (§ 250). The explanation



Fig. 99.

nation is to be found in the fact that the middle portions of the waves on striking the balloon are retarded, the outer portions, so to speak, getting ahead and curving around, so that the waves are converted from a convex into a concave form. A balloon filled with hydrogen gas would act in the opposite manner, the middle of the waves getting ahead.

188. Wind Refraction. — It is a matter of common observation that sounds heard with the wind are louder than those heard against it. The reason is to be found in the fact that the velocity of the sound in the first case is increased by that of the wind. But the velocity of the wind is less near the earth's surface than a little above. Hence, the part of the sound waves touching the earth travels more slowly than that above; the waves are thus deflected downward and the sound is condensed along the earth's surface. In the second case, the lower part of the sound wave will be less retarded by the wind than the upper part and hence will get ahead of it. The wave is thus deflected upward.

Problems.

1. Why are there echoes in very large halls?
2. Why are multiple echoes often heard near a forest?
3. A steamer passing near a tall cliff blows a short sharp whistle, and 2 sec. later the echo is heard. The temperature of the air is 20°C . What is the distance of the steamer from the cliff?
4. It is claimed that a speaker cannot enunciate more than five syllables per second and be clearly understood, owing to the residual impression that each sound leaves on the ear. If the temperature is 20°C ., how far distant must a reflecting surface be to reëcho clearly a word of three syllables?
5. At what distance is an observer from a reflecting surface which repeats a sound after 3 sec., the temperature of the air being 21°C .?

V. FORCED AND SYMPATHETIC VIBRATIONS.

189. Forced Vibrations. — **Experiment.** — Suspend a heavy weight by a cord or wire so that it will vibrate nearly as a seconds pendulum. From this weight suspend by a short thread a small weight, as a bullet. Set the system vibrating. The large ball by its superior energy impresses its own period of vibration on the small one and forces it to vibrate with it.

The experiment is an illustration of *forced vibrations*, which in general may be defined as vibrations not agreeing in period with the natural period of the vibrating body, but with that of the periodic force acting on it.

190. Illustrations. — The sounding-board of a piano and the membrane of a banjo are forced into vibration by the strings stretched over them. Two clocks which have nearly the same rate when on separate stands, will keep exact time together when they are placed on the same shelf, because each pendulum supplies a periodic force which acts on the other. They therefore exercise mutual control. The two prongs of a tuning-fork naturally vibrate at slightly different rates on account of unavoidable differences between them; but since they are connected at the stem, the faster one tends to accelerate the slower, and the slower to retard the faster, with the result that they agree in rate. The top of a wooden table may be forced into vibration by pressing against it the stem of a vibrating tuning-fork, and the loudness of the sound is greatly increased. This is a case of forced vibrations, and the table will respond to a fork of any pitch.

191. Sympathetic Vibrations. — **Experiment.** — Place near each other on the table two mounted tuning-forks tuned to exact unison. Keep one of them in vibration for a few seconds, and then stop it. The other one will be heard to sound loudly.

This experiment illustrates *sympathetic vibrations* in bodies having the same natural vibration period as the periodic force. In the case of the forks, the pulses in the air reach the second fork at intervals corresponding to its vibration period, and their effect is cumulative, each impulse arriving just in time to add to the movement. If the forks differ in period, the impulses from the first will not produce cumulative effects on the second, and the second fork will fail to respond.

In *forced vibrations* a vibrating body is compelled to surrender its preference for a particular mode and rate of vibration, and to adopt with more or less accuracy those imposed upon it by some external periodic force. But when there is equality of period between the periodic force and the natural vibration of the body, the co-vibration of the two is known as *resonance*.

192. Illustrations.—Resonance may be mechanical as well as sonorous. A heavy weight suspended by a rope may be set swinging through a wide amplitude by tying to it a thread and pulling gently on it when the weight is moving in the direction of the pull. Each effort then adds to the accumulated motion; the series of small impulses at the right intervals unite to produce a large movement.

If two heavy pendulums, suspended side by side on knife-edges on the same stand, are carefully adjusted to swing in the same period, and one of them is set swinging, it will cause the other one to swing, and will give up to it nearly all its motion.

Many years ago a suspension bridge at Manchester, in England, was destroyed by its vibration reaching an amplitude which exceeded the limits of safety. The cause was the regular tread of troops keeping time with what proved

be the natural rate of vibration of the bridge. Since the custom has always been observed of breaking up when bodies of troops cross a bridge.

Release the wires of a piano by pressing the loud pedal, a note sung near it will be echoed by the wire which gives one of the same pitch. The "sound of the sea" heard when a sea shell is held to the ear is a case of resonance. The mass of air in the shell has a vibration rate of its own, and it amplifies any faint sound of the same period. A vase with a long neck will also exhibit resonance.



Fig. 100.



Fig. 101.

193. Air Resonators. — Experiment. — Hold a vibrating fork over the mouth of a cylindrical jar (Fig. 100). Pour in water slowly, and note that, as the air column becomes shorter, the sound grows louder till a certain length is reached, after which it becomes weaker. Forks of different pitch are tried, each will be found to have a different length of air column for reinforcing its sound.

When the prong at *a* (Fig. 101) moves to *b*, it makes half a vibra-

tion, and generates half a sound wave. The pulse it sends down the tube AB is reflected from the bottom. Now, if AB is one-fourth of



Fig. 102.

in the air outside the tube, and so to increase their amplitude, but will reduce it instead.

Unless the length of the air column is large in comparison with its diameter, it will be somewhat less than one-fourth the wave length of the sound reënforced. The box on which a tuning-fork is mounted (Fig. 102) is a resonator, designed to increase the volume of sound.

194. The Helmholtz

Resonator.—The resonator devised by Von Helmholtz, for the pur-

pose of picking out the overtones (§ 215) in a composition, is spherical in form, with two short tubes opposite sides (Fig. 103). The larger opening, A , is the mouth of the resonator; the smaller one, B , fits in the ear. These resonators are made of thin brass or glass and their pitch is determined by their size. When one



Fig. 103.

of them is held to the ear, it strongly reënforces any sound agreeing with it in pitch, but is silent to others.

VI. INTENSITY AND LOUDNESS.

Intensity - loudness
Key - timbre
1 degree of audibility
{ amplitude (and distance) density = air
195. The Physical Intensity of a sound varies as the energy of the vibrating particles of the medium. The *loudness* of a sound depends on the individual sensitiveness of the ear, and on the extent of the physical disturbance reaching the ear drum. The loudness of a sound also involves the pitch. Intensity refers to the mechanical action that gives rise to the sound, while loudness refers to the sensation produced.

196. Effect of Amplitude. — **Experiment.** — Strike one of the prongs of a tuning-fork a slight blow. The sound emitted is feeble, and the prongs when touched to water disturb it but little. Now strike the fork a sharp blow; the sound is much louder, and when the prongs touch the water, it is thrown about more vigorously.

When the amplitude of vibration of the fork is large, a correspondingly large amplitude of vibration is imparted to the air. Since a vibrating body, like a pendulum, has a constant period nearly independent of amplitude, the mean velocity of the vibrating air particles must vary as the amplitude. But the energy of the movement varies as the square of the velocity (§ 85); hence, the intensity of sound varies as the square of the amplitude of vibration.

197. Effect of Density of Medium. — **Experiment.** — Fill a large bell-jar with hydrogen or coal gas. Raise the jar, keeping the mouth downward, and ring within it a small bell. The sound is much fee-

bler than when the jar is filled with air. Now fill a large jar with carbon dioxide, and ring the bell in it. The sound is louder than in air.

The loudness of a sound depends, therefore, on the density of the medium at the place where the vibration is imparted to it.

The energy of the wave motion set up by the bell in the light gas is less than that in the dense one, and there is a corresponding difference in loudness. If a tuning-fork were used in conducting the experiment, the duration of vibration would be found to be longer in the rarer gas, so that the total amount of energy absorbed by the medium from the fork would be the same in each case, as required by the doctrine of Conservation of Energy (§ 87). On high mountains, where the air is quite rare, conversation is carried on with difficulty, and the firing of a gun produces little noise; while one fired below may be heard as a loud report, even at great elevations.

198. Effect of Distance. — As the sound waves move outward from the vibrating body, each spherical layer of air imparts its energy to the enveloping one. Since these layers are surfaces of spheres, the number of particles composing them increases as the squares of their radii. Hence, the energy of the individual particles must decrease in like ratio, that is, *the intensity of sound varies inversely as the square of the distance from the source.*

During the vibratory movement of the air, some of the mechanical energy is transformed into heat by friction and viscosity and is dissipated. Hence, the actual decrease in intensity of sound is greater than that given by the theoretical law of inverse squares.

199. The Speaking Tube. — The weakening of sound from the enlargement of the sound waves as they recede from the source would evidently not take place if they were confined within a tube. Under such a condition the sound waves would not be propagated as concentric spheres, but the successive layers of air affected would be of equal mass, and the sound would be conveyed with little loss of intensity. Tubes used in this way are called *speaking tubes*. Long galleries, water pipes, and sewers act as speaking tubes.

200. Effect of Area of Vibrating Body. — Experiment. — Compare the sound of a small tuning-fork with that of a large one of the same pitch (§ 205). The large one produces the louder sound.

In order that a vibrating body may be a source of sound, the condensations and rarefactions in the air must be well marked. When the object is small, its surface is insufficient to affect a large quantity of air. Hence *the intensity of sound depends on the area of the sonorous body.*

Illustrations of this fact are found in many stringed musical instruments, where the sound is intensified by placing two or more strings side by side when they are of small diameter; and, secondly, by placing a sounding-board beneath them to be set in motion by the string. The loud sound produced by many wind instruments is explained by the fact that the air within the broad aperture opposite the mouthpiece is a vibrating body of large area.

VII. INTERFERENCE AND BEATS.

201. Interference. — Experiment. — Hold a vibrating tuning-fork over a cylindrical jar, serving as a resonator. Turn it slowly around its axis and notice that when the edge of the prong is toward the jar

the sound is nearly inaudible. When in one of these positions cover one prong with a pasteboard tube (Fig. 104). The sound is restored to nearly maximum intensity.



Fig. 104.

The explanation of this experiment is found in the fact that when the two prongs of the fork approach each other a condensation is produced in the air between them, and at the same time two rarefactions are started from the backs of the prongs. These opposite movements

communicated to the air meet along surfaces extending outward from the edges of the fork and there neutralize each other. This explanation is supported by the fact of the restoration of the sound on cutting off one set of waves by the paper cylinder.

Interference is the superposition of two similar sets of waves traversing the medium at the same time. If two sound waves of equal length and amplitude meet in opposite phases, the condensation of one corresponding with the rarefaction of the other, the sound at the place of meeting is extinguished by destructive interference; if their phases are not precisely opposite or their amplitudes not equal, the extinction of the sound is not quite complete, or the interference is partial. One of the two series of similar waves may be direct and the other reflected.

202. Beats. — **Experiment.** — Select two large tuning-forks of the same pitch. When they are set vibrating, the sound is smooth as if only one fork were vibrating. Stick a piece of wax to a prong of one of the forks; the sound will be pulsating or throbbing.

Experiment. — With glass tubes and jet-tubes set up the apparatus of Fig. 105. Provide one tube with a paper slider so that its length may be varied. When the gas flame is turned down to proper size, the tube gives off a continuous sound, and we have what is known as a *singing flame*. By moving the slider, the tubes may be made to yield the same tone, the combined sound being smooth and steady. Now change the position of the slider, and the sound throbs and pulsates in a very disagreeable manner.

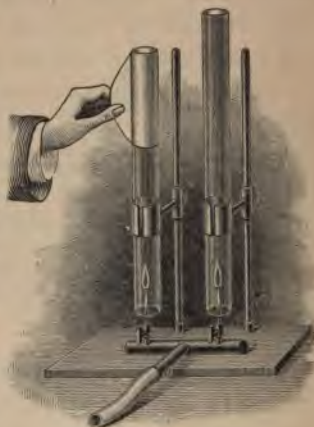


Fig. 105.

Both of these experiments are illustrations of the interference of two sets of sound waves. The outbursts of sound, followed by comparative silence, are called *beats*.

203. Number of Beats. — Let two sounds be produced by forks making, for example, one hundred and one hundred and twenty vibrations per second respectively. Then, in each second the latter fork gains twenty vibrations on the former; and there must be twenty times during each second when they are vibrating in the same phase, and twenty times in opposite phase. Hence, interference and subsidence of sound must occur twenty times during the second, and twenty beats are produced. / Therefore, the number of beats per second is equal to the difference of the vibration rates of the two sounds. /

VIII. PITCH.

204. Musical Sounds are those which are pleasant to the ear, and are caused by regular periodic vibrations. A *noise* is a disagreeable sound, either because the vibrations producing it are not periodic, or because it is a mixture of discordant sounds (§ 223), like the clapping of the hands.

Experiment.—Attach a rose burner to a metal pipe about 15 in. long, and connect it with the gas service by a rubber tube. Light the gas and notice the rustling sound attending its burning. Now hold a large tin tube, several feet long, over the burner. At a certain position of the flame within the tin tube, a sound like that of an organ pipe will be obtained. With tubes of different lengths, the pitch will be different.

The experiment shows that the rustling of the flame is caused by the mixing of many different sounds. If these sounds were not present they could not be reënforced by the different air columns.



Fig. 106.

205. Pitch.—**Experiment.**—Mount on the axle of a whirling machine (Fig. 106) or on the armature of a small electric motor a cardboard disk (Fig. 107) provided with several concentric rows of equidistant holes differing in number, or several toothed wheels differing in the number of

teeth. When rotating rapidly blow a stream of air from the tube *T* against one of the circles of holes in *D*, or press a thin card *C* against one of the toothed wheels *W*. In either case a distinct note is heard, different for each series of holes, or for each toothed wheel, or for any change in speed of rotation.

In each case, waves are produced in the air which, following each other with definite rapidity, give that characteristic to the sound which is called *pitch*. The perforated disk is called a *siren*, and the toothed wheel is a form of a *Savart's wheel*.

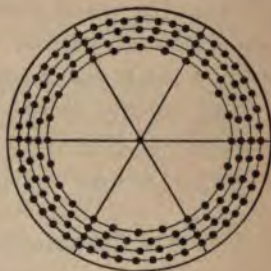


Fig. 107.

Either of these devices may be used in measuring the pitch of a note. If the number of holes in the circle or of teeth in the wheel be multiplied by the number of revolutions made per second, the product will be the frequency or vibration rate of the sound produced. Audible sounds have a lower limit of about sixteen vibrations a second, and an upper limit of about forty thousand. Most musical sounds are comprised between twenty-seven and four thousand vibrations a second.

206. Relations between Pitch, Wave Length, and Velocity.— If a tuning-fork makes 256 vibrations per second, and in that time a sound wave travels in air 344 m., then the first wave formed will be 344 m. from the fork on the completion of the 256th vibration. Hence, in 344 m. there would be 256 waves, and the length of each is $\frac{344}{256}$ m., or 1.344 m. In general, if l = wave length, v = velocity, and n = vibration rate, then

$$l = \frac{v}{n}, v = nl, \text{ and } n = \frac{v}{l}. \quad (25)$$

✓ **207. Intervals.** — A *musical interval* is the relation between two sounds expressed by the ratio of their frequencies. Many of these intervals have definite names. When the ratio is 1 it is called *unison*, 2 an *octave*, $\frac{3}{2}$ a *fifth*, $\frac{4}{3}$ a *fourth*, $\frac{5}{4}$ a *major third*, $\frac{6}{5}$ a *minor third*, $\frac{2}{1}$ a *chromatic semitone*. Any three notes whose frequencies are as 4:5:6 form a *major triad*, and together with the octave of the lowest a *major chord*. Any three notes whose frequencies are as 10:12:15 form a *minor triad*, and together with the octave of the lowest a *minor chord*.

208. The Diatonic Scale or Gamut. — This is a series of eight notes which succeed each other with gradually increasing pitch, the two extremes being an octave apart. The first, or lowest note, is called the *keynote*, and the last is regarded as the keynote of another set of eight notes. In this way the series is repeated till the limit of pitch is reached, or a sufficiently extended scale is obtained. The tones comprised in each octave are named C, D, E, F, G, A, B, C'. The keynote may be given any pitch at pleasure. Physicists have agreed to assign to C, known as "middle C," 256 vibrations per second. In music the standard of pitch is variable; in the United States piano manufacturers agreed in 1892 to adopt as their standard $A = 435$.

The relative values of the notes composing the gamut is shown in the following table:—

Name . . .	C	D	E	F	G	A	B	C'
Vibration No.	256	288	320	$341\frac{1}{3}$	384	$426\frac{2}{3}$	480	512
Vibration ratio	C	$\frac{3}{2}C$	$\frac{4}{3}C$	$\frac{4}{3}C$	$\frac{3}{2}C$	$\frac{5}{3}C$	$\frac{15}{8}C$	2C
Intervals . .		$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{11}{10}$

An inspection of this table shows that the intervals between the successive tones are not equal, but are of

three kinds,— $\frac{2}{3}$, called a *major tone*; $\frac{1}{9}$, a *minor tone*, and $\frac{1}{12}$, a *major semitone*,—and that they succeed each other in a definite order. If a note be raised by a chromatic semitone, $\frac{2}{24}$, it is said to be *sharpened*, and if lowered by $\frac{1}{24}$, to be *flattened*.

209. The Tempered Scale.—If C were always the keynote, the diatonic scale would be sufficient for all purposes except for minor chords; but if some other note be chosen for the keynote, in order to maintain the same order of intervals new and intermediate notes will have to be introduced. For example, let D be chosen for the keynote, then the next note will be $288 \times \frac{2}{3} = 324$ vibrations, a number differing slightly from E. Again, $324 \times \frac{1}{9} = 60$, a note differing widely from any note in the series. In like manner, if other notes are taken as keynotes, and a scale is built up with the order of intervals of the diatonic scale, many more new notes will be needed. This interpolation of notes for both the major and minor scales would increase the number in the octave to seventy-two.

In instruments with fixed keys such a number is unmanageable, and it becomes necessary to reduce the number by changing the value of the intervals. Such a modification of the notes is called *tempering*. Of the several methods proposed by musicians, that of *equal temperament* is the one generally adopted. It makes all the intervals from note to note equal, interpolates one note in each whole tone of the diatonic scale, and thus reduces the number of notes in the octave to twelve. Each interval is a semitone and equals $\sqrt[12]{2}$ or 1.05946. The only accurately tuned interval in this scale is the octave; the thirds are sharp, and the fifths flat. The following table shows

the differences between the diatonic and the equally tempered scales:—

	C	D	E	F	G	A	B	C'
Diatonic . .	256	288	320	341.3	384	426.7	480	512
Tempered . .	256	287.3	322.5	341.7	383.6	430.5	483.3	512

Problems.

1. If a bell is struck with a hammer, the sound gradually "dies away." Explain.

2. Why is the pitch of the sound emitted by a phonograph raised by increasing the speed of the cylinder?

3. What is the length of the sound waves in air produced by a tuning-fork vibrating 320 times per second, the temperature being 20° C.?
107.6 cm

✓ 4. What is the vibration rate of a reed that produces waves 134.53 cm. long in the air, the temperature being 20° C.?
256 vib.

5. Water is poured into a cylindrical jar till the air column reinforces the sound of a tuning-fork. The length of the air column is then 33.63 cm. and the temperature of the air 20° C. What vibration rate of the fork do these facts show?
256 vib.

✓ 6. A siren has 24 holes in the plate and makes 1000 revolutions in 1 min. What is the frequency of the note emitted?
40 vib.

✓ 7. The wheel of Savart's apparatus has 300 teeth, and makes 77 revolutions per minute. What is the frequency of the note emitted?

✓ 8. Calculate the length of the shortest air column in a cylindrical jar that will strongly reinforce the sound of a tuning-fork having a vibration rate of 512, the temperature being 16° C.
16.7 cm

9. How many beats are produced per second by the two notes "middle C" and D of the same octave in the diatonic scale (physicists' pitch)?
32 beats per sec

10. Calculate the pitch of D sharp and E flat in the diatonic scale (physicists' pitch).
*D sharp 309
E flat 307.2*

1. Taking C (256) as the keynote, calculate the frequency of the triad. What notes on the diatonic scale give this chord? ✓
2. Calculate the frequency of the note a fourth above "middle (physicists' pitch). What is its wave length when the temperature of the air is 16°C ? $341\frac{1}{3} \approx 7100.7 \text{ cm}$ ✓
3. Standard pitch is based on $A = 435$. Calculate the frequency "middle C" diatonic scale and "middle C" equally tempered scale.
4. Calculate the interval between a minor tone and a major tone.
5. How many vibrations give the major third, the fifth, and the seventh respectively of $E = 320$ (diatonic scale)?
6. Calculate the wave length of the note E which is a third above "middle C," the temperature being 10°C . Use physicists' pitch.
7. A tuning-fork is held over a long glass tube partly filled with cold water. The maximum reinforcement of sound occurs when the air column in the tube above the water is 52 cm. long. Find the vibration number of the fork. 159.8

IX. VIBRATIONS OF STRINGS.

10. **Mode of Vibration.** — Strings when used for the production of sound, are fastened at their ends, stretched to the proper tension, and made to vibrate transversely either by drawing a violin bow across them, striking them with a light hammer, or plucking them with the fingers. On examination of any stringed musical instrument, as a violin, will make it evident that by varying the tension, the length, or the mass per unit length of the wires or strings, tones of any desired pitch may be secured.

11. **Laws of Strings.** — In order to study the laws governing the vibration of strings, an instrument called a *monometer* is used. It consists of a thin wooden box (Fig.

108), near the ends of which are fixed bridges, *A* and *D*. Wires or strings may be stretched lengthwise of the box by attaching them to the pins set in the frame at one end and to the weights at the other, the wires passing over

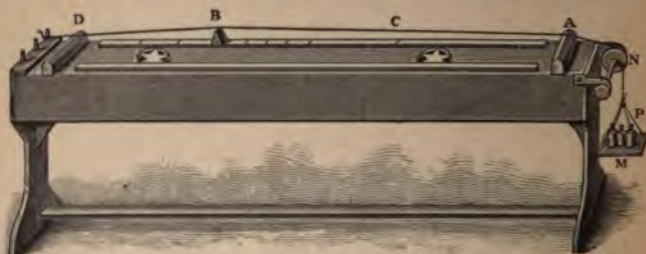


Fig. 108.

pulleys, as at *N*. By means of a movable bridge, *B*, the length of a wire may be shortened at pleasure. Below the wires there is a scale of equal parts.

Experiment.—Stretch two similar wires on the sonometer and tune them to unison by varying the weights. With the movable bridge shorten one of them successively to $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{2}{5}$, etc. The successive intervals between the notes given by the long wire and the shortened one will be $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, etc., and the notes emitted by the wire of variable length will be those of the diatonic scale. Hence,

✓ **THE LAW OF LENGTHS.** — *The tension and the diameter being constant, the vibration number varies inversely as the length.*

Experiment.—Stretch two similar wires with unequal known tensions. Shorten the one of lower pitch till it is in unison with the other. The ratio of the lengths will be that of the square root of the tensions. Hence,

✓ **THE LAW OF TENSIONS.** — *The length and the diameter being constant, the vibration number varies as the square root of the tension.*

If, for example, the tensions are as four to one, the lengths will be as two to one for unison. The double length reduces the vibration number to one-half, and this reduction is offset by the double frequency due to the quadrupling of the tension.

Experiment.—Stretch equally two wires differing in diameter and material, that is, in mass per unit length. Bring them to unison with the movable bridge. The ratio of the lengths will be the inverse of that of the square roots of the masses per unit length. Hence,

THE LAW OF MASSES.—*The length and tension being constant, the vibration number varies inversely as the square root of the mass per unit length.*

212. Applications.—In the piano, violin, harp, and other stringed instruments, the pitch of each string is determined partly by its length, partly by its tension, and partly by its size. The tuning is done by varying the tension.

X. OVERTONES AND HARMONIC PARTIALS.

213. Fundamental Tone.—**Experiment.**—Fasten a silk thread to one prong of a large tuning-fork (Fig. 109). Set the fork vibrating and apply tension to the thread till it vibrates as a single spindle.



Fig. 109.

The experiment illustrates the manner in which a string or wire vibrates when emitting its lowest tone. The

fundamental tone of a vibrating body is the lowest *tone* that it can yield. It is produced when the body vibrates as a whole, or in the smallest number of segments possible.

214. Nodes and Ventral Segments. — **Experiment.** — Proceeding as in the last experiment, it will be found that under a suitable



Fig. 110.

tension the thread can be made to vibrate in a number of parts (Fig. 110), giving it the appearance of a succession of spindles.

Experiment. — Stretch a wire on the sonometer with a thin slip of cork strung on it; then touching the cork lightly at one-third or one-fourth, or any aliquot part from one end (Fig. 111), bow or pluck the



Fig. 111.

shorter portion. The wire will vibrate in equal segments. This may be made more evident by placing narrow V-shaped pieces of paper, or *riders*, on the wire before bowing it. Some of them will be thrown off, and others will remain on, marking the places of maximum and of minimum vibration respectively.

The intermediate points of minimum vibration and the loops are called *nodes*; the vibrating portions between the nodes are called *loops* or *ventral segments*; and the middle points of the loops are called the *antinodes*. The experiment also illustrates *stationary waves*. The nodal points are caused by the bowed segment sending out waves along the wire, which interfere with similar waves reflected from the opposite end. The distance between two nodes is half a wave length.

215. Overtones and Harmonics. — **Experiment.** — Stretch a wire on the sonometer and set it in vibration by plucking or bowing it near one end. The tone heard most distinctly is its fundamental. Touch the wire lightly at its middle point. Instead of stopping the sound, a note an octave higher will be heard, showing that the wire is vibrating in two parts. If the wire be again plucked, both sounds can be heard together. Touching the wire one-third from the end brings out a tone an octave and a fifth higher, showing that the wire vibrates in thirds the same time that it is vibrating as a whole. With a long string, it is possible to prove that a still further subdivision of a vibrating string takes place. In conducting such experiments, care must be exercised in selecting the point at which the string is plucked, for it is evident that there can be no node at that point.

The tones produced by sounding bodies vibrating in parts are called *overtones* or *partial tones*. If the vibration rate of an overtone is an exact multiple of the fundamental it is called a *harmonic partial*. In strings the overtones are usually harmonics, but in vibrating plates and membranes they are generally not. The overtones are named *first*, *second*, *third*, etc., in the order of their vibration rates as compared with that of the fundamental. The frequency of an overtone is found by multiplying the fundamental by a number one greater than the number of the overtone. For example, the frequency of the first

overtone of $C = 256$ is $256 \times 2 = 512$, that of the second is $256 \times 3 = 768$, and so on.

XI. VIBRATION OF AIR IN PIPES.

216. Gases as Sources of Sound. — It was seen in the use of the resonator that gases can be thrown into vibration when they are confined in tubes or globes, and that they thus become sources of sound. Such a column of gas can be set in vibration in two ways: by a vibrating tongue, as in reed instruments; or by a stream of air striking against the edge of a lateral opening in the tube, as in the whistle, flute, etc.

217. Laws for Air Columns. — **Experiment.** — Fit a cork piston in a glass tube whose length is about 30 cm. and diameter 2.5 cm. With a piece of brass tubing flattened at one end direct a stream of air across the mouth of the tube. If the position of the piston, as well as the force of the blast, be right, the tube will yield a pure tone. If we shorten or lengthen the air column by moving the piston, the pitch of the tone will rise or fall accordingly. If we determine by trial the different lengths necessary to give the gamut, a comparison of them will give the continued ratio, $1 : \frac{2}{3} : \frac{4}{5} : \frac{3}{4} : \frac{5}{6} : \frac{8}{7} : \frac{9}{8} : \frac{16}{15} : \frac{1}{2}$ (§ 208), showing that,

The frequency varies inversely as the length of the air column.

Experiment. — Prepare two glass or paper tubes, 20 and 10 cm. long, respectively, and about 2 cm. in diameter. Hold the hand over one end of the shorter tube, and blow across the open end so as to produce its lowest pure tone. A comparison of this tone with that obtained by blowing across one end of the longer open tube will show that the pitch is the same. Hence,

For the same pitch, the open pipe is twice the length of the closed one.

State of Air in Sounding Tubes. — Experiment. —

ying an organ pipe, made either of glass or with one glass side (12), lower into it, as it emits its fundamental, a membrane covered with fine sand. The sand will vibrate the least at the middle of the tube, and most at the ends. The vibration of the air in the pipe is longitudinal. A *node* is a place of least motion and greatest change of density; an *antinode* is a place of greatest motion and least change of density. The closed end of a tube is necessarily a node, and the open end an antinode.

open pipes, for the fundamental tone, there is a node at the middle and an antinode at each end. In stopped pipes there is a node at the closed end and an antinode at the other end.

A pipe or tube acts as a resonator, and for the fundamental tone the length of the closed tube is one-fourth of a wave length; since the open pipe has a node in the middle, its length is but half a wave length.

Overtones. — Experiment. — Blow a strong stream across the end of the long tube used in Art. 217, and notice that notes of higher pitch than the fundamental are produced. These are *overtones*, caused by the air vibrating in parts or segments. In proof of this, insert a piston in the tube, and by means of it shorten the air column till it emits its fundamental the overtone previously obtained.



Fig. 112.

Since there can be no motion of the air at the surface of the piston, it must mark a node; and since the tone is changed by the presence of the piston, there must have been a node at that point before the introduction of the piston.

When an open pipe yields its fundamental there is a

*open pipe 1/4 wave length
stopped pipe 1/2 wave length*

node at the middle; hence to yield higher tones there must be added one, two, three, or more nodes so placed that there will be an antinode at each end. *A*, *B*, and *C*, of Fig. 113, illustrate the division into segments of an open pipe. If *A* gives the fundamental, then *B* must give a tone an octave higher or the first overtone, since the first node is one-fourth from the end. In *C*, the node is one-

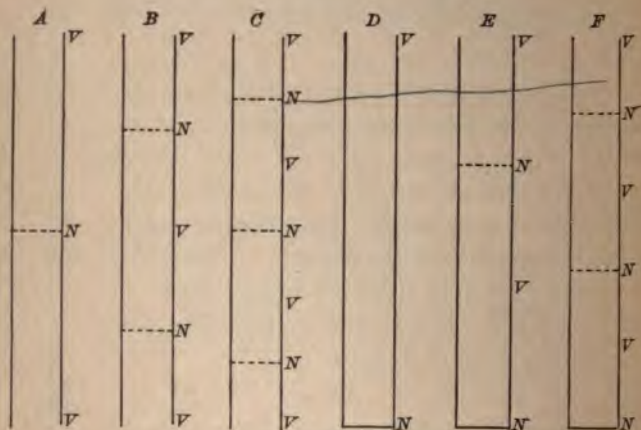


Fig. 113.

sixth from the end, and the frequency is therefore three times that of *A*, or the overtone is the second. Hence,

In open pipes the complete series of overtones is possible.

For stopped pipes, the open end is always an antinode, and the closed end a node; then by adding successively a node we have the conditions shown in tubes *D*, *E*, *F*, of Fig. 113. If *D* is the fundamental, then the vibration rate of *E* is three times that of *D*, giving the second overtone, since the node is one-third from the end. In like manner *F* is the fourth overtone, the node being one-fifth from the end. Hence,

in closed pipes only those overtones are possible whose vibration rates are 3, 5, 7, etc., times the fundamental.

Problems.

1. How can any particular overtone be excluded from the complex of a vibrating string?
2. If the E string of a violin is 15 inches long, by how much must it be shortened to sound G? *2.5 in*
3. A string stretched by a weight of 25 lb. sounds the note E. At what tension must be given to it to sound the C below? *16 lb* ✓
4. A wire on a monochord is 3 ft. long and vibrates in unison with tuning-fork marked C (256). By shortening the wire a foot what frequency will the emitted note have? *384*
5. A wire stretched on the sonometer is provided with a movable peg. The wire is 120 cm. long and gives the note C (256). What changes must be given the wire successively by means of the bridge to produce the notes of the gamut?
5. A string is stretched by a weight of 9 kgm. What tension must be given to it so that it will emit a note a fifth higher? *20.25 kgm*
7. Two wires of the same material and length are stretched by equal weights. The first wire has twice the diameter of the second. What change must be made in the tension to put the wires in unison? ✓
8. A wire under tension vibrates with a frequency of 256. What would be the frequency, if the wire were half as long, twice as thick, under one-quarter of the tension? *4 times*
9. A string stretched by a weight of 16 kgm. gives a certain note. A second string of the same length and material gives the same note when stretched by a weight of 25 kgm. What are the relative diameters of these strings? *first is $\frac{4}{5}$ of second* ✓
10. A steel and a silver wire of the same diameter and length are stretched with equal tensions. Their specific gravities are 7.8 and 10.6 respectively. What will be the frequency of the silver wire, that of steel being 200? *171.5* ✓
1. A cord stretched with a force of 10 lb. gives the note C. What must be the stretching force to cause it to give the note G? *22.5 lb*

12. What is the length of a closed organ pipe that sounds E (320) when blown with air at 20°C .? *76.9 cm*

✓ 13. A closed organ pipe is 1 ft. long. What is the frequency of the fundamental note, the temperature of the air being 15°C .? *380*

14. Calculate the frequency of the first four overtones of an open organ pipe whose frequency is 64. *128 192 256 320*

✓ 15. The lowest note on a pipe organ is C (16). Calculate the length of an open pipe that gives this note when the temperature is 22°C . *32*

NOTE:— If the following problems are found to be too difficult, the teacher may omit them at his discretion. The same remark applies to the last problems in several other sets.

16. A certain stopped pipe gives C (256) when blown with air at 10°C . What will be its frequency when blown with air at 20°C .?

[Find the length of the wave at 10°C . This value combined with v at 20°C . will determine n .]

17. A closed organ pipe sounds the note E (320). What notes are its two lowest overtones?

18. An open organ pipe sounds C (256). What change must be made in its length in order that it may give the note E instead? *change*

19. A closed organ pipe is 2 ft. long. What is the frequency of the first three overtones, the temperature of the air being 18°C .?

20. An open and a closed organ pipe are each 3 ft. 9 in. long. Calculate the wave lengths of their fourth overtones respectively.

XII. QUALITY OF SOUND. *depends on overtones*

220. Definition of Quality.—Two of the essential characteristics of musical sounds have already been considered, namely, pitch and loudness. There is a third important difference between musical sounds. We easily perceive that one sound differs from another not only in being more acute or grave, louder or softer, but also in respect to the character of the sound. We have no difficulty in distinguishing the notes of a piano from those of a violin,

even though they may be of the same pitch and apparent loudness. Similar differences enable us to distinguish one voice from another in speech and in song. Even the untrained musical ear can readily appreciate differences in the character of the music produced by different instruments of the same class. All such differences, not assignable to pitch or loudness, are included under the term *quality*. Those characteristics of a sound that enable one to refer it to its source are called its *quality*.

221. Quality due to Overtones.—Experiment.—Pluck the wire of a sonometer at the middle and compare the sound with that given when the wire is plucked near one end. In the first case the first and second overtones are necessarily lacking (§ 215), while in the second case the whole series is probably present.

Although these two notes are of the same pitch, yet there is a marked difference in their quality, caused by the difference in their overtones. The sound waves in each case are the result of compounding the fundamental with the overtones present. Pitch depends on the wavelength; loudness on the amplitude; and quality depends upon the only other physical difference between aerial sound waves, namely, their vibrational form, due to the relative phase and intensity of the overtones. When the fundamental is relatively strong, the overtones being few and weak, the tone is full and mellow; but when the fundamental is weak, and the overtones numerous and strong, the tone is metallic.

Differences in quality are to be referred to the series of overtones present in each case. Voices differ for this reason. Violins differ in sweetness of tone, because the sounding-boards of some bring out the overtones differently from others. Voice culture consists in training and

developing the vocal organs and resonant cavities, to the end that purer overtones may be secured, and greater richness may thereby be imparted to the tones. Often in the reflection of sound by distant objects its character is greatly changed, on account of the partial or complete suppression of the fundamental or some of the overtones in the process of reflection.

222. Analysis and Synthesis of Sound.—Helmholtz constructed a series of resonators of different sizes, like those of Fig. 103, one to reënforce a certain fundamental tone and one for each of its overtones. By placing these resonators successively to the ear, when the fundamental tone is sounded, the presence of any of its overtones is indicated by the resonator responding. Proceeding in this way, Helmholtz demonstrated that quality of sound is determined by the overtones present; that to give a tone richness and sweetness the first four or five overtones are essential.

Helmholtz also proved that a sound of any quality may be built up by combining a fundamental tone with overtones. To do this he employed a set of tuning-forks, kept in vibration by electromagnets (§ 455). The set consisted of ten forks, nine of which sounded the overtones of the tenth. Each fork was provided with a resonator to strengthen its sound. By means of a set of keys any particular resonator could be brought into action, and the sound of the fork made audible at a distance.

XIII. HARMONY AND DISCORD.

223. Consonance and Dissonance.—Experiment.—Tune two wires on the sonometer to unison. By the movable bridge raise the pitch of one wire a fifth. When the wires are plucked, the combination

sound is smooth and agreeable. Now move the bridge till the interval is a second; the sound of the two wires will be disagreeable.

When two notes differ in pitch and their combination agreeable to the ear, they are said to be *consonant*; when disagreeable, *dissonant*.

Helmholtz concluded that dissonance is due to beats. He even expressed their relation quantitatively to the effect that dissonance occurs when the number of beats per second is between 10 and 70, and that maximum discord is caused by 30 or 32 a second. Later investigators have shown that the number of beats that produce the greatest discord is not constant, but depends upon the pitch. Beats may be regarded as the physical aspect of discord. They are still recognized as one cause of it, but psychologists are now agreed that discord cannot be defined by beats.

Problems.

1. What are the three characteristics of a musical sound?
2. Why is there a difference in quality in the sounds given by open and closed pipes?
3. Why are overtones not always harmonics?
4. Why is the tone of a violin richer than that of a flute?
5. The hammer of a piano commonly strikes the wire at a distance of about one-seventh its length from one end. Why is this better than striking it at the middle?

XIV. VIBRATING RODS, PLATES, AND BELLS.

224. Vibration of Rods.—Rods of metal, of wood, and of glass may be made to vibrate either transversely or longitudinally. A rod may be vibrated transversely by fixing it and drawing a violin bow across the free end, or by

striking it with a suitable hammer; it may be set in longitudinal vibration by clamping at the middle and stroking lengthwise with a cloth dusted with powdered resin. A moist cloth is better for glass. The jews'-harp, the music-box, and the coiled-wire gong of a clock are illustrations of the transverse vibration of rods or plates clamped at one end. The tuning-fork may be regarded as an elastic bar free at the ends, and supported in the middle by a stem which is subject to all the motion of the middle of the ventral segment, giving it an up and down movement which is transmitted to the supporting sounding-board (Fig. 114). The overtones are of high pitch and feeble intensity, and soon vanish, leaving the tone pure.



Fig. 114.

225. Vibration of Plates. — Experiment. — Support a brass or



Fig. 115.

glass plate, as shown in Fig. 115. Scatter a little fine sand evenly over it, touch the plate at some point with the finger, while a violin bow is drawn across the edge. The plate is thrown into vibration, the sand arranging itself in symmetrical figures whose complexity increases as the pitch becomes higher.

These sand figures make it clear that the plate vibrates transversely in segments, the sand being thrown to places of least vibration, which lie between parts having opposite motions. The arrangement of nodal lines is determined by the relative position of the point bowed to that pressed by the finger. This method of studying vibrating bodies was first employed by Chladni, and the figures are called Chladni's figures.

Vibration of Bells. — **Experiment.** — Draw a violin bow across the edge of a large bell or goblet half full of water. It will yield a musical sound, and at the same time the surface of the water will be greatly agitated in sections corresponding to the several segments into which the vibrating body is divided.

For the fundamental tone the bell divides into four segments, the pitch rising with the number of segments. Powdered sulphur sifted evenly on the water will make the position of the nodes more conspicuous.

XV. GRAPHIC AND OPTICAL METHODS.

226. **Graphic Methods** of studying sound are of service in determining the vibration rate of sounding bodies. In one of the simplest a small style is attached to the vibrating body and traces its movements upon a piece of smoked paper or glass, which moves uniformly beneath it. Generally, the paper is wrapped

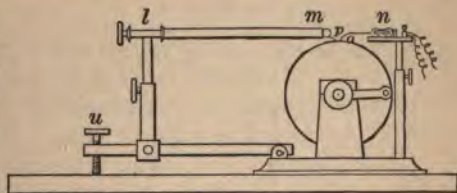


Fig. 116.

around a cylinder, mounted on an axis, one end of which has a screw thread on it, so that when the cylinder turns

it also moves axially (Fig. 116). If the vibrating body is a fork, the beats of a seconds pendulum may be marked on the paper by electric sparks from the style, and the number of sinuosities in the line traced on the cylinder between the marks may be counted. The rate of the fork is thus easily measured.

227. Manometric Flames.—Experiment.—A box with mirror faces is mounted so as to turn on a vertical axis (Fig. 117). In front of



Fig. 117.

these revolving mirrors is supported a short cylinder which is divided into two parts by a partition of gold-beater's skin. Illuminating gas is admitted to one compartment; a small gas jet is connected with the same one and a speaking-tube with the other.

In a darkened room, the image seen in the revolving mirror will be a smooth band of light. Now sound a heavy C-fork in front of the mouthpiece, or produce there any pure tone; the

appearance in the rotating mirror will be that of the upper band of Fig. 118. A condensation entering the box acts on the membrane, compressing the gas, thereby extending the flame; a rarefaction entering it produces an opposite effect. Hence a serrated band is seen in the mirror. Now sound a C'-fork, a strong tone an octave higher than that first used; the appearance is the second band of Fig. 118, differing from the last in having teeth half as wide. If we connect two mouthpieces with the box, using a T-pipe, and sound the C-fork in front of one, and the C'-fork in front of the other, we obtain the last band of Fig. 118, the short tongues of which are due to

C' , the octave of C . The same figure is obtained by singing the vowel sound o on the note $B\flat$, showing that this sound is composed of the fundamental combined with the first overtone.

The experiment shows the possibility of analyzing sounds by the flame pictures they produce. This method was invented by the late Rudolph Koenig of Paris, and it has the great advantage of being independent



Fig. 118.

of hearing. If this box, or manometric capsule, as it is called, be attached to a Helmholtz resonator, the flame will respond whenever any sound is produced that affects the resonator. With a complete set of resonators, each with its manometric capsule, a most efficient apparatus is provided for the analysis of sounds.

	Light Waves	Sound Waves
Wavelength	Ethert (continuous)	Molecular Shocks
Frequency	16,000 m p s.	1100 ft per sec in air & increasing
Velocity	Trillions per sec	16 to 11,000 per sec
Nature	Transverse	Longitudinal

CHAPTER V.

LIGHT.

I. NATURE AND PROPAGATION OF LIGHT.

228. **Light**, as distinguished from the sensation of seeing, is a periodic or undulatory disturbance in a medium which is assumed to exist everywhere in space, even penetrating between the molecules of ordinary matter. This medium is known as the *ether*. Light waves do not consist of alternate condensations and rarefactions, as in sound, but of periodic *transverse* disturbances. These disturbances are probably not transverse movements of the ether itself, but transverse alterations in the electrical and magnetic condition of the ether. But whatever may be the nature of the medium, light is a wave motion in it, and the vibrations are transverse.

229. **Definition of Terms.**—In general, when light falls on a body, a part of it is reflected, a part is transmitted, and the rest is absorbed. *Transparent* bodies allow light to pass through them with so little loss that objects can be easily distinguished through them, as glass, air, pure water. *Translucent* bodies transmit light so imperfectly that objects cannot be seen distinctly through them, as horn, oiled paper, very thin sheets of metal or wood. *Opaque* bodies transmit no light, as blocks of wood or iron. This classification is one of degree; no sharp line of separation between these classes can be drawn.

No body is perfectly transparent. If several layers of glass are put together, the distinctness of vision through them diminishes with the increase in the number of layers; stars which are invisible at the foot of a mountain are often visible at the top. (Why?)

230. Speed of Light. — Until 1676 it was believed that light travelled instantaneously. In that year Roemer, a Danish astronomer, was engaged at the Paris observatory in observing the eclipses of Jupiter's moons. Confining attention to the one nearest the planet, because its eclipses occurred most frequently, he found that the time of the observed eclipses did not correspond with the computed values derived from observations extending over a long period. He found the interval between two successive eclipses at E and E'' (Fig. 119)

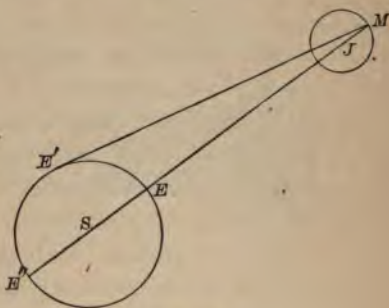


Fig. 119.

the same; but for intermediate positions this interval was greater while the earth was receding from Jupiter in going from E through E' to E'' , and less while approaching Jupiter again. It was greatest at E' when the earth was moving directly away from the planet. The sum of all these excesses from E to E'' amounts to 16 min. 38 sec., or 998 sec. In accounting for these facts Roemer advanced the theory that the increase is caused by the time taken by light to traverse the added distance when the earth is moving away from Jupiter, and that 998 sec. represent

the time required by light to traverse the diameter of the earth's orbit. This gives for the speed of light about 186,000 miles a second.

In 1849 Fizeau determined the time required for light to travel a known distance on the earth's surface. In 1850 Foucault showed not only that light takes a measurable time to travel short distances, but that the speed of light varies with the medium. These experiments have been repeated in a modified form by Michelson and Newcomb in our own country, and the results, as summarized by Professor William Harkness, show that the velocity of light is about 186,337 miles (299,877 km.) a second.

c P
d
231. Light travels in Straight Lines. — If an opaque object, as a book, is between the eye and a lamp, it hides the lamp from view. From such facts as this we learn that *light is propagated in straight lines*. Other facts to be considered in a subsequent article (§ 265) make it necessary to add the restriction, in a medium having the same physical properties in all directions. Rays of light are the directions in which light is propagated. These directions are radii of the spherical waves and normals to the wave fronts. When the source is at a great distance, the rays are sensibly parallel, and a number of them taken together constitute a *beam of light*; for example, in the case of light from the sun or stars, the distance is so great that the rays are considered to be parallel. Rays of light proceeding outward from a point form a *diverging pencil*; and rays proceeding toward a point, a *converging pencil*.

232. Shadows. — **Experiment.** — Hold a ball or disk between the flame of a lamp and a white screen. From a part of the screen the light will be wholly cut off, and surrounding this area is one from

ch the light is excluded in part. If three small holes be made in screen, one where the screen is darkest, one in the part where the en is less dark, and one in the lightest part, it will be found, on cing through them, that the flame of the lamp is wholly invisible ough the first, part is visible through the second, and the whole e through the third.

The space behind the object from which the light is eluded is called the *shadow*. The figure on the screen a section of the shadow. The darker part of the dow, called the *umbra*, is caused by the total exclusion the light by the opaque object; and the lighter part, *penumbra*, by its partial exclusion.

When the luminous object is a point, as *L* (Fig. 120), n the shadow will be bounded by the cone of rays,

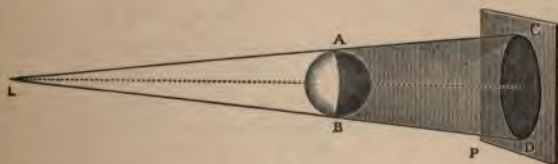


Fig. 120.

B, tangent to the object, and will have but one part, umbra. When the luminous body has magnitude, as (Fig. 121), then the space *ABDC* behind the opaque ly receives no light, and the parts between *AC* and



Fig. 121.

AC' , and between BD and BD' , receive some light, the amount increasing as AC' and BD' are approached. [The student should draw figures for the cases when the luminous body is larger than the opaque body, and when of the same size.]

233. Images by Small Apertures. — Experiment. — Support in vertical planes two sheets of cardboard, A and B (Fig. 122). In the

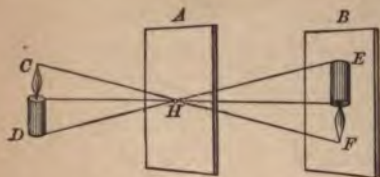


Fig. 122.

centre of A cut a round hole about 2 mm. in diameter; and place in front of it, at a distance of a few centimetres, a lighted candle, CD . An inverted image of the flame will be found on the other sheet. If a triangular or a square hole is used, an equally distinct image of the candle will be formed, showing that the image is independent of the shape of the opening. Again, if the aperture is gradually enlarged, the image loses in distinctness of outline, gains in brightness, and gradually assumes the shape of the aperture.

The experiment shows that the definition and the brightness of the image are independent of the shape of the aperture, but are affected by its size. To understand the origin of this image it must be borne in mind that each point of the object is the vertex of a cone of rays passing through the aperture and forming an image of the aperture on the screen. These images will be symmetrically placed with reference to the points emitting the light, and consequently will build up a figure of the same form as the luminous object. Now it is evident that these numerous pictures of the aperture overlap in forming a picture of the object, the number at any one place determining the brightness. The edge of the picture will, therefore, be less bright than the other portions, and these differences

will be more noticeable, the larger the opening. In the case of a very large opening, the overlapping of the images of the aperture destroys all resemblance of the image to the object, the resulting image having the shape of the aperture.

234. Illustrations. — When the sun shines through the small chinks in the foliage of a tree, there may be seen on the ground a number of spots of light, either round or oval. These are images of the sun. During partial eclipses of the sun such figures assume a crescent shape.

In the photographer's camera, and in the eye, we have further examples of the formation of images by small apertures. In these cases, as will be seen in subsequent articles, the definition is improved by means of a lens (§ 269).

II. PHOTOMETRY.

235. The Intensity of illumination is the quantity of light received on a unit of surface. Everyday experience shows that it varies, not only with the nature of the source, but also with the distance at which the source is placed.

236. Law of Intensity. — **Experiment.** — Cut from cardboard three squares, 4 cm., 8 cm., and 12 cm. on a side, respectively, mounting them on wire supports. The centres of these screens should be at the same distance above the table as the source of light. Use a lamp giving a small, flat flame, and set it with the edge of the flame toward the surface of the largest screen, and distant about one metre. Interpose the medium-sized screen so that it exactly cuts off the light from the lateral edges of the largest. If the source were a point, the light would now be cut off from the whole screen. In like manner place the smallest screen with reference to the intermediate one. If these screens are placed with care, it will be found that their distances from the light are as 1:2:3. Now as each screen exactly cuts off the light from the one next to it in the series, it follows that

each receives the same amount of light from the source when it is not intercepted. The surfaces of these screens are as 1:4:9, and hence the amount of light per unit of surface must be inversely as 1:4:9, the squares of 1, 2, and 3, respectively.

Any disturbance propagated in spherical waves must have its energy distributed over a continually increasing area as the radius increases; and since this area increases as the square of the radius, it follows that the energy per unit of area must decrease at the same rate. Therefore, *the intensity of the illumination varies inversely as the square of the distance from the source of light.* The intensity of illumination is further decreased because of absorption by the turbid medium through which light is transmitted.

237. A Photometer is an instrument for comparing the intensity of one light with that of another assumed as a standard. The principle applied is that the ratio of the intensities of two lights equals the square of the ratio of the distances at which they give equal illuminations. The standard in general use is the light emitted by a sperm candle of the size known as "sixes," when burning 120 grains per hour. The illuminating power of a light is expressed by stating the number of times it is greater than the standard candle.

238. The Bunsen Photometer.—In this photometer, a screen of paper, *A* (Fig. 123), having a translucent spot, made by applying a little hot paraffin, is supported on a graduated bar between the standard candle, *B*, and the light to be measured, *C*. When the two surfaces are equally illuminated, the spot is scarcely distinguishable from the surrounding paper. By moving *B*, this condition is easily secured. Then the candle power of *C* equals $\overline{AC}^2 \div \overline{AB}^2$.

Since the spot never appears exactly like the surrounding paper, it is better to place a pair of mirrors, forming a V, opening toward the edge of *A*. By looking



Fig. 123.

them, both sides of *A* can be seen at once. *B* is then adjusted to give these images the same appearance.

Problems.

1. What is the important difference in the manner of propagation between sound and light?
2. Why is one unable to see distinctly when one passes quickly from a bright room to a dark one?
3. Why cannot one see through a bent tube?
4. In the formation of an image by a small aperture, why does the image lose in distinctness as the aperture is enlarged?
5. If the diameter of the earth is 8000 mi., that of the sun 866,000 mi., and the distance of the earth from the sun 93,000,000 mi., what is the length of the earth's shadow (umbra)? *867,133 miles* ✓
6. If the diameter of the moon is 2100 mi., what is the length of the moon's shadow (umbra) if the sun's diameter is 866,000 mi., and the moon's distance from the sun, 93,000,000 mi.? *326,069 miles* ✓
7. In an attempt to determine the height of a church steeple, it was found that the point of the shadow on the ground was 40 ft. ✓

from the base of the steeple, while that of a vertical pole 10 ft. long was 2 ft. from the foot of the pole. What was the height of the steeple?

8. In the use of a Bunsen photometer to measure the candle power of an incandescent lamp, it was found that for equal illumination on both sides, the distance of the standard candle was 10 cm. from the screen, and the lamp was 38 cm. What was the candle power of the lamp? *14.44 cp.*

9. Two lamps, one of 5 candle power and the other 20, are placed 150 cm. apart. Find the position of a screen between the two lights that will be equally illuminated by them. *100 cm.*

10. A lamp of 10 candle power is placed 5 ft. from a screen, and a second lamp of 20 candle power is placed 10 ft. from the screen. Compare the intensities of illumination. *2:1*

11. How close to a wall must a 20 candle power lamp be placed to produce the same illumination of the wall as a 2000 candle power electric light distant 1000 ft.? *100 ft.*

III. REFLECTION OF LIGHT.

239. Regular Reflection. — When a beam of light falls on a polished surface AC (Fig. 124), the greater part

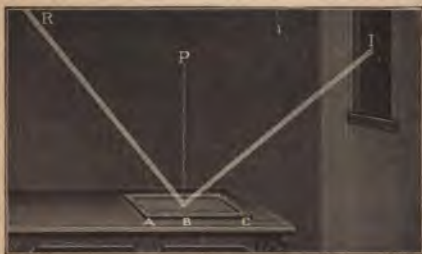


Fig. 124.

of it is reflected in a definite direction. The angle that the incident ray makes with the normal PB to the reflecting surface at the point of incidence is called the *angle of incidence*, as IBP ; and the angle

between the reflected ray and this normal is the *angle of reflection*, as GBP .

240. Law of Reflection. — Experiment. — A semicircular board is provided with two arms pivoted at the centre, one carrying a lighted

le and a convex lens (§ 269), the other an oiled-paper screen and (Fig. 125). A plane mirror is mounted at the centre of the semicircle, with its reflecting surface parallel to the diameter. A scale on the edge of the semicircle has its zero in a normal to the mirror. The lens, with a wire bent across it, is so placed that the shadow of

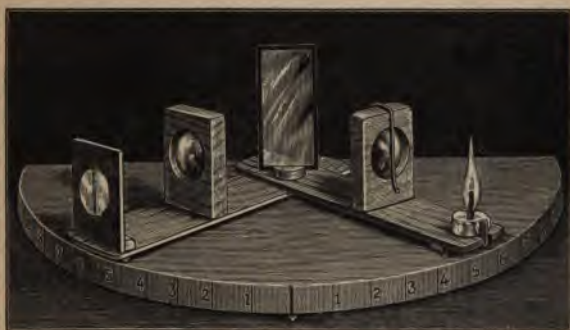


Fig. 125.

wire, after reflection from the mirror, is focussed sharp and clear on the screen by the second lens. Now give the candle-arm any inclined position and move the screen-arm till the shadow of the wire is across the middle of the screen. The arms will be found on measurement to make equal angles with the normal to the mirror.

Hence, the angle of reflection is equal to the angle of incidence, and the two angles lie in the same plane.

241. Diffused Reflection.—**Experiment.**—Fill a large glass jar with smoke. Cover the mouth with a piece of cardboard, in which is a hole about 1 cm. in diameter. With a small hand-glass reflect a beam of sunlight into the jar through the hole in the cover. The whole interior of the jar will be illuminated.

The small particles of smoke floating in the jar furnish great many surfaces. The light falling on them is reflected in as many different directions, the result being seen in the diffusion of the beam.

All reflecting surfaces, to a greater or less extent, scatter light in the same way as these smoke particles, on account of the irregularities of their surfaces. Figure 126 illustrates the difference between a perfectly smooth reflector



Fig. 126.

and reflectors as they actually are, more or less irregular according to the degree of polish.

It is by diffused reflection that objects become visible to us. Perfect reflectors would be invisible. The trees, the ground, the grass, and particles floating in the air, reflect the light from the sun in every direction, and thus fill the space about us with light. Aeronauts tell us that when they reach very high altitudes, the sky grows black, owing to the absence of floating particles to diffuse the light.

242. A Mirror is any smooth surface. A *plane mirror* is one whose reflecting surface is plane. A *spherical mirror* is one whose reflecting surface is a portion of the surface of a sphere.

243. Image of a Point in a Plane Mirror. — Let A be a luminous point in front of a plane mirror MN (Fig. 127). Any ray AB incident on the mirror is reflected in the direction BD , making the angle of reflection FBD equal to that of incidence FBA . (See Appendix.) In like

a second ray, as AC , is reflected along CE . If CE are produced, they meet at A' . Join A and A' and AA' is perpendicular to MN ,¹ and A' is as far from MN as A is in front. Since AB and AC are

rays diverging from A , incident on a mirror, it follows that the rays reflected from A , incident on a mirror, must be reflected as if they came from a point as far back as A is in front. The eye placed at E receives the rays as if they came from A' . At A' is accord-

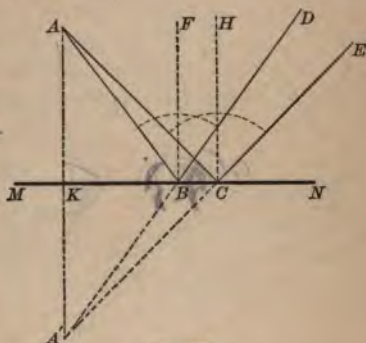


Fig. 127.

ed the image of A in the mirror MN , and is a *virtual image*, because the light only *apparently* comes from it. Therefore, *the image of a point in a plane mirror is virtual, and is as far back of the mirror as the object is in front*; it may be found by drawing from the object perpendicular to the mirror, and producing it till it is doubled.

Image of an Object. — Since the image of an object is an emblem of the images of its points, it follows that the image of an object can be located by finding those points. Let AB (Fig. 128) represent an object in front of the mirror MN . Drop perpendiculars from the

$ABK = DBN = A'BK$. $\therefore ABN = A'BN$. Angle $ACK = A'CK$. Hence, the triangles ABC and $A'BC$ are equal, and

In the triangles AKB and $A'KB$, it follows that the angles $AKB = A'KB$, and $AK = A'K$.

points of the object to the mirror, and produce them till their length is doubled. Then $A'B'$ is the image of AB . It is evidently virtual, erect, and of the same size as the object.

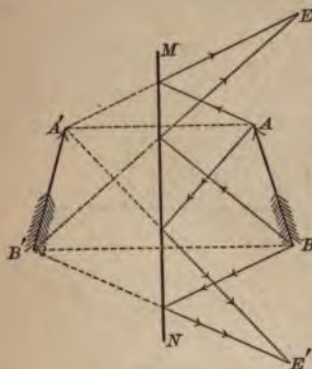


Fig. 128.

The rays which form the image for one observer are not those which form it for another. Let E and E' represent two different observers. To find the path of the rays which enter the eye at E , draw lines from A' and B' to E . The intersections of these lines with MN are the points of incidence of the rays from A and B which are reflected to E .

In like manner we may trace the path of the rays for the position E' .

245. Experimental Proof of the Position of the Image.—

Experiment.—Support a pane of window glass in a vertical position. On one side of it place a lighted candle, and on the other a tumbler of water, each at the same distance from the glass, and in a line perpendicular to it. An image of the candle will be seen in the tumbler of water, showing that the image is as far back of the mirror as the object is in front.

The experiment also explains how many optical illusions, such as Pepper's ghost, are produced. A large sheet of unsilvered plate glass, with its edges hidden from view by curtains, is so placed that the audience have to look obliquely through it to see the actors on the stage. Other actors, strongly illuminated and out of view by the audience, are seen by reflection in the glass and appear as ghosts on the stage. The magic cabinet and the head

without a body are also illusions produced by the aid of mirrors.

246. Multiple Reflection. — **Experiment.** — Support two mirrors so that their reflecting surfaces form an angle. If a lighted candle be placed between them, several images may be seen in the mirrors; three when at right angles, the number increasing with the diminution of the angle. When the mirrors are parallel, all the images are on a straight line perpendicular to the mirrors.

These several images are caused by the successive reflection of the light from the mirrors; the image in one mirror serves as an object for the second mirror, and the image in the second becomes in turn an object for the first mirror.

In Fig. 129 the two mirrors

are at right angles. O' is the image of O in AB , and is found as directed in § 244.

O'' is the image of O' in AC , and is found by the perpendicular $O'O''$. O'' is the image of O in AC , and

since the mirrors are at right angles, O''' is also the image of O'' in AB . O''' is situated

behind the plane of both mirrors, and no images of it are formed. All the

images are situated in the circumference of the circle whose centre is A and radius AO . If E is the position of the eye, then O' and O'' are each seen by one reflection, and O''' by two reflections, and for this reason it is less bright.

To trace the path of a ray from O''' , draw $O'''E$, cutting AB at b , and from the intersection b draw bo'' , cutting AC at a . Join aO ; the path is $OabE$.

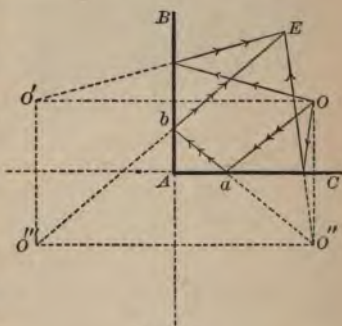


Fig. 129.

247. Illustrations. — The double image of a bright star and the several images of a gas-jet in a thick mirror (Fig. 130) are examples



Fig. 130.

of multiple reflection, the front surface of the mirror and the metallic surface at the back serving as parallel reflectors. Geometrically the number of images is infinite; but on account of their faintness only a limited number is visible. The *kaleidoscope*, a toy invented by Sir David Brewster, is an interesting application of the same principle. It consists of a tube containing

three mirrors extending its entire length, the angle between any two of them being 60° . One end of the tube is closed by ground glass, and the other by a cap with a round hole in it. Pieces of colored glass are placed loosely between the ground glass and a plate of clear glass parallel to it. On looking through the hole at any source of light, multiple images of these pieces of glass are seen, symmetrically arranged around the centre, and forming beautiful figures, which vary in pattern with every change in the position of the objects.

248. Deviation by Revolving Mirror. — If a ray of light is incident on a plane mirror, and the mirror is turned through an angle, the angular deviation of the ray will be

double that of the mirror. Let the ray AM be normal to the mirror (Fig. 131); it will then be reflected back on itself. Turn the mirror through the angle θ (theta); the normal AM is turned through the same angle, so that the angles of incidence and reflection are now both equal to θ . The deviation of the reflected ray is the angle AMB or 2θ .

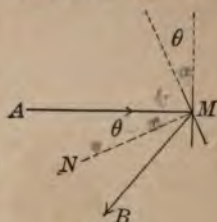


Fig. 131.

This principle is applied in measuring small movements of rotation, as in reflecting galvanometers; for not only is the deviation doubled, but the ray of light acts as a long weightless pointer, thus adding greatly to the sensitiveness of the instrument.

249. Spherical Mirrors. — A mirror is *spherical* when its reflecting surface is a portion of the surface of a sphere. If the inner surface is polished for reflection, the mirror is *concave*; if the outer surface, it is *convex*. Only a small portion of a spherical surface cut off by a plane is used as a mirror, and its boundary is therefore circular. The



Fig. 132.

centre of the mirror is the centre of curvature of the sphere of which the reflecting surface is a part. The middle point of the reflecting surface is the *pole* or *vertex* of the mirror, and the straight line passing through the centre of cur-

vature and the pole of the mirror is its *principal axis*. Any other straight line through the centre and intersecting the mirror is a *secondary axis*.

In Fig. 132, MN is the spherical mirror,¹ C is the centre, A is the pole, BA the principal axis, and DE and HK secondary axes. The angle MCN measures the *aperture* of the mirror.

The difference between a plane mirror and a spherical one is that the normals to a plane mirror are all parallel lines, while those of a spherical mirror are the radii of the surface, and all pass through the centre of curvature.

250. A Focus is the point common to the paths of all the rays of a pencil of light after incidence. It is a *real* focus if the rays and waves of light actually pass through the point, and *virtual* if they only appear to do so.

251. Principal Focus of Spherical Mirrors.—**Experiment.**—Let the rays of the sun fall on a concave spherical mirror. Hold a graduated ruler in the position of its principal axis, and slide along it a small strip of cardboard. Find the point where the image of the sun is smallest. This will mark the principal focus, and it is a real one. If a convex spherical mirror be used, the light will be reflected as a broad pencil diverging from a point back of the mirror. The focus is then a virtual one.

When a pencil of parallel rays is incident on a concave spherical mirror, parallel to its principal axis, the point to which they converge after reflection is called its *principal focus*. In the case of a convex spherical mirror, the principal focus is the point on the axis behind the mirror from which the reflected rays diverge. The distance of the principal focus from the mirror is its *principal focal length*.

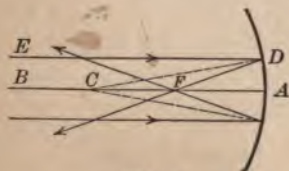
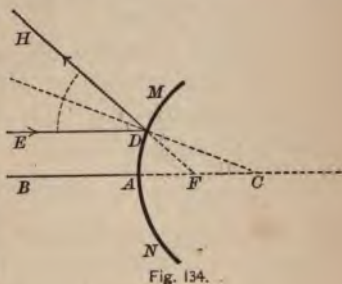


Fig. 133.

¹ The figures of mirrors in this chapter are sections made by a plane passing through the principal axis.

Let MN (Fig. 133) be a concave mirror whose centre is C and principal axis is AB . Let ED be a ray parallel to BA . Then CD is the normal at D ; and CDF , the angle of reflection, must equal EDC , the angle of incidence. Since the ray BA is normal to the mirror, it will be reflected back along AB . The reflected rays DF and AB have a common point F , which is the principal focus. The triangle CFD is isosceles with the sides CF and FD equal. (Why?) But when the point D is near A , FD is sensibly equal to FA ; F is therefore the middle point of the radius CA . Other rays parallel to BA will pass after reflection nearly through F (§ 255). Hence, *the principal focus of a concave spherical mirror is real and is half way between the centre of curvature and the vertex.*

Let MN (Fig. 134) be a convex spherical mirror. ED and BA are rays parallel to the principal axis. Their common point F , after reflection, is back of the mirror and halfway between A and C . (Why?) Hence, *the principal focus of a convex spherical mirror is virtual and halfway between the centre of curvature and the mirror.*



252. Conjugate Foci. — If a diverging pencil of light is incident on a spherical mirror, it is focussed after reflection at a point on the axis which passes through the radiant point or source of light; after reflection the rays diverge from this focus as a new radiant point. Thus, in Fig. 135, the rays BA and BD diverge from B ; BA is reflected back along its own path (why?), and BD is reflected

along DH , making the angles of incidence and reflection equal to each other. After reflection they both pass through B' and diverge from it. B and B' are *conjugate foci*. Rays diverging from either point will converge to the other.

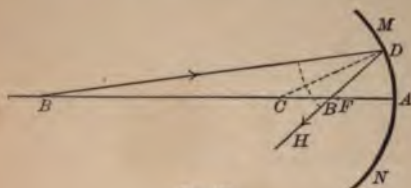


Fig. 135.

In Fig. 136, the rays BA and BD diverge from B as the radiant point; after reflection they diverge from B'

behind the reflecting surface, and B' is a virtual focus. B and B' are again conjugate foci.

In the first case the source of light is farther from the mirror than the centre of curvature, and the focus is real; in the second case it is nearer the mirror than the principal focus, and the focus is virtual.¹

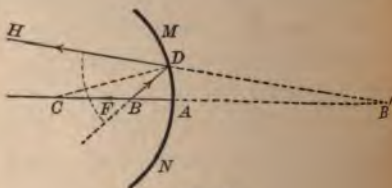


Fig. 136.

253. Formation of Images.—**Experiment.**—Darken the room and support on the table a spherical mirror, a lamp, and a small screen. The lamp and screen must be placed so that the screen will not cut off too much light from the mirror. Place the lamp anywhere beyond the

¹ In Fig. 135, CD bisects the angle BDH . Hence, $\frac{BD}{B'D} = \frac{BC}{B'C}$. If D is close to A , we may, without sensible error, place $BD = BA$ and $B'D = B'A$. Put $BA = p$, $B'A = p'$, $CA = r = 2f$. Then $BC = p - r$, $B'C = r - p'$, and $\frac{p}{p'} = \frac{p-r}{r-p'}$, from which $\frac{1}{p} + \frac{1}{p'} = \frac{2}{r} = \frac{1}{f}$. By measuring p and p' , we may compute r and f . For the convex mirror, p' and r are negative.

focus, and move the screen till a clear image of the flame is formed on it. Notice the size and position of the image, and whether it is erect or inverted. When the lamp is between the focus and the mirror, an image of it cannot be obtained on the screen, but it can be seen by looking into the mirror. The same is true for the convex mirror, whatever be the position of the lamp; in these last cases the image is a virtual one.

The experiment illustrates the several relative positions of the object and its image for a concave mirror, all depending on the position of the object with respect to the mirror:—

First.—When the object is at a finite distance beyond the centre of curvature, the image is real, inverted, smaller

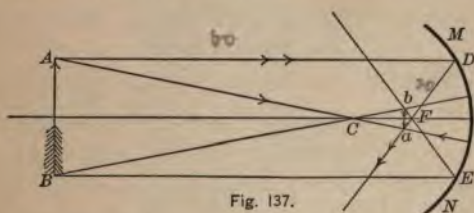


Fig. 137.

than the object, and between the centre of curvature and the focus (Fig. 137).

Second.—When a small object is at the centre of curvature, the image is real, inverted, of the same size as the object, and at the centre of curvature.

Third.—When the object is between the centre and the focus, the image is real, inverted, larger than the object, and is beyond the centre.

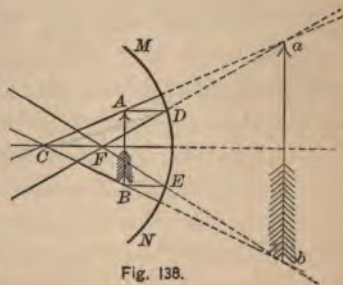


Fig. 138.

Fourth. — When the object is at the principal focus, the rays are reflected parallel to the principal axis, and no image is formed.

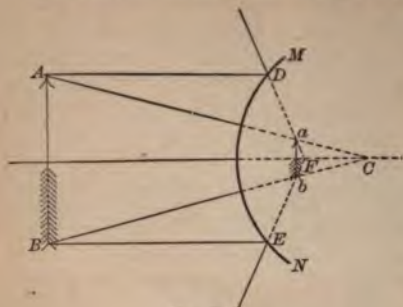


Fig. 139.

Fifth. — When the object is between the principal focus and the mirror, the image is virtual, erect, and larger than the object (Fig. 138).

When the mirror is convex, the image is always virtual, erect, and smaller than the object (Fig. 139).

254. Construction for Images. — The geometrical construction for images in spherical mirrors consists in finding conjugate focal points. For this purpose it is necessary to trace only two rays for each point of the object, one along the secondary axis through it, and the other parallel to the principal axis. The first ray is reflected back on itself, and the second through the principal focus. The intersection of the two reflected rays from the same point of the object locates the image of that point.

To illustrate: In Fig. 137, AC is the path of both the incident and the reflected ray, while the ray AD is reflected through the principal focus F . Their intersection is at a . The rays BC and BE are reflected similarly through b . Hence, ab is the image of AB . In Fig. 138, the ray AC along the secondary axis, and AD reflected back through F as DE , must be produced to meet back of the mirror at the virtual focus a . A and a are conjugate foci; also B and b , and ab is a virtual image.

Concave Mirror has Convergent Effects } Real Image
 Convex Mirror has Divergent Effects } Virtual Image

The construction for the convex mirror (Fig. 139) is the same. From the point A draw AC along the normal or secondary axis, and AD parallel to the principal axis. The latter is reflected so that its direction passes through F . The intersection of these two lines is at a . The image ab is virtual and erect.

255. Spherical Aberration. — **Experiment.** — Bend a strip of bright tin or polished brass into as true a semicircle as possible. Place it on a sheet of paper with its concave surface toward a candle or lamp (Fig. 140). The light focuses on curved lines.

Experiment. — Project the image of a candle or a lighted lamp on a screen with a concave spherical mirror. The edge of the image will be indistinct; that is, not sharply defined.

Now cover up the reflecting surface, exposing only the central portion. The image will be less bright (why?), but the definition will be sharper.

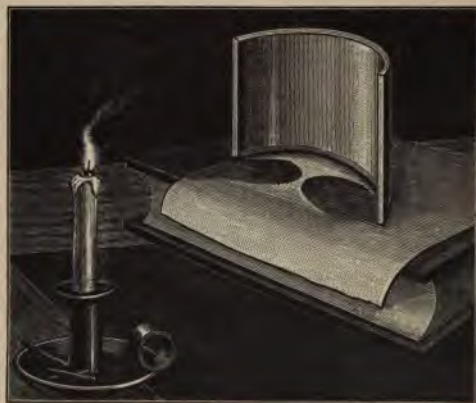


Fig. 140.

Rays incident near the margin of the mirror cross the principal axis at points nearer the mirror than F , the principal focus.

This distribution of the focus is known as *spherical aberration*, and the curve obtained in the first experiment is called the *caustic by reflection*. This curve may also be exhibited by allowing sunlight to fall on a cup partly full

of milk, or on a plain gold ring supported on a white surface. If parallel rays incident on a concave mirror of

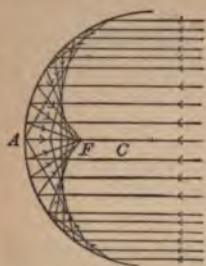


Fig. 141.

wide aperture are traced (Fig. 141), it will be seen that the caustic is formed by their intersection after reflection. If the aperture of the mirror does not exceed 10° , this confusion of focus is small, and may be neglected. By decreasing the curvature of the mirror from the vertex outward, the aberration may be corrected. This is accomplished in the parabolic mirror, a form

used as a reflector in lighthouses, in the headlights of locomotives, and for astronomical purposes.

Problems.

1. A plane mirror is placed at an angle of 45° with a horizontal plane. Find, by geometrical construction, the position of the image of a vertical object placed in front of this mirror.

2. Find, by construction, the number of images of a point placed between two plane mirrors, inclined to each other at an angle of 60° .

3. In Fig. 137, instead of finding the image of AB as there shown, take five equidistant points on AB , two of these being the extremities; then, as in § 252, find the conjugate foci of these points. Join these foci to form an image of AB . Why is this image distorted?

4. The radius of curvature of a concave spherical mirror is 30 cm. If a pencil of light diverge from a point 90 cm. in front of it, at what point will it focus? *18 cm. in front*

5. The principal focal length of a concave mirror is 20 cm. What is the position of the image when the object is placed 30 cm. in front of the mirror? *60 cm. in front*

6. Show, by diagram, the effect on the image in a convex spherical mirror of moving the object closer to the mirror.

7. An object placed 60 cm. in front of a concave spherical mirror gave an image 20 cm. in front of the mirror. What is the radius of curvature? What is the principal focal length? $r=30\text{ cm}$ $f=15\text{ cm}$ ✓

8. What radius of curvature must be given to a concave spherical mirror in order that an object placed 36 cm. in front of the mirror may give an image 72 cm. in front? 48 cm ✓

9. If the distance of an object from a convex spherical mirror is equal to the mirror's radius of curvature, where will the image be? ✓

10. The focal length of a concave spherical mirror is 24 in. An object is placed 20 in. in front of the mirror. Find the position of the image. $17\frac{1}{2}\text{ in}$ ✓

11. Show from the relation $\frac{1}{p} + \frac{1}{p'} = \frac{2}{r}$, that, if the object is placed at the centre of curvature of a concave spherical mirror, the image will also be at the centre of curvature.

12. In a concave spherical mirror, where must the object be placed so that the image will be situated half way between the centre of curvature and the principal focus? $\frac{3r}{2}$ ✓

IV. REFRACTION OF LIGHT.

256. Refraction.—**Experiment.**—Place a rectangular tank, provided with a glass face, so that the light from a candle passing over the upper edge of one end just illuminates the whole of the opposite end (Fig. 142). The bottom of the tank lies wholly in the shadow cast by the end. Now fill the tank with water. The shadow no longer covers the whole bottom, since the rays are bent at the surface of the water, as in *B*.

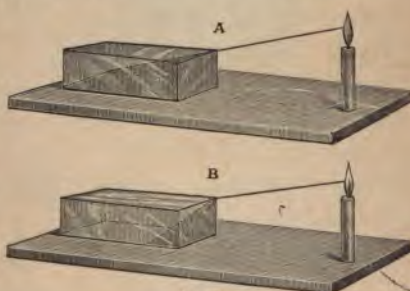


Fig. 142.

This change in the course of light in passing from one medium to another is called *refraction*.

257. Its Cause.—The investigations of Foucault, Michelson, and others, show that light has a less velocity in

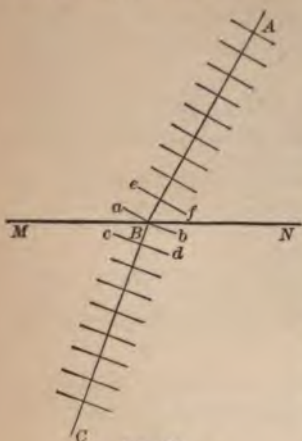


Fig. 143.

glass, water, etc., than it has in air. Now if a beam of light is incident obliquely on the surface MN of water (Fig. 143), all parts of a wave do not enter the water at the same time. Let the parallel lines perpendicular to AB represent the waves. Then one part of f will reach the water first and will travel less rapidly. The other portions, on entering, will be retarded in succession, the result being that the wave is swung around; that is, the

direction of propagation BC , perpendicular to the wave front, is changed; or, in other words, the ray is refracted.

258. Terms Defined.—Let BA (Fig. 144) represent a ray of light in air incident obliquely at A upon the surface MN of another medium, as water. Draw the normal DE to the refracting surface. The angle BAD between the incident ray and the normal to the surface at the point of incidence is the angle of incidence; and the angle CAE between



Fig. 144.

refracted ray and the normal is the angle of refraction. Produce BA to H . The angle CAH measures the deviation of the light from its original course, and is called the *angle of deviation*. With A as a centre, describe a circle. Draw BF and CG , perpendicular to DE . Then the ratio $\frac{BF}{AB}$ is the sine of the angle BAD , and the ratio $\frac{CG}{AC}$ is the sine of CAE . The ratio of these sines is called the *index of refraction*; and, since $AB = AC$, this index is also $\frac{BF}{CG}$.

59. Laws of Refraction. — Experiment. — A rectangular glass jar is provided with a circular protractor scale on one face, and a hinged cardboard cover to the top (Fig. 145). Fill the jar with water exactly to the horizontal diameter of the circle. With a plane mirror, reflect a strong beam of light through the slit at an angle as to be incident on the water exactly back of the point of emergence. Read the angles of incidence and refraction on the circular scale, and find the ratio of their sines (see Appendix). Move the slit and obtain other angles for comparison. These ratios will be found to be constant.



Fig. 145.

The experiment illustrates the following laws : —

1. *When a beam of light passes obliquely from a less refractive medium to a more highly refractive medium, it is bent toward the normal; when it passes in a reverse direction, it is bent away from the normal.*

2. *Whatever the angle of incidence, the ratio of the sines*

of the angles of incidence and refraction is constant for the same two media.

III. The planes of the angles of incidence and refraction coincide.

These laws were first discovered by Snell, a Dutch physicist, in 1621.

260. Indices of Refraction.—The *absolute index of refraction* is the ratio of the sines of the angles of incidence and refraction when the ray passes from a vacuum into the substance. The *relative index of refraction* is the index for light passing from one substance into another; it is found by dividing the absolute index of the latter by the former. The larger the index of refraction, the greater is said to be the *optical density* of the substance.

The following table gives the indices of a few substances relative to air:—

Water	1.333	Crown glass	1.51
Alcohol	1.36	Flint glass	1.54 to 1.71
Carbon bisulphide . .	1.64	Diamond	2.47

For the purposes of this book, the refractive index for water may be taken to be $\frac{4}{3}$; for crown glass, $\frac{3}{2}$; for flint glass, $\frac{8}{6}$; and for diamond, $\frac{5}{2}$.

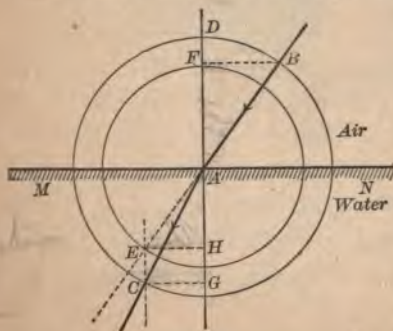


Fig. 146.

261. Construction for the Refracted Ray.—Let *MN* (Fig. 146) be the surface separating the two media, air and water; and let *BA* be a ray of light incident at *A*; it is required to draw the

fracted ray. With A as a centre, and with radii whose ratio is $\frac{4}{3}$, the index of refraction, draw two concentric circles. Through E , one of the intersections of BA with the smaller circle, draw EC parallel to the normal AD , cutting the larger circle at C . Draw AC . This will be the refracted ray, because $\frac{BF}{CG} = \frac{4}{3}$, a fact easily shown.¹

When the ray passes into a medium of smaller optical density, then the intersection of the incident ray with the *larger* circle must be used; through this point draw a parallel to the normal, cutting the *smaller* circle. The line through the intersection with the smaller circle and the point of incidence is the refracted ray.

It should be observed that it does not matter whether the intersections of the incident ray with the circles be taken before the ray enters the medium or afterwards.

Problems.

1. Trace a ray of light from air into flint glass.
2. Trace a ray of light from glass² into air.
3. Trace a ray of light from air into alcohol (index = $\frac{4}{3}$).
4. Trace a ray of light from air into diamond.
5. Trace a ray of light from water into flint glass.
6. Trace a ray of light from water into air.

262. Refraction through a Parallel Plate.—Experiment.—Draw a straight line on a sheet of paper. Place a piece of thick plate

¹ The triangles EAH and ABF are similar. Hence $\frac{BF}{EH} = \frac{AB}{AE}$. But $EH = CG$, and $\frac{AB}{AE} = \frac{4}{3}$, by construction. Therefore $\frac{BF}{CG} = \frac{4}{3}$, the index of refraction.

² By "glass" crown glass is to be understood, and the index is taken $\frac{4}{3}$.

glass over the line, covering a portion of it. Look obliquely through the glass; the line will appear broken at the edge of the plate, the part under the glass appearing laterally displaced.

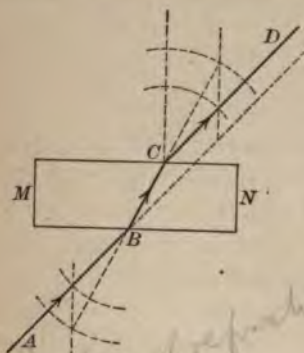


Fig. 147.

To explain this, let MN (Fig. 147) represent a thick plate of glass, and AB a ray of light incident obliquely upon it. If the path of the ray be determined, the emergent ray will be parallel to the incident ray. Hence, the apparent position of an object viewed through a plate is at one side of its true position.

263. A Prism. — Optically, the portion of a transparent substance lying between two intersecting planes is a *prism*, and the angle between these planes is the *refracting angle* of the prism. A beam of light incident on a prism is bent away from the refracting angle, and consequently the apparent position of an object seen through it is displaced toward the refracting edge.

Let ABC (Fig. 148) represent a section of a glass prism

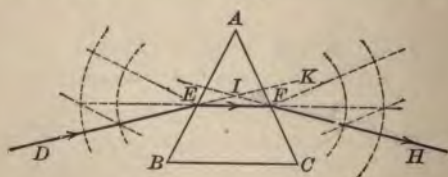


Fig. 148.

made by a plane perpendicular to the refracting edge A . Also, let DE be a ray incident on the face BA . This ray

refracted along EF , and entering the air at the F will be refracted again, taking the direction FH .

The Angle of Deviation. — Experiment. — Make a small hole at the centre of a cardboard. Around this hole, on a screen, the sheet of cardboard, and a Bunsen burner shown in Fig. 149. The burner should have a glass chimney of asbestos lined in sodium nitrate; the light should be of one color. Place the prism back of the screen, with its refracting edge up.



Fig. 149.

A on the screen is the illuminated spot when the prism is in place, and B will be the spot when the light goes through the prism. The angle APB is roughly the deviation.

The angle of deviation is the angle included between the incident ray and the emergent ray, as KIH (Fig. 148). By using different rays through the prism, and by using prisms with different angles, the deviation will be found to vary with the angle of the prism, with the index of refraction, and with the angle of incidence. The least deviation for any prism occurs when the angles of incidence and emergence are equal.

Phenomena of Refraction. — Light in passing from water into air is refracted from the normal, and consequently objects under water appear elevated above their real position. A familiar instance is a coin in an empty glass, at the bottom of which is hidden from view, becoming visible when the

cup is filled with water. The light from the coin is bent on leaving the water so that it reaches the eye. The



Fig. 150.

apparent shoaling of a pond of water is explained in the same way. Light coming to the eye from a star will be gradually bent, describing a curve (Fig. 150), because the refractive index of the air is greater near the earth than higher up. Since the direction in which the star is seen is that of a tangent to the curve at the eye, the effect will be to increase the apparent altitude of the

star. For the same reason the sun is visible when it is actually below the horizon.

266. Total Reflection. — **Experiment.** — Using the apparatus of § 259, place the cardboard against the end so that the slit is close to the bottom of the jar (Fig. 151). By means of mirrors, send a beam of light upward through the water and incident on the surface just back of the centre of the circular scale. The light will be reflected from the surface back into the water as by a mirror.



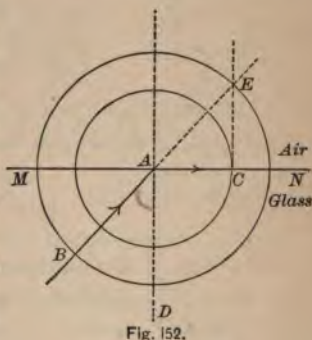
Fig. 151.

When a beam of light passes from one medium into another of smaller optical density, it is refracted from the normal, the angle of refraction being always greater than the angle of incidence. Hence, by gradually increasing the angle of incidence, L

can be found in which the angle of refraction is 90° ; the rays pass off in the surface. When the angle of incidence in the rarer medium is a right angle, the angle of incidence is called the *critical angle*. This angle with the substance, being $48\frac{1}{2}^\circ$ for water, 41° for glass, and 24° for diamond. When the angle of incidence exceeds the critical angle, as in the experiment, it suffers *total internal reflection*.

Construction for the Critical Angle. — Let MN (Fig. 152) separate two media, and glass. With A as centre, draw two concentric circles the ratio of their radii being the index of refraction, the issuing ray must lie

Hence, draw the normal, cutting the larger circle at E , and the line BA through E will be the required ray (§ 261), the angle being the critical angle.



Illustrations of Total Reflection. — Of all the rays issuing from a point at the bottom of a pond and incident on the surface, only those within the cone whose semi-angle is $48\frac{1}{2}^\circ$ can pass into the air, on account of the separation of the refracted from the totally reflected rays by total internal reflection (Fig. 153). Hence, an observer under water sees all objects out-



side as if they were crowded within this cone, and beyond this cone he sees by reflection objects lying on the bottom of the pond.

Total reflection in glass is best shown by means of a prism whose cross-section is a right-angled isosceles triangle (Fig. 154). A ray incident normally on either face about the right angle enters the prism without refraction, and is incident on the hypotenuse at an angle of 45° , which is greater than the critical angle (§ 266). Hence, the ray suffers total reflection and leaves the prism at right angles to the incident ray. Such a prism makes the best possible reflector in an optical instrument where it is desirable to change the direction of the light by 90° .

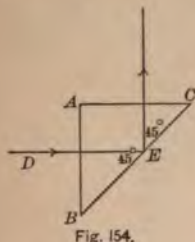


Fig. 154.

Problems.

1. Find, by geometrical construction, the magnitude of the critical angle for water. Use a protractor to measure the angle.
2. Find, by construction, the magnitude of the critical angle for glass. Use a protractor to measure the angle.
3. Trace a ray of light through a glass prism, angle 60° , the incident ray making an angle of 75° with the face of the prism. Explain the difficulty encountered.
4. Construct the angle of least deviation for an equilateral glass prism.

V. LENSES.

269. A Lens is a portion of a transparent substance bounded by two curved surfaces, or one plane and one curved surface. Those most commonly met with have spherical surfaces (Fig. 155), and are classified as follows:—

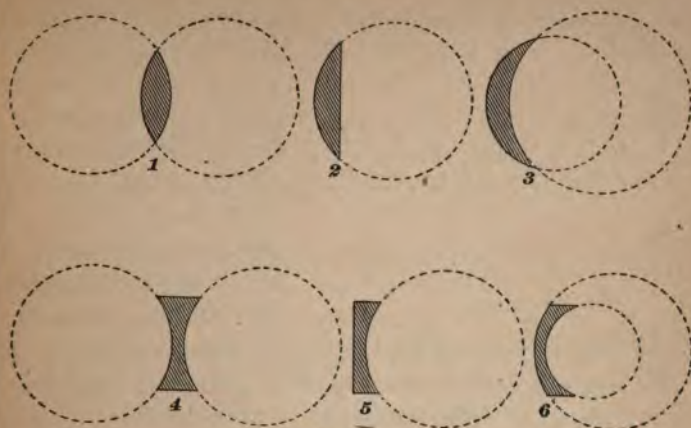


Fig. 155.

- | | |
|--|---|
| 1. Double-convex,—both surfaces convex . . . | } Converging lenses,
thicker at the middle
than at the edges. |
| 2. Plano-convex,—one surface convex, one
plane | |
| 3. Concavo-convex,—one surface convex, one
concave | |
| 4. Double-concave,—both surfaces concave, . | } Diverging lenses,
thinner at the middle
than at the edges. |
| 5. Plano-concave,—one surface concave, one
plane | |
| 6. Convexo-concave,—one surface concave, one
convex | |

The concavo-convex and the convexo-concave lenses are frequently called *meniscus* lenses. The double-convex lens may be regarded as the type of the converging class of lenses, and the double-concave lens of the diverging class.

270. Terms Defined.—The centres of the spherical surfaces bounding a lens are the *centres of curvature*. The *optical centre* is a point such that any ray passing through it and the lens suffers no change of direction. In lenses

whose surfaces are of equal curvature, it is their centre of volume, as O , in Fig. 156. In plano-lenses, the optical

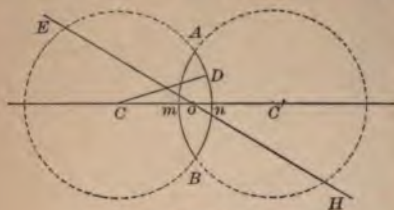


Fig. 156.

centre is the middle point of the curved face. The straight line, CC' , through the centres of curvature, is the *principal axis*, and any other straight line through the optical centre

traces, as EH , is a *secondary axis*. The normal at any point of the surface is the radius of the sphere drawn to that point; thus CD is the normal to the surface ADB at D .

271. Tracing Rays through Lenses. — Let MN represent a lens whose centres of curvature are C and C' , and AB

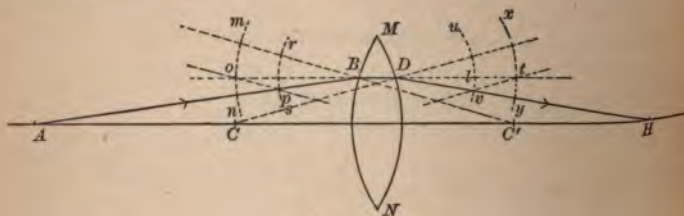


Fig. 157.

the ray to be traced through it (Figs. 157, 158). Draw the normal, $C'B$, to the point of incidence. With B as a centre, draw the arcs mn and rs , making the ratio of their radii equal the index of refraction, $\frac{3}{2}$.¹ Through p , the intersection of AB with rs , draw op parallel to the normal, $C'B$, and cutting mn at o . Through o and B draw oBD ;

¹The figures of lenses in this chapter are sections made by planes through the centre of curvature, and the index is taken at $\frac{3}{2}$.

ll be the path of the ray through the lens (§ 261).
t will again be refracted; to determine the amount,
ne normal

l the aux-
ircles, xy
as before.
gh the in-
on of BD
ed with
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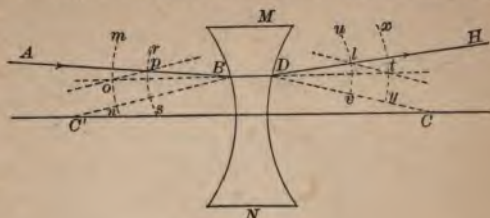


Fig. 158.

the normal CD , cutting uv at l . Through D and
 DH ; this will be the path of the ray after emer-
(Compare this procedure with that of § 263.)
ld be noticed that the convex lens bends the ray
the principal axis, while the concave lens bends it
com it.

Principal Focus of a Lens.—**Experiment.**—Let the rays
in fall on a convex lens parallel to its principal axis. Hold
the lens a sheet of paper, moving it till the round spot of light
est and brightest. If held steadily, a hole may be burned in
r. This spot marks the *principal focus* of the lens, and its
from the optical centre is the *principal focal length*.

res 159 and 160 illustrate the method of finding the
focus geometrically.

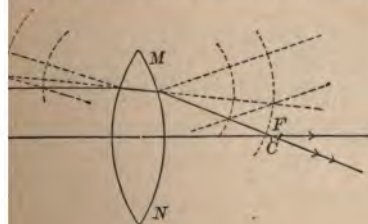


Fig. 159.

In the double-con-
vex lens, it is real,
and at the centre of
curvature, *if the
index of refraction
is $\frac{3}{2}$* . For the plano-
convex lens, the

ngth is twice the radius of curvature for the same

refractive index. Convex lenses are often called *burning glasses*, because of their power to focus the *heat rays*, as

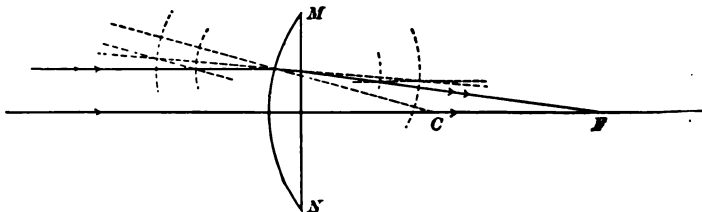


Fig. 160.

shown in the experiment. Figure 161 shows that parallel rays are rendered diverging by a double-concave lens, the

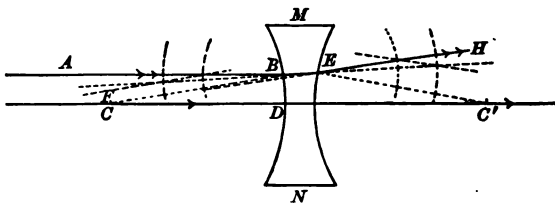


Fig. 161.

principal focus being virtual, and at the centre of curvature, when the refractive index is $\frac{3}{2}$. Concave lenses increase the divergence of light, whereas convex lenses decrease it.

273. Conjugate Foci. — Lenses resemble spherical mirrors in this respect, that if a pencil of light diverges from a point and is incident on the lens, it is focussed at a point on the axis through the radiant point. These points are called *conjugate foci*, for the same reason as in mirrors. If p and p' are the distances of these points from the lens, and f is the principal focal length, then the relation between the three quantities is expressed by the equation

$+\frac{1}{p'} = \frac{1}{f}$.¹ For converging lenses p' must be considered negative when the image is virtual; and for diverging lenses both p' and f are to be treated as negative. By measuring p and p' , the focal length of a lens may be computed.

Diverging lenses always increase the divergence of the rays incident upon them, and hence the focus of such lenses is always virtual. With converging lenses the results vary with the angle of divergence. Hence, the following cases arise:—

First.—When the incident rays diverge from a point more than twice the focal distance from the lens.

In Fig. 157 the two rays diverging from A focus at H , less than twice the focal distance.

Second.—When the incident rays diverge from a point twice the focal distance from the lens.

(The student should draw a figure, and show that the focus is real, and at twice the focal distance.)

Third.—When the incident rays diverge from a point less than twice and more than once the focal distance.

(The student should draw a figure and show that the focus is real and at more than twice the focal distance.)

Fourth.—When the rays diverge from the principal focus.

¹ In Fig. 163, from the similar triangles AOB and aOb , we have $\frac{B}{b} = \frac{KO}{OL}$. If E and I be connected by a straight line, this line may be taken as approximately equal to AB , and to pass through O . From the

similar triangles EFI and aFb we have $\frac{EI}{ab} = \frac{OF}{FL}$. Hence, $\frac{KO}{OL} = \frac{OF}{FL}$.

But $KO = p$, $OL = p'$, $OF = f$. Then $FL = p' - f$, $\frac{p}{p'} = \frac{f}{p' - f}$, and

$+\frac{1}{p'} = \frac{1}{f}$. By measuring p and p' , we may compute f . For the diverging lens f and p' are negative.

p' = distance from eye from lens
a = distance from lens

(The student should draw a figure and show that the emergent rays are parallel.)

Fifth. — When the rays diverge from a point between the principal focus and the lens.

Figure 162 shows that the divergence of the rays is not

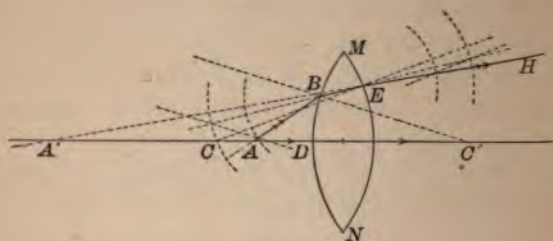


Fig. 162.

wholly overcome by the lens, but that they leave the lens as if they emanated from a point farther from the lens than the actual radiant point. Hence, the focus is virtual.

274. Formation of Images. — The image of an object formed by a lens may be found by means of the images of its points. The work is considerably lessened by observing the following method of construction : —

Draw secondary axes through the extremities of the object. These will be rays of light which suffer no change of direction. (Why?) Also through these extremities draw rays parallel to the principal axis, and find by construction their path in the lens (§ 261). On leaving the lens they will pass through the principal focus. (Why?) The image of each extremity will be the intersection of the two rays drawn from it. The image of any point is always on the secondary axis passing through it.

To illustrate, let AB be the object and MN the lens (fig. 163). Rays along the secondary axes pass through the lens without deviation. The rays AD and BH , parallel to the principal axis, are refracted on entering the lens along DE and HI respectively, and pass through F , the principal focus, after leaving the lens. The intersec-

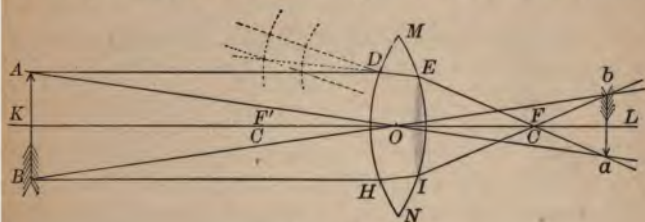


Fig. 163.

tion of Aa with $ADEa$ is the image of A , and that of BIb with Bb is the image of B . If other rays from A and B be drawn, they will focus at a and b respectively. Hence, ab is the image of AB .

The student should draw figures and establish the following propositions for converging lenses:—

I. When the object is at a finite distance greater than twice the focal distance, the image is real, inverted, situated beyond the principal focus, and is smaller than the object (Fig. 163).

II. When the object is at twice the focal distance, the image is real, inverted, situated at twice the focal distance, and is of the same size as the object.

III. When the object is at less than twice and more than the focal distance, the image is real, inverted, situated beyond the principal focus, and is larger than the object.

IV. When the object is at the principal focus, the light leaves the lens in parallel rays, no image being formed.

V. When the object is between the principal focus and the lens, the image is virtual, erect, and enlarged (Fig. 164).

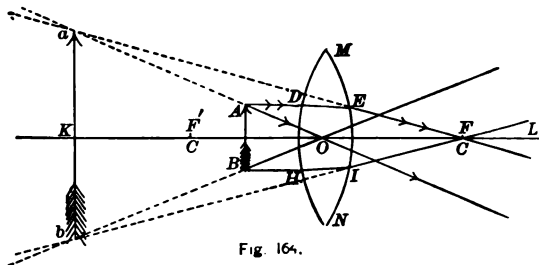


Fig. 164.

The images formed by diverging lenses are always virtual, erect, and smaller than the object (Fig. 165).

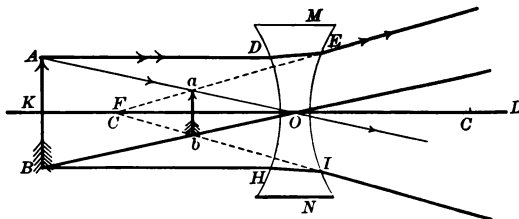


Fig. 165.

The distance of the object from the lens affects only the size of the image.

275. Experimental Illustrations. — **Experiment.** — Arrange in line on the table a lamp, a converging lens, and a screen. Give the lamp successively the positions described in the preceding article, adjusting the screen each time till a sharply defined image is obtained. The results will be in harmony with the preceding propositions. If the lamp is placed at the focus of the lens, only a blurred image is obtained; if between the focus and the lens, no image is formed on the screen, but a magnified image of the lamp may be seen on looking through the lens.

If, after focussing the image on the screen, the eye be placed in line with the object and the lens, and the screen be removed, an inverted image of the object can be seen suspended in mid-air.

The projection of images on a screen by means of a lens is its application in the projection lantern (§ 299), the compound microscope (§ 296), the telescope (§ 297), and the photographer's camera (§ 300). The formation of virtual images by a converging lens is applied in the simple magnifier and in the eyepieces of telescopes and compound microscopes.

276. Spherical Aberration.—If rays from any point be drawn to different parts of a lens, and their directions be determined after refraction, it will be found that those incident near the edge of the lens cross the principal axis, after emerging, nearer the lens than those incident nearer the middle. (The student should convince himself of this by drawing a figure.) The principal focal length for the marginal rays is therefore less than for central rays. This indefiniteness of focus is called *spherical aberration by refraction*, the effect of which is to lessen the distinctness of images formed by the lens. In practice an annular screen, called a *diaphragm*, is used to cut off the marginal rays; this renders the image sharper in outline, but less bright. In the large lenses used in telescopes the curvature of the lens is made less toward the edge, so that all parallel rays are brought to the same focus.

Problems.

1. Compare, by geometrical construction, the focal lengths of two convex lenses of the same radius of curvature, one to be of glass, the other of diamond.
2. A lamp placed 90 cm. from a convex lens gives an image 45 cm. from the lens. What is the focal length?

3. The radius of curvature of a double convex lens of crown glass is 20 in. Find the position of the image of an object distant 30 in. from the lens. *60 in*

4. Show, by geometrical construction, the effect on the image of varying the distance of the object from a concave lens.

✓ 5. An object is placed 75 cm. from a concave lens whose focal length is 50 cm. Find the position of the image. *30 cm*

✓ 6. By means of a convex lens of 8 in. focal length, it is required to project an object on a screen 22 ft. distant from the lens. How close to the lens must the object be placed? *8.25 in*

✓ 7. An object is placed at a distance of 25 cm. from a convex lens of 50 cm. focal length. Where will the image be formed? *50 in*

VI. DISPERSION.

277. Analysis of White Light. The Solar Spectrum. —

Experiment. — Close the opening of a porte-lumière¹ with a piece of



Fig. 166.

tin, in which is cut a very narrow vertical slit. Let the ribbon of sunlight issuing from the slit be incident obliquely on a glass prism (Fig. 166). A many-colored band, gradually changing from red at one end through orange, yellow, green, blue, to violet at the other, appears on the screen. If a converg-

¹ The porte-lumière is a device by means of which a beam of sunlight can be reflected horizontally into a darkened room through an opening in the shutter. In the simplest form it is a hinged mirror set outside of the window, and so arranged that its position can be changed from within.

ing lens of about 30 cm. focal length be used to focus an image of the slit on the screen, and the prism be placed near the principal focus, the colored images of the slit will be more distinct.

This experiment, though not original with Sir Isaac Newton, was first explained by him in 1666. It shows that white or colorless light is a mixture of an infinite number of differently colored rays, differing in refrangibility, the red being least and the violet most refrangible. The brilliant band of light consists of an indefinite number of colored images of the slit; it is called the *solar spectrum*, and the opening out or separating of the beam of white light is known as *dispersion*.

278. Synthesis of Light. — Experiment. — Project a spectrum of sunlight on the screen. Now place a second prism like the first behind it, but reversed in position (Fig. 167). There will be formed a colorless image, slightly displaced on the screen.



Fig. 167.

The second prism reunites the colored rays, making the effect that of a thick plate of glass (§ 262). The recombination of the colored rays into white light may also be effected by receiving them on a concave mirror or a large convex lens.

279. Chromatic Aberration. — Experiment. — Close the opening

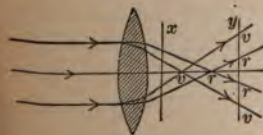


Fig. 168.

of the porte-lumière with a piece of cardboard in which is a small round hole. Project an image of this aperture on the screen, using a double-convex lens for the purpose. The round image will be bordered with the spectral colors.

This defect in lenses is known as *chromatic aberration*. It is caused by the lens refracting the rays of different

colors to different foci. The violet rays, being more refrangible than the red, will have their focus nearer to the lens than the red, as shown in Fig. 168, where v is the principal focus for violet light and r for the red. If a screen were placed at x , the image would be bordered with red, and if at y with violet.

280. The Achromatic Lens.—**Experiment.**—Project a spectrum of sunlight on the screen, using a prism of crown glass, and note the length of the spectrum when the prism is turned to give the least deviation. Repeat the experiment with a prism of flint glass having the same refracting angle. The spectrum formed by the flint glass will be about twice as long as that given by crown glass, while the position of the middle of the spectrum on the screen is about the same in the two cases. Now use a flint glass prism whose refracting angle is half that of the crown glass one. The spectrum is nearly equal in length to that given by the crown glass prism, but the deviation of the middle of it is considerably less. Finally, place this flint glass prism in a reversed position against the crown glass one. The image of the aperture is no longer colored, and the deviation is about half that produced by the crown glass alone.

The above facts suggested to Dolland, an English optician, in 1757, that by combining a double-convex lens of crown glass with a plano-concave lens of flint glass the dispersion by the one would neutralize that due to the other, while the refraction would be reduced about half (Fig. 169). Such a lens or system of lenses is called *achromatic*, since images formed by it are not fringed



Fig. 169.

with the spectral colors.

281. Dispersion by a Globe of Water.—**Experiment.**—Fill an air-thermometer bulb, about 4 cm. in diameter, with clear water. Cover the opening of the porte-lumière with a large sheet of white cardboard, in which is a circular hole about 3.75 cm. in diameter. Support the bulb a short distance from the opening so that the cylind-

sunlight is incident upon it. There will appear on the cardboard one or more circular spectra, resembling rainbows.

To understand this experiment it must be kept in mind when light passes from one medium to another, part of the light is always reflected. So in the case of the globe of water, part passes through it and part enters and is internally reflected from the back surface, forming the colored image on the cardboard screen.

The circle whose centre is O (Fig. 170) represents the globe of water, and SS' rays of sunlight incident upon

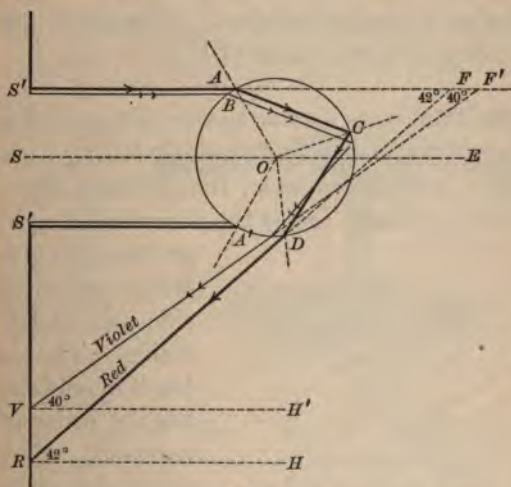


Fig 170.

Of all the rays entering the globe, it is shown in mathematical optics that the *red* rays incident in the immediate vicinity of $59^{\circ} 23' 30''$ from the axis SE , as they pass together after reflection and subsequent refraction—that is, they are parallel on leaving the sphere, and have sufficient intensity to produce a colored image

on the screen. The violet rays at $58^{\circ} 40'$ from the axis, as at B , produce a violet image for a like reason. Between these positions the other colors arrange themselves in order. Since the sphere is symmetrical with respect to SO , it follows that the colored images will appear as circles. In this way the inner colored ring is produced in the above experiment. The angle $S'FR$ is the deviation of the red rays, and equals $42^{\circ} 1' 40''$; the deviation of the violet rays $S'F'V$ is $40^{\circ} 17'$. Hence the red circle on the screen has a radius of about 42° , and the violet one about 40° . The second colored band is caused by rays that are twice internally reflected. The violet part has a radius of about 54° and the red 51° .

282. The Rainbow is a solar spectrum formed by spherical raindrops dispersing the sunlight falling upon them. Usually two bows are visible, the *primary* and the *secondary*. The *primary bow* is the inner and brighter one,

and is distinguished by being red on the outside and violet on the inside. The *secondary bow* is much fainter, and has the order of colors reversed. Figure 171 shows the relative position of the sun, the observer, and the raindrops which form the bows. An observer at E , with his back to the sun, receives red light

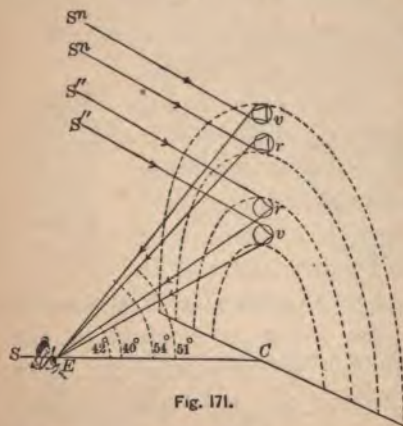


Fig. 171.

from every raindrop situated 42° from the line SC , drawn

ugh the sun and the eye, and violet from those 40° ant. The drops, being approximately spheres, send light on all sides, and hence all of those situated in a e about C , and distant 42° , will send red light to the and give the appearance of a red circle, while those at will send violet light, the other colors arranging them- es between these limits. Again, those drops which 51° from C will also send red light to the eye, the being twice reflected internally, and those 54° will violet light, thus forming the secondary bow.

3. The Spectroscope (Fig. 172) is an instrument for ing spectra. In one of its simplest forms it consists prism, a tel- pe, and a tube d the *colli-* r, carrying djustable slit he outer end a converging at the other ender paral- he diverging



Fig. 172.

coming from the slit. The slit must therefore be ed at the principal focus of the collimating lens. To k the deviation of the spectral lines, there is provided he supporting table a divided circle, which is read by aid of the vernier attached to the telescope arm.

4. Kinds of Spectra.—**Experiment.**—Place a lighted candle np in front of the slit of the spectroscope. The spectrum will be to pass from red at one end, through the various colors, to violet e other, without any interruptions.

Experiment. — Over the tube of a Bunsen burner place a short cylinder of asbestos paper into which has been fused a salt of either sodium, lithium, or strontium. The heat of the flame vaporizes these salts. View this flame through the spectroscope, and instead of a bright band there will be seen one or more bright colored lines, depending on the nature of the salt, sodium giving a bright yellow line, lithium a bright carmine line, strontium a cluster of lines in the orange-red, etc. We infer that the lights produced by an incandescent vapor are of certain particular wave lengths only.

Experiment. — Project a spectrum of the electric arc on the screen, using a hollow prism with plane glass sides and filled with carbon bisulphide. The spectrum will be composed of colors from red at one end through all the intermediate spectral tints to violet at the other, without any interruptions or gaps; that is, it will be *continuous*. With a clean glass rod introduce a few crystals of sodium nitrate into the arc, and lengthen it so as to reduce the intensity of the continuous spectrum. The spectrum will now consist of a few bright colored lines, one red, one yellow, three green, and one violet, the yellow being especially prominent. Now substitute for the lower carbon in the lamp a larger one with a cup-shaped depression in its upper end. Reverse the direction of the current through the lamp, making the lower carbon the positive. Place a few crystals of sodium nitrate in the carbon cup immediately after it has been heated by the current, and close the circuit. The heat will vaporize the sodium, and the intensely hot arc will be surrounded with an atmosphere of sodium vapor at a lower temperature. This outer vapor will absorb the light emitted by the hotter vapor within, and dark bands will appear in place of the bright ones in the faint continuous spectrum, particularly in the red and yellow.

These experiments illustrate the three kinds of spectra, namely, the *continuous*, the *discontinuous* or *bright-lined*, and the *absorption* or *reversed* spectra. Solids, liquids, and dense vapors and gases, when heated to incandescence, give continuous spectra; rarefied gases and vapors heated to incandescence give discontinuous spectra; and gases and vapors absorb light of the same refrangibility as they emit at a higher temperature.

285. The Fraunhofer Lines.—If sunlight be analyzed with a spectroscope, a number of dark lines will be seen to cross the spectrum (Fig. 173). This discovery was made by Wollaston in 1802. Fraunhofer

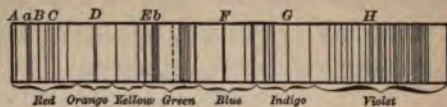


Fig. 173.

was the first to notice that some of these lines coincide in position with the bright lines of certain artificial lights. He mapped no less than 576 of them, and designated the more important ones by the letters *A, B, C, D, E, F, G, H*, the first in the extreme red and the last in the extreme violet. For this reason it is customary to refer to them as the Fraunhofer lines. In recent years Lockyer, Rowland, Langley, and many others, employing greatly improved apparatus, have found the number of these lines to be practically unlimited.

The explanation of these dark lines was first suggested by Stokes in 1852, and the theory was fully established by Kirchhoff in 1859. In the last experiment it was shown that sodium vapor absorbs that part of the light of the electric arc which is of the same refrangibility as the light emitted by the vapor itself. Similar experiments with other substances show that every substance has its own absorption spectrum. These facts suggested the following explanation of the Fraunhofer lines: The heated nucleus of the sun gives off light of all degrees of refrangibility. Its spectrum would therefore be continuous, were it not surrounded by an atmosphere of metallic vapors and of gases, which absorb or weaken those rays of which their spectra consist. Hence, the parts of the spectrum which would have been illuminated by those particular rays have their brightness diminished, since the rays from the

nucleus are absorbed, and the illumination is due to the less intense light coming from the vapors. These absorption lines are not lines of no light, but are lines of diminished brightness, appearing dark by contrast with the other parts of the spectrum.

286. Spectrum Analysis consists in detecting the presence of substances by the spectra of their heated vapors. The great delicacy of the method is exhibited in the statement made by Professor Swan, that he was able to detect by its spectrum the presence of $\frac{1}{2500000}$ th part of a grain of sodium.

The applications of the spectroscope are many and various. By an examination of their absorption spectra, normal and diseased blood are easily distinguished, the adulteration of substances is detected, and the chemistry of the stars is approximately determined.

VII. COLOR.

287. The Color of light depends on its wave length, extreme red being due to the longest waves, and extreme violet to the shortest. The unit employed in measuring wave lengths of light is the *tenth-metre*, of which 10^{10} are required to make a metre. The following are the values for the principal Fraunhofer lines in air at 20° C. and 760 mm. pressure:—

<i>A</i> Dark Red	7621.31	<i>E</i> ₁ Light Green	5270.52
<i>B</i> Red	6884.11	<i>E</i> ₂	5269.84
<i>C</i> Orange	6563.07	<i>F</i> Blue	4861.51
<i>D</i> ₁ Yellow	5896.18	<i>G</i> Indigo	4293.
<i>D</i> ₂	5890.22	<i>H</i> ₁ Violet.	3968.

In white light the number of colors is infinite, and they pass into one another by imperceptible gradations of shade

refrangibility. Color stands related to light in the way that pitch does to sound. In artificial lights the colors are either feeble or wanting, as can be seen by an examination of their spectra. Hence, artificial lights are not white, but each one is characterized by a color that predominates in its spectrum.

8. Color of Opaque Bodies.—**Experiment.**—Paste a small angular strip of white paper on a sheet of black cardboard. View the strip through a glass prism, holding its edges parallel to the length of the strip. The image is a spectrum, colored like that produced by sunlight, but less bright. If a red strip of paper, similarly pasted, is examined in the same way, the spectral image is red at one end, while the colors belonging to the other end are dim or wanting. In like manner if a blue strip is examined, the spectral image is blue, the other colors being mostly wanting.

Experiment.—Project the solar spectrum on a white screen. (Why?) Hold pieces of colored paper or cloth successively in different parts of the spectrum. A strip of red flannel appears brilliantly red in the red part of the spectrum, and black elsewhere; a blue ribbon is only in the blue part of the spectrum, and a piece of black paper is black in every part of the spectrum.

These experiments show that the color of a body is due both to the light that it receives and the light that it reflects; a body is red because it reflects chiefly, if not wholly, the red rays of the light incident upon it, the others being absorbed wholly or partly at its surface, and that it cannot be red if there is no red light incident upon it. In like manner a body is white if it reflects all the rays in about equal proportions, provided white light is incident upon it. Therefore it appears that bodies have no color of their own, since they can exhibit no color not already present in the light which illuminates them. This truth is illustrated by the difficulty experienced in matching colors by artificial lights, and by the changes in shade some fabrics

undergo when taken from sunlight into gaslight. Most artificial lights are deficient in blue and violet rays; and hence all complex colors, into which blue or violet enters, as purple and pink, change their shade when viewed by them.

289. Color of Transparent Bodies. — **Experiment.** — Project the spectrum of the sun or of the arc light on the screen. Hold across the slit a flat bottle or cell filled with a solution of ammoniated oxide of copper. The spectrum below the green will be cut off. Substitute a solution of picric acid and the spectrum above the green will be cut off. Place both solutions across the slit and the green alone remains. It is the only color transmitted by both solutions. In like manner blue glass cuts off the less refrangible part of the spectrum, ruby glass cuts off the more refrangible, and the two together cut off the whole.

It thus appears that the color of a transparent body is determined by the colors that it absorbs. It is colorless if

it absorbs all colors in like proportion, or absorbs none; but if it absorbs some colors more than others, its color is due to the mixed impression produced by the transmitted radiations.



Fig. 174.

290. Mixing Colors. — **Experiment.**

— Cut out of colored paper several colored disks, about 15 cm. in diameter, with a hole at the centre for mounting them on the spindle of a whirling machine (Fig. 174), or for slipping them over the handle of a heavy spinning top. Slit them along a radius from the circumference to the centre, so that two or more of them can be placed together, exposing any proportional part of each one as desired (Fig.

175). Select seven disks, whose colors most nearly represent those of

er spectrum; put them together so that equal portions of the re exposed. Clamp on the spindle of the whirling machine ate them rapidly. When viewed in a strong light the color ll white or uniform gray.

method of mixing colors is based on the physio- fact that a sensation lasts than the impression pro- it. Before the sensation by one impression has the disk has moved, so

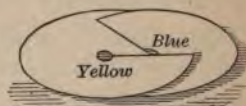


Fig. 175.

different impression is produced. The effect is lent to superposing the several colors on one r, as was done in the recomposition of white light).

ed, green, and violet disks, or red, green, and blue re used, exposing equal portions, gray or white is ed on rotating them rapidly. If any two colors



Fig. 176.

standing opposite each other in Fig. 176 are used, the result is white; and if any two alter- nate ones are used, the result is the intermediate one. By using the red, the green, and the violet disks, and exposing different proportions, it has been found possible to produce any color of the spectrum.

This fact suggested to Dr.

the theory that there are only three primary color ons, and that our recognition of different colors is

the excitation of these three in varying degrees. us evident that a mixture of colored lights is very nt from a mixture of pigments (§ 292).

291. Complementary Colors. — Any two colors whose mixture produces on the eye the impression of white light are said to be *complementary*. Thus, red and bluish green are complementary; also orange and light blue. When complementary colors are viewed next to each other, the effect is a mutual heightening of color impressions.

Experiment. — Complementary colors may be seen by retinal fatigue. Cut some design out of paper, and paste it on red glass. Project it on a screen in a dark room. Look steadily at the screen for several seconds, and then turn up the lights. The design will appear on a pale green ground.

The explanation is that the portion of the retina on which the red light falls becomes tired of red, and refuses to convey as vivid a sensation of red as of the other colors when less intense white light is thrown on it. But it retains its sensitiveness in full for the rest of white light, and therefore conveys to the brain the stimulus of white light with the red cut out; that is, of the complementary color, green.

292. Mixing Pigments. — **Experiment.** — Draw a broad line on the blackboard with a yellow crayon. Over this draw a similar band with a blue crayon. The result will be a band distinctly green.

The yellow crayon reflects green light as well as yellow, and absorbs all the other colors. The blue crayon reflects green light along with the blue, absorbing all the others. Hence, in superposing the two chalk marks, the mixture absorbs all but the green. The mark on the board is green, because that is the only color that survives the double absorption. In mixing pigments, the resulting color is the residue of a process of successive absorptions. If the spectral colors, blue and yellow, are mixed, the product is white instead of green.

VIII. INTERFERENCE AND DIFFRACTION.

Newton's Rings. — Experiment. — Press together at their two small pieces of heavy plate glass, using a small iron clamp for purpose. Then look obliquely at the glass; curved bands of color may be seen surrounding the point of greatest pressure.

This experiment is a modification of one performed by Hooke. It was afterwards repeated by Newton while attempting to determine the re-

lationship between the colors in the spectrum and the thickness of the air-film. Each used a plano-convex lens of long focus resting

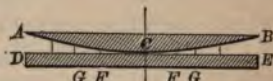


Fig. 177.

on a plate of plane glass. Figure 177 shows a section of the apparatus. Between the lens and the plate there is a crescent-shaped film of air, very thin, and quite similar to the one formed between the glass plates in the above experiment.

If the glasses are viewed by reflected light, there is a dark spot at the point of contact, surrounded by several colored rings (Fig. 178); but if viewed by transmitted light, the colors are complementary to those seen



Fig. 178.

by reflection (§ 291). The explanation of the phenomenon is to be found in the interference of two sets of waves, one reflected internally from the curved surface, ACB , and the other from the surface DCE , on which it presses. If light of one color is incident on AB , a portion will be reflected from ACB , and another portion from DCE . Since

light reflected from DCE has travelled farther by twice the thickness of the air-film than that from ACB , and the

film gradually increases in thickness from C outward, it follows that at some places the two reflected portions will meet in like phase, and at others in opposite phase, causing a strengthening of the light at the former, and extinction of it at the latter. If red light be used, the appearance will be that of a series of concentric circular red bands separated by dark ones, each shading off into the other. If violet light be employed, the colored bands will be closer together on account of the shorter wave length. Other colors will give bands intermediate in diameter between the red and violet. From this it follows that if the glasses be illuminated by white light, at every point some one color will be destroyed, and the others will be either weakened or strengthened, depending on the thickness of the air-film at the point under consideration, the color at each point being the result of mixing a large number of colors in unequal proportions. Hence, the point C will be surrounded by a series of colored bands.¹

The colors of the soap bubble, of oil on water, of heated metals which easily oxidize, of a thin film of varnish, and of the surface of very old glass, are all caused by the interference of light reflected from the two surfaces of a very thin film.

294. Diffraction.—**Experiment.**—Place across the opening of the porte-lumière two superposed pieces of perforated cardboard. The projected images of the very small holes, as one piece is moved across the other, are fringed with the spectral colors.

¹ The light from ACB differs in phase half a wave length from that reflected from DE , because the former is reflected in an optically dense medium next to a rare one, and the latter in an optically rare medium next to a dense one. This phase difference is additional to the one above described.

periment. — Transparent *diffraction gratings* are made by ruling a fine diamond point a number of equidistant parallel lines very close together on glass. Substitute this for the prism in projecting the spectrum of sunlight or of the arc light on the screen (§ 284). There will be seen on the screen a central image of the slit, and on either side of it a series of spectra. Cover half of the length of the slit with red glass and the other half with blue. There will now be a series of red images and also a series of blue ones, the red ones being farther from the central image than the blue. Lines ruled close together on smoked glass may be used instead of a "grating."

These experiments illustrate a phenomenon known as *interference*. The colored bands are caused by the interference of the waves of light which are propagated in all directions from the fine openings, the effects being visible because the transparent spaces are so small that the intensity of the direct light from the source is largely reduced. Diffraction gratings are also made to operate by reflected light. Striated surfaces, like mother-of-pearl, changeable colors, and the plumage of many birds, owe their beautiful iridescent colors to interference of light by diffraction.

IX. OPTICAL INSTRUMENTS.

5. **The Simple Magnifier**, or *simple microscope*, is a single-convex lens, usually of short focal length. The

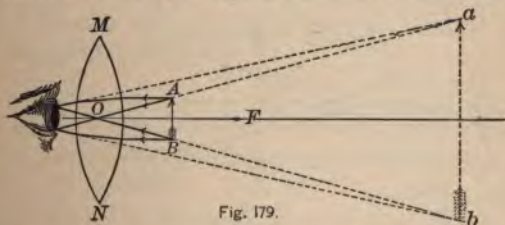


Fig. 179.

It must be placed nearer the lens than its principal focal length. The image is then virtual, erect, and enlarged. *B* is the object in Fig. 179, the virtual image is *ab*;

and if the eye be placed near the lens on the side opposite the object the impression received will be projected outward in the direction in which the light enters the eye, and the virtual image will be seen in the position of the intersection of the rays produced, as at ab .



Fig. 180.

296. The Compound Microscope (Fig. 180) is an instrument designed to obtain a greatly enlarged image of very small objects. In its simplest form it consists of a converging lens MN (Fig. 181), called the *object glass*, and another converging lens RS , called the *eyepiece*. The two lenses are mounted in the ends of the tube of Fig. 180. The object is placed

on the stage just under the objective, and a little beyond its principal focus. A real image ab (Fig. 181) is formed slightly nearer the eyepiece than its focal length. This

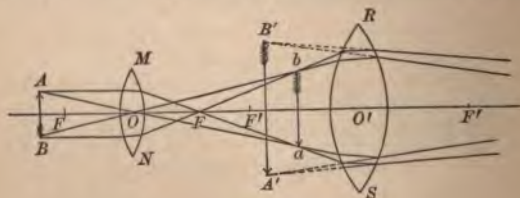


Fig. 181.

image formed by the objective is viewed by the eyepiece, and the latter gives an enlarged virtual image. (Why?) Both the objective and the eyepiece produce magnification.

The Astronomical Telescope.—The system of lenses refracting astronomical telescope (Fig. 182) is to that of the compound microscope. Since it is used to view distant objects, the objective MN is of large aperture and long focal length. The real image formed by it is the object for the eyepiece, which again forms a virtual image for the eye of the observer. The

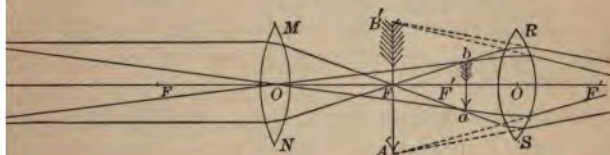


Fig. 182.

magnification is the ratio of the focal lengths of the objective and the eyepiece. The objective must be large, for the purpose of collecting enough light to permit large magnification of the image without too great loss in brightness.

The figure shows that the image in the astronomical telescope is inverted. In a terrestrial telescope the image is made erect by introducing near the eyepiece two doublet lenses, in such relation to each other and to the intermediate image that a second real image is formed like the first, and the final image is erect.

Galileo's Telescope.—This earliest form of telescope gives an erect image by the use of a diverging lens for the eyepiece (Fig. 183). This lens is placed between the eye and the real image, ab , which would be formed by the objective if the eyepiece were not interposed. The eye is practically at the image ab , and the rays of light issuing from it slightly divergent for distant objects.

The image is therefore at $A'B'$ instead of at ab , and it is erect and enlarged. This telescope is much shorter than the astronomical telescope, for the distance between the

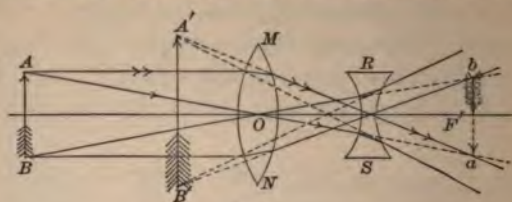


Fig. 183.

lenses is the difference of their focal lengths instead of their sum. In the *opera glass* two of Galileo's telescopes are attached together with their axes parallel.

299. The Projection Lantern is an apparatus by which a greatly enlarged image of an object may be projected on a screen. Its three essentials are a strong light, a con-

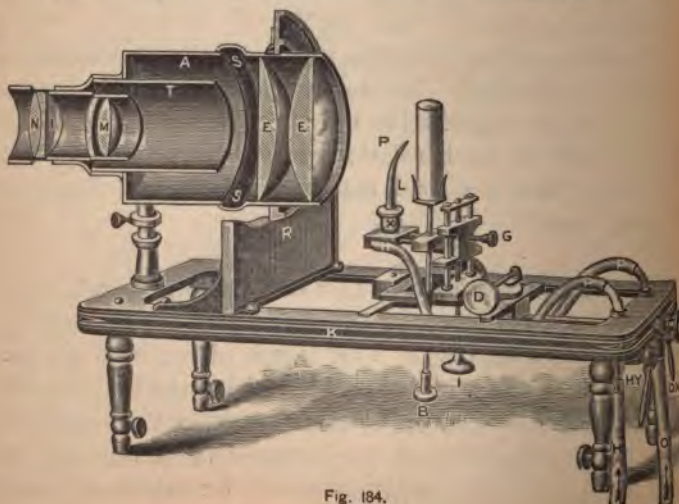


Fig. 184.

ser, and an objective. The light may be the calcium light, as shown in Fig. 184, the electric arc light, or a gas oil burner. The condenser *E* is composed of a pair of plano-convex lenses, with their convex surfaces turned toward each other. It has for its chief purpose the concentration of the light on the object by refraction, so as to bring as much as possible on the screen. The object, commonly a drawing or a photograph on glass, is placed at the condenser at *SS*, where it is strongly illuminated. The objective, *MN*, is an achromatic combination, acting as a converging lens to project on the screen a real, inverted, and enlarged image of the object.

100. **The Photographer's Camera**, a section of which is shown in Fig. 185, consists of a box *BC*, adjustable in length, blackened inside, and provided at one end with a double achromatic lens *L*, and at the other with a holder for the sensitive

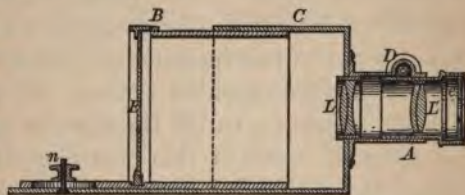


Fig. 185.

plate. If by means of the rack and pinion *D*, the lens be properly focussed for an object in front of it, an inverted image will be formed on the plate *E*. The light acts on the salts contained in the sensitized film on the plate, producing in them a modification which, by the processes of "developing" and "fixing," becomes a permanent negative picture of the object.

101. **The Eye.** — The eye is like a small photographic camera, with a converging lens, a dark chamber, and

a sensitive screen. Fig. 186 is a vertical section through the axis. The outer covering, or *sclerotic coat* *H*, is a thick opaque substance, except in front, where it is extended



Fig. 186.

as a transparent coat, called the *cornea* *A*. Behind the cornea is a diaphragm *D*, constituting the colored part of the eye, or the *iris*. The circular opening in the iris is the *pupil*, the size of which changes with the

intensity of light. The pupil of the cat's eye is not round, but elongated. Between the cornea and the crystalline lens *E* is a transparent fluid called the *aqueous humor*, while the large chamber behind the lens is filled with a jellylike substance called the *vitreous humor*. The *choroid coat* lines the walls of this posterior chamber, and on it is spread the *retina*, a membrane traversed by a network of nerves, branching from the *optic nerve* *M*. The choroid coat is filled with a black pigment, which serves to darken the cavity of the eye, and to absorb the light reflected internally.

302. Production of Sight. — Rays of light diverging from the object enter the pupil and form an inverted image on the retina, precisely as in the photographic camera. In place of the sensitized plate is the sensitive retina, from which the sensation of sight is carried to the brain along the optic nerve.

In the camera the distance between the lens and the screen or plate must be adjusted for objects at different distances. In the eye the corresponding distance is fixed, and the adjustment for distinct vision is made by changing the curvature of the front surface of the crystalline lens. The curvature is increased by relaxing the ligament which connects it with the choroid, and the focal length is thus diminished. This capability of the lens of the eye to change its focal length for objects at different distances is called *accommodation*.

303. The Blind Spot.—There is a small depression where the optic nerve enters the eye. The rest of the retina is covered with microscopic rods and cones, but there are none in this depression, and it is insensible to light. It is accordingly called the *blind spot*. Its existence can be readily proved by the help of Fig. 187. Hold the book



Fig. 187.

with the circle opposite the right eye. Now close the left eye and turn the right to look at the cross. Move the book toward the eye from a distance of about a foot, and a position will readily be found where the black circle will disappear. Its image then falls on the blind spot. It may be brought into view again by moving the book either nearer the eye or farther away.

304. Defects of the Eye.—A normal eye is one which, in its passive condition, focuses parallel rays on the retina. If such rays focus in front of the retina, because the eyeball from front to back is too long, the eye is *myopic*,

giving rise to *near-sightedness*. By intercepting the rays with a diverging lens of suitable focal length, the image may be made to fall on the retina and the vision becomes distinct.

If, on the other hand, the eyeball from front to back is too short, the focus for parallel rays will be back of the retina, and the eye is *hypermetropic*. One is then unable to see near objects distinctly, but can see distant ones. This kind of eye must not be confused with a *presbyopic* one, where through some functional defect the crystalline lens loses the power of adjusting itself to rays of considerable divergence. This defect is known as *far-sightedness*, and is corrected by means of a converging lens. Since the normal eye adapts itself without painful effort to the distinct vision of small objects, like the letters on this page, at a distance of 25 cm. (10 in.), it is customary to speak of this distance as that of normal distinct vision. Eyes are classified as *near-sighted* when the distance of distinct vision is less than 25 cm., and *far-sighted* when greater.

Some persons are unable to see certain colors correctly. This defect is known as *color blindness*. It was first observed by John Dalton and is therefore sometimes described as daltonism. Color blindness is due to some defect in the retina, so that it is not equally sensitive to all of the three primary colors (§ 290). In the case of Dalton, the red was not perceived at all; the result was that red objects appeared black, bluish-green and white objects appeared alike, and yellow objects looked green.

temperature - heat pressure in air
is no air pressure within (Barometer)
in them for air within gets heated
words against the use of ~~heat~~
CHAPTER VI.

HEAT.

I. HEAT AND TEMPERATURE.

5. **Sensations of Heat and Cold.**—If one takes hold of an iron rod that has just been removed from the fire, it feels *hot*; on the other hand, if one touches a piece of ice, it feels *cold*. The cause of these sensations is said to be *heat*. Iron feels hot because it imparts heat to the hand, and ice feels cold because the hand loses heat to the ice.

6. **Nature of Heat.**—For a long time it was believed that heat was a subtle and weightless fluid that caused all natural phenomena by entering bodies and possibly coming out with them. This fluid was called *caloric*. About the beginning of the last century certain experiments of Count Rumford and Sir Humphry Davy demonstrated that the caloric theory of heat was no longer tenable; finally about the middle of the century, when Joule demonstrated that a definite amount of mechanical work is equivalent to a definite amount of heat, it became evident that *heat is a form of energy*. The modern *kinetic* theory, briefly stated, is as follows: The molecules of a body possess a certain amount of independent motion, generally irregular. Any increase in the energy of this motion manifests itself by the body becoming warmer, and any decrease by its becoming cooler. The heating or the cooling of a body, by whatever process, is but the transference or transformation of energy.

307. Temperature.—It is a matter of common observation that if we place a mass of hot iron in contact with a cold one, the latter becomes warmer and the former cooler, the heat flowing from the hot body to the cold one. The two bodies are said to differ in *temperature* or “heat level,” and when they are brought in contact there is a flow of heat from the one of higher temperature to the one of lower till thermal equilibrium is established. *Temperature* may be defined as the thermal condition of a body which determines the transfer of heat between it and any body in contact with it. This transfer is always from the body of higher temperature to the one of lower. *Temperature* may be considered as a measure of the degree of hotness; it depends solely on the kinetic energy of the molecules of the body. *Temperature* must be distinguished from quantity of heat. A pint of water in a vessel may be at a much higher temperature than the water in a lake, yet the latter contains a vastly greater quantity of heat, owing to the greater quantity of water.

308. Measuring Temperature.—**Experiment.**—Select three basins *A*, *B*, and *C*. Fill *A* with hot water, *B* with cold water, and *C* with tepid water. Hold one hand in *A*, and the other in *B* for a few seconds; then transfer both to *C*. The water of *C* will feel cold to the hand from *A* and warm to the hand from *B*.

Experiment.—Hold the hand successively against a number of the various objects in the room, at about the same height from the floor. Metal, slate, or stone objects will feel colder than those of wood, even when side by side and of the same temperature.

It is therefore evident that the sense of touch cannot be depended on to give accurate information regarding the relative temperatures of bodies, and some method independent of bodily sensations must be resorted to for their

measurement. The one most extensively used is the regular increase in the length or volume of a substance depending a rise in its temperature. This method is employed by the common mercurial thermometer.

II. THE THERMOMETER.

A **Thermometer** is an instrument for measuring temperatures. The common *mercurial thermometer* consists of a capillary glass tube of uniform bore, one end of which is blown a bulb, either spherical or cylindrical (Fig. 188). Part of the mercury is expelled by heating, and while in this position the open end of the tube is immersed into a vessel of pure mercury. As the thermometer cools, mercury is forced into the tube by atmospheric pressure. Enough mercury is introduced to fill the bulb and part of the tube at the lowest temperature which the thermometer is designed to measure. The thermometer is now applied to the bulb till the external mercury fills the tube; the end is then sealed in the blowpipe flame. The mercury contracts as it cools, leaving a vacuum at the top of the tube.

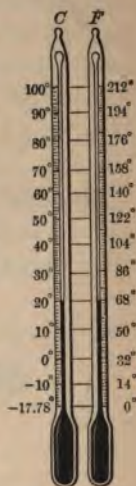


Fig. 188.

Necessity of Fixed Points.—It is evident that no thermometers are likely to have bulbs and stems of equal capacity. Consequently, equal increments of temperature will not produce equal changes in the height of mercury. If, then, the same scale were attached to thermometers, their indications would differ so widely that the results would be worthless. Hence, if thermometers are to be compared, corresponding divisions on the

has high B.P. & then the regulation
 is such that the thermometer

scale of different instruments must indicate the same temperature. This may be done by graduating every thermometer by comparison with a standard, an expensive proceeding and for many purposes unnecessary, since mercury has a nearly uniform rate of expansion. If two points are marked on the stem, the others can be obtained by dividing the space between them into the proper number of equal parts. Careful investigations have made it certain that under a constant pressure the temperature of *melting ice* and that of *steam* are invariable. Hence, the temperature of melting ice and that of steam under a pressure of 76 cm. of mercury have been chosen as the fixed points on a thermometer.

311. Marking the Fixed Points.—The thermometer is packed in finely broken ice, as far up the stem as the mercury extends. The containing vessel (Fig. 189) has an



Fig. 189.

Fig. 190.

opening at the bottom to let the water run out. After standing in the ice for several minutes the top of the thread of mercury is marked on the stem. This is called the *freezing point*.

The *boiling point* is marked by observing the top of the mercurial column when the bulb and stem are enveloped in steam (Fig. 190) under an atmospheric pressure of 76 cm. (29.922 in.). If the pressure at the time is not 76 cm., then a correction must be applied, the

amount being determined by the approximate rule that

the temperature of steam rises $0^{\circ}.1$ C. for every increase of 2.71 mm. in the barometric reading, near 100° C.

312. Thermometer Scales. — The distance between the fixed points is divided into equal parts called *degrees*. The number of such parts is wholly arbitrary, and several different scales have been introduced. Three of these are in use at the present time: the *Fahrenheit*, the *Centigrade*, and the *Réaumur*. The Fahrenheit scale was introduced by Fahrenheit about 1714, and is the one in common use in all English-speaking countries. For some unknown reason he marked the freezing point at 32° above the zero of the scale, and the boiling point at 212° , dividing the space between into 180 equal parts. The Centigrade scale was designed by Celsius about 1742. It differs from the Fahrenheit in making the freezing point 0° and the boiling point 100° , the space between being divided into 100 equal parts. This is the one in general use among scientific men. The Réaumur scale marks the freezing point 0° and the boiling point 80° . This is the household scale on the con-

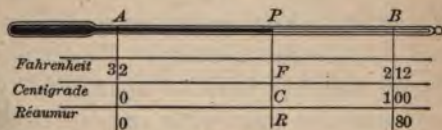


Fig. 19i.

continent of Europe; in this country its use is restricted to breweries. Each of these scales is extended beyond the fixed points as far as desired. The divisions below 0° are read as negative; for example, -10° signifies 10 degrees below zero. The reading according to any particular scale is indicated by affixing the initial letter of the name; for example, 5° F., 5° C., and 5° R. signify 5 degrees above zero on the Fahrenheit, Centigrade, and Réaumur scales respectively.

313. Comparison of Scales.—In Fig. 191 AB is a thermometer with three scales attached, P is the head of the mercury column, and F , C , and R are the readings on the scales respectively. On the Fahrenheit scale $AB = 180$ and $AP = F - 32$, since the zero is 32 spaces below A ; on the Centigrade $AB = 100$ and $AP = C$; on the Réaumur $AB = 80$ and $AP = R$. Then the ratio of AP to AB is $\frac{F - 32}{180} = \frac{C}{100} = \frac{R}{80}$. By substituting the reading on any one scale in this equation the equivalent on either of the other scales is easily obtained. For example, if it is required to express 68° F. on the Centigrade scale, then $\frac{68 - 32}{180} = \frac{C}{100}$ and $C = 20^\circ$.

314. Limitations of the Mercurial Thermometer.—Since mercury freezes at $-38^\circ.8$ C., it is evident that it cannot be used as the thermometric substance below this temperature. For temperatures below -38° C. alcohol is substituted for mercury. Under a pressure of one atmosphere mercury boils at about 350° C. For temperatures approaching this value and up to about 550° C. the thermometer stem is filled with pure nitrogen under pressure. The pressure of the gas keeps the mercury from boiling (§ 337). For high temperatures the mercury thermometer must be calibrated by comparison with an air thermometer, or by reference to the temperature at which water boils under known high pressures.

315. The Air Thermometer was invented by Galileo about 1593 for the use of physicians. In its early form it consisted of a glass bulb on the end of a tube of small bore, supported vertically in front of a scale. By warming the

part of the air is expelled, and then the stem is filled in a liquid, as colored water, alcohol, or mercury. When the air cools it contracts and the liquid is pushed up the stem from atmospheric pressure.

192 The bottle containing the liquid is used as a support. If the temperature of the liquid column is depressed; and if the temperature falls, the column rises. The instrument is remarkable for its sensitive-ness, that is, for the large movements of the column for small changes of temperature; but it is greatly modified in construction, making it quite complex in plan, it is only a rough scope, because its readings change with every change in barometric pressure.

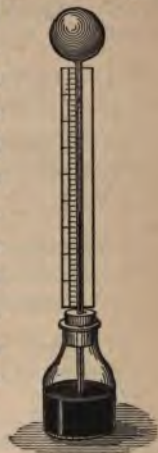


Fig. 192.

Questions and Problems.

In view of the fact that alcohol expands quite irregularly, what is the best way to graduate an alcohol thermometer?

When the bulb of a thermometer is plunged into hot water, the liquid at first falls. Why?

What will be the effect on the distance between the fixed points of a thermometer if the tube has a very small bore? If it has a large bore?

Why is a thermometer with a cylindrical bulb preferable to one with a spherical bulb?

What would be the effect on the readings of a mercurial thermometer, if, after graduation, the bulb should contract?

What would be the nature of the error in a thermometer if the tube tapered outward from the bulb?

In the air thermometer, should the cork fit the bottle air-tight?

Give two reasons why mercury is the most suitable of all known liquids for use in thermometers.

9. Express in the Fahrenheit scale the following: 120°C. , -40°C. , 25°C. , 5°C. *248° -40° 77° 41°*

10. Express in the Fahrenheit scale the difference between 60°C. and 60°F. *80° F*

11. Express in the Centigrade scale the following: 68°F. , -13°F. , 50°F. , -22°F. *20° -25° 10° -30°*

12. It is claimed that Olszewski has obtained a temperature of -271.3°C. What temperature would this be on the Fahrenheit scale? *-456.3° F*

✓ 13. Express the following temperatures in the Fahrenheit scale: boiling point of nitrogen -195.5°C. , melting point of hydrogen -257°C. , alcohol flame 1705°C. *-319.9° -430.6° 310°*

14. When the barometer reading is 72 cm., what is the boiling point of water? *98.52° C*

✓ 15. When the boiling point of water is 95°C. , what is the barometric pressure? *624.5 mm*

16. A thermometer provided with both a Centigrade and a Fahrenheit scale was used in taking the temperature of a room. The sum of the readings was found to be 88. What was the reading of each scale? *20° C 68° F*

17. What is the temperature of an oil bath, when the reading on a Fahrenheit thermometer standing in the oil is twice that of a Centigrade? *160° C 320° F*

18. The boiling point of water according to a certain thermometer is found to be 98°C. , when the barometer pressure is 745 mm. What is the error of the thermometer at this boiling point? What is the correct temperature when this thermometer reads 30°C. , assuming the zero point correct?

[Find the true boiling point, then compare the false scale with the true, just as the Centigrade scale is compared with the Fahrenheit in § 313.] *BP in mm by 0.95° C 30.29° C*

19. If a thermometer reads 1°C. in melting ice, what is the correct temperature when this thermometer reads 22°C. , assuming that the boiling point is right? *21.21° C*

20. If the reading of the true boiling point of water is 98° , at what will the thermometer read when placed in a bath whose temperature is 30° C., assuming the zero point right? *30.6°C*

III. EXPANSION.

316. **Expansion of Solids.**—**Experiment.**—A metallic rod *S* (Fig. 193) is supported horizontally in such a manner that the end *A* rests firmly against a support, while the end *B* rests against

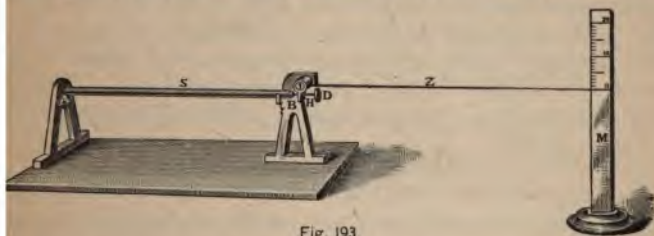


Fig. 193.

the short arm of a bent lever *Z*, the long arm moving over a scale *M*. Place a spirit lamp under *S*. The pointer *Z* will move forward on *M*, showing that the rod is increasing in length.

Experiment.—Rivet together at short intervals a strip of sheet iron and one of copper (Fig. 194). Support the ends and place a spirit lamp under the middle. This composite bar will bend into an arc with copper on the convex side, showing that

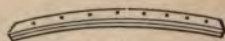
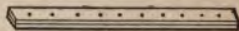


Fig. 194.

the two metals expand unequally and that the copper expands more than the iron.



Fig. 195.

Experiment.—Fig. 195 illustrates a piece of apparatus known as Graveland's ring. It consists of a metallic ball that at ordinary temperatures will just pass through the ring. Heat the ball in boiling water. It will now rest on the ring and will

fall through until it has cooled.

These experiments show that solids expand in every direction when heated and contract when cooled, the amount varying with the substance. Stretched india-rubber and iodide of silver are exceptions to this law; for within a certain range they contract when heated and expand when cooled.

317. Expansion of Liquids. — Experiment. — Prepare several glass tubes of the same bore, closing them at one end with a blow-pipe. Fill each of them to the height of about 15 cm., but with different liquids, as water, alcohol, glycerine, etc., each colored with an aniline dye. Support the tubes in a vessel of hot water. The liquids will rise in the tubes, but not equally. If placed in ice-water they contract unequally.

The experiment shows that liquids, like solids, expand when heated and contract when cooled, the amount depending on the nature of the substance. It also shows that the expansion of liquids is greater than that of glass, otherwise there would have been no apparent increase in their volume. Some liquids do not expand when heated at certain points

on the thermometric scale. Water, for example, on heating from 0°C. to 4°C. contracts, but above 4°C. it expands.

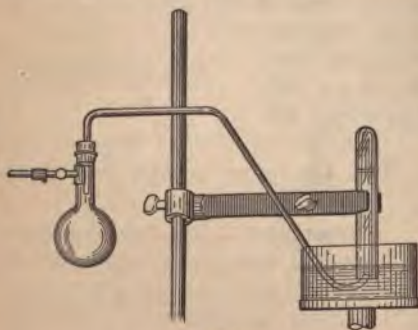


Fig. 196.

318. Expansion of Gases. — Experiment. — Fit a bent delivery-tube to a small Florence flask (Fig. 196). Fill the flask with air and place the upturned

end of a delivery-tube under an inverted graduated glass cylinder filled with water. Heat the flask by immersing it in a vessel of hot water.

air will expand and escape through the delivery-tube into the receiver; note the amount. Now refill the flask with some other gas, and repeat the experiment. If the work is carefully done, it will be seen that the amount of gas collected each time is constant, and that the expansion is the same.

The investigation of gases by Gay-Lussac, Charles, Laplace and many others has proved that all gases expand very nearly alike at atmospheric pressure, approaching equality as the pressure is diminished. Gases which are easily liquefied, as carbon dioxide, show the greatest variation in their coefficient of expansion.

9. Coefficient of Expansion. — It appears from the foregoing experiments that substances when heated expand in every direction. This expansion in volume is called *cubical expansion*, in distinction from *linear expansion*, or expansion in length, and *superficial expansion*, or expansion in area. The *coefficient of linear expansion* is the fraction of its length which a body expands when heated from 0° C. to 1° C.; the *coefficient of superficial expansion* is the fraction of its area which a body expands when heated from 0° C. to 1° C.; and the *coefficient of cubical expansion* is the fraction of its volume which a body expands when heated from 0° C. to 1° C. Since the expansion of most substances is found to be nearly constant for each degree of temperature, it is customary to determine the mean coefficient for a change of several degrees. If l_1 and l_2 represent the lengths of a metallic rod at the temperatures t_1 and t_2 respectively, then $\frac{l_2 - l_1}{t_2 - t_1} = \frac{l_2 - l_1}{t}$ is the expansion for 1°, in which t is the difference of temperatures. If α represents the mean coefficient of expansion, then $\alpha = \frac{l_2 - l_1}{l_1 t}$; whence $l_2 = l_1(1 + \alpha t)$. In like

manner for volumes, if k is the coefficient of cubical expansion, v_1 and v_2 the volumes at the temperatures t_1 and t_2 respectively, then $k = \frac{v_2 - v_1}{v_1(t_2 - t_1)} = \frac{v_2 - v_1}{v_1 t}$; whence $v_2 = v_1(1 + kt)$.

In the case of solids, superficial and cubical expansion are obtained by computation from the linear expansion, the coefficient of the former being twice the linear, and that of the latter three times.

320. Law of Charles.—It was shown by Charles, in 1787, that the volume of a given mass of any gas under constant pressure increases by a constant fraction of its volume at zero for each rise of temperature of 1°C . The investigations of Regnault and others show that the law is not rigorously true, and that the accuracy of Charles's law is about the same as that of Boyle's law. The coefficient of expansion k of dry air is 0.003665, or about $\frac{1}{273}$. This fraction may be considered as the coefficient of any true gas.

321. The Absolute Scale.—The law of Charles leads to a fourth scale of temperature called the *absolute scale*. By this law the volumes of any mass of gas, under constant pressure, at 0°C ., and at any other temperature $t^\circ \text{C}$., are connected by the following relations (§ 319):—

$$v = v_0(1 + \frac{1}{273}t) = \frac{v_0(273 + t)}{273}.$$

At any other temperature, t' , the volume becomes

$$v' = \frac{v_0(273 + t')}{273}.$$

Then

$$\frac{v}{v'} = \frac{273 + t}{273 + t'}.$$

Suppose now a new scale is taken, whose zero is 273 centigrade divisions below the freezing point of water, so that temperatures on this scale are denoted by T . Then $273 + t$ will be represented by T , and $273 + t'$ by T' , and

$$\frac{v}{v'} = \frac{273 + t}{273 + t'} = \frac{T}{T'}$$

the volumes of the same mass of gas under constant pressure are proportional to the temperatures on this new scale. The point 273° below 0° C. is called the *absolute zero*, and the temperatures on this scale, *absolute temperatures*. Up to the present it has not been found possible to cool a body to the absolute zero; but by evaporating liquid hydrogen under very low pressure, a temperature estimated to be within 9° of the absolute zero has been attained by Professor Dewar.

322. The Law of Boyle and Charles Combined.—If v , p , and T denote the volume, pressure, and absolute temperature of a given mass of gas, then by Boyle's law (§ 161) $v \propto \frac{1}{p}$ when T is constant; and by the law of Charles, $v \propto T$, when p is constant. Therefore when T and p both vary, v varies directly as T and inversely as p , or $v \propto \frac{T}{p}$. Whence $pv \propto T$, or $pv = \text{constant} \times T$. This relation is known as the "gas equation" and is written

$$pv = RT. \quad (26)$$

R is the constant which converts a proportionality into an equality. It follows that not only is the volume of a given mass of gas under constant pressure proportional to its absolute temperature (§ 321), but the product of the pressure and volume of a given mass of gas is proportional to its absolute temperature.

To illustrate the use of the above relation: If 20 cm³. of gas at 20° C. is under a pressure of 76 cm. of mercury, what will be the pressure when its volume is 30 cm³. and temperature 50° C.?

From (26), $\frac{pv}{T}$ is a constant,

or
$$\frac{pv}{T} = \frac{p'v'}{T'}.$$

Hence
$$\frac{76 \times 20}{273 + 20} = \frac{p \times 30}{273 + 50},$$

from which
$$p = 55.85 \text{ cm.}$$

323. Force of Contraction and Expansion. — Experiment. — Fill a small test-tube about one quarter full of water, and close the end by fusion. Lay it in an empty sand bath on the ring of an iron stand. Apply heat, and stand at a safe distance. In a few minutes there will be a loud report, caused by the bursting of the tube.

The force of expansion or of contraction of a substance is evidently equal to the force necessary to compress or expand it to the same extent by mechanical means, and hence can be computed by proceeding in the manner illustrated in the following example: A bar of malleable iron, one square inch in cross-sectional area, if placed under the tension of a ton, increases in length 0.0001 of itself. The coefficient of linear expansion of iron is 0.0000122. Since $0.0001 \div 0.0000122 = 8+$, a change of temperature of about 8° C. will produce the same change in the length of the bar as a force of one ton.

324. Applications of Expansion. — Many familiar phenomena are accounted for by expansion or contraction attending changes of temperature. If hot water is poured

into a thick glass tumbler, the glass will probably break because of the strain produced by the sudden expansion of its inner surface. The principle of unequal expansion is employed in thermometers, in the compensated clock pendulum (§ 77), and in the balance wheel of a watch. Glass and platinum have nearly the same coefficient of expansion. For that reason platinum is in great demand in the manufacture of incandescent electric lamps, since it does not crack the glass when it cools. Iron tires are fitted to wheels and then expanded by heating so that they slip on easily; on cooling, they contract and compress the wheel. The rivets which hold together the plates of steam boilers are inserted red-hot, and hammered down. The contracting rivets press the plates together with great force. In all heavy iron structures, such as railroad bridges, a certain freedom of motion of the parts must be provided for; otherwise, the changes in length attending variations in temperature would have a disastrous effect. Sidewalks of artificial stone should have spaces left for expansion to prevent "buckling." Crystalline rocks, on account of unequal expansion in different directions, are slowly disintegrated by changes of temperature; and for the same reason quartz crystals, when strongly heated, fly in pieces.

Problems.

1. Account for the fact that a glass stopper can ordinarily be loosened by warming the neck of the bottle.
2. Why must temperature be considered in preparing a table of densities?
3. In determining the mean coefficient of linear expansion of a brass rod the following data were obtained: length at 18°C. , 59.8 cm.; length at 98°C. , 59.8888 cm. Calculate the coefficient.

4. An iron bar is 1 m. long at 0°C . What will be its length at 30°C ., the coefficient of linear expansion being 0.0000122?

5. A steel tape 2 m. long is correct at 16°C . What is the error made in measuring with it a distance of 50 m. at a temperature of 24°C ., the linear coefficient of expansion being 0.00001322?

6. The coefficient of linear expansion of zinc is 0.00002976. Find the increase in area of a zinc disk 1 ft. sq. in, heating it from 20°C . to 40°C .

7. An aluminium cube is 5 cm. on each edge at a temperature of 10°C . Calculate the change in volume produced by heating it to 100°C ., the coefficient of linear expansion being 0.00002221.

8. The coefficient of linear expansion of glass is 0.0000086. If a specific gravity bottle holds 50 cm^3 . at 15°C ., what will be its capacity at 25°C .?

9. The coefficient of linear expansion of steel is 0.00001322, and that of zinc is 0.00002976. Calculate the length of a zinc rod that will have the same increase in length as a steel one 100 cm. long.

10. The length of the iron bridge across the Menai Straits is 461 m. Find the change in length of this iron tube between -10°C . and 35°C ., the mean coefficient of expansion of iron being 0.0000122.

11. A mass of air has a volume of 500 cm^3 . at 5°C . What will be its volume at 45°C ., the pressure remaining the same?

12. If the volume of a quantity of air at 20°C . is 200 cm^3 ., at what temperature will its volume become 300 cm^3 ., the pressure remaining the same?

13. 200 cm^3 . of air are heated from 0°C . to 30°C ., and at the latter temperature the volume is found to be 222 cm^3 . What is the resulting coefficient of expansion?

14. What will be the volume of air measuring 10 cu. ft. at 0°C ., if the temperature be raised to 273°C ., and the pressure be doubled?

15. A litre glass flask of air at 0°C . is heated to 30°C . How many cm^3 . of air escape at 30°C ., neglecting the expansion of the glass?

6. How much gas must be collected at a temperature of 20°C . 74 cm. barometric pressure to give 100 cm^3 . at 0°C . and 76 cm. pressure? 110.72 cm^3

7. A flask is filled with air at 20°C . and 74 cm. pressure and stoppered. If the flask be then heated to 100°C ., under what pressure will the air in the flask be, assuming that the flask does not expand? 94.2 cm

8. A litre flask filled with air at 0°C . is heated to 100°C . What volume of air measured at 0°C . will escape? 268.1 cm^3

9. A vessel of air at 0°C . is heated to 67°C ., when it is found that m^3 . of air at the latter temperature have escaped. What was the capacity of the vessel at 0°C .? 81.5 cm^3

10. A quantity of gas is collected in a graduated tube over mercury. The volume of the gas at 20°C . is 50 cm^3 ., and the level of the mercury in the tube is 20 cm. above the outside mercury level. The barometer reads at 75 cm. Find the volume that the gas would occupy at 0°C . 76 cm. pressure. 33.1 cm^3

The pressure of the gas in the tube is $75 - 20 = 55\text{ cm}$.]

11. 100 cm^3 . of oxygen are collected at 20°C . and 74 cm. pressure. What pressure must the gas be subjected to maintain the volume unchanged if the temperature is raised to 30°C .? 76.5 cm

12. If a volume of air measured at 0°C . and 76 cm. pressure is put under a pressure of 80 cm. of mercury, to what temperature must the gas be raised to keep its volume constant? 120.4°C

13. A litre flask contains 1.29 gm. of dry air at 0°C . and 76 cm. barometric pressure. At what temperature will a litre of air weigh 1.29 gm. if the pressure is 72 cm.? 60.63°C

IV. MEASUREMENT OF HEAT.

325. **Unit Quantity of Heat.**—For the purpose of measuring the quantity of heat gained or lost by a body when its temperature or its state changes, it is necessary to adopt

a unit of heat. The one most commonly used in connection with the metric system is the quantity of heat that will raise the temperature of one gramme of water one degree Centigrade. It is called a *calorie*. There is no agreement as to where the one degree shall be on the scale between the freezing and the boiling points. An exact definition requires the degree to be specified, because it is known that the heat required to raise a gramme of water from 4° to 5° is not the same as the quantity required to raise it from 14° to 15° , for example. The difference, however, is small; and in this book the heat necessary to raise a gramme of water through one degree at different temperatures will be assumed to be the same.

above water equivalent

326. Thermal Capacity.—The number of calories required to raise the temperature of a body through one degree Centigrade is the *thermal capacity* of the body. The thermal capacities of equal masses of different substances differ widely. Thus, if 100 gm. of water at 0° be mixed with 100 gm. at 100° , the temperature of the whole mass will be very nearly 50° . But if 100 gm. of copper at 100° be cooled in 100 gm. of water at 0° , the final temperature will be about $9^{\circ}.1$. The heat lost by the copper in cooling through $90^{\circ}.9$ is sufficient to raise the same mass of water through only $9^{\circ}.1$.

327. Specific Heat.—The thermal capacity of a unit mass of a substance is its *specific heat*. In the metric system the number of calories necessary to raise the temperature of one gramme of the substance through one degree Centigrade, at any temperature, is its specific heat at that temperature. Specific heat varies a little with the temperature, but for most purposes it may be assumed

10

constant. The specific heat of mercury is 0.033; this means that the heat which will raise 1 gm. of mercury through 1° C. will raise 1 gm. of water through only 0.033 C.

The following table gives the specific heat of several substances at the mean temperatures of the second column, and in terms of water at 15° C.

Water	5°	1.0041
Water	15°	1.0000
Water	20°	0.9987
Ice	- 10°	0.502
Paraffin	10°	0.694
Copper	50°	0.092
Zinc	50°	0.093
Iron	15°	0.109
Platinum	50°	0.032
Mercury	20°	0.033

328. Specific Heat by Method of Mixtures. — Experiment. —

Take a known number of grammes of lead shot in a test-tube, closing the end loosely with a plug of cotton. Suspend the test-tube for several minutes in boiling water. The temperature of the shot will then be that of the water. Now pour the shot quickly into a beaker containing a known quantity of water at the temperature of the room or a little lower. Stir gently with a thermometer for a few seconds, and record the temperature. The mass of the water in grammes, multiplied by the gain in temperature, will be the number of calories of heat gained by the water, and this same quantity is lost by the shot in cooling from the temperature of boiling water to that of the final temperature of the beaker and its contents. This number, divided by the product of the number of grammes of shot and its fall in temperature, will be the specific heat of lead.

The experiment illustrates the process of obtaining specific heat by the "method of mixtures." In practice it is necessary to take into account the thermal capacity of the vessel containing the water, since the vessel and its

contents change temperature at the same time. This is done by ascertaining what mass of water will have the same thermal capacity as the vessel and adding this amount to the mass of water in the vessel. The mass of water having the same thermal capacity as the vessel is known as the "water equivalent" of the vessel. It may be found either by experiment, or by multiplying the mass of the vessel, expressed in grammes, by the specific heat of the material of which it is made.

In the treatment of problems under this subject, the principle applied is that the gain or loss of heat by the water (or other liquid) heated or cooled is equal to the heat lost or gained by the body introduced. The loss or gain of heat on the part of the body is equal to the product of its mass, its specific heat, and its change of temperature.

✓ To illustrate: 20 gm. of iron at 98°C . are placed in 75 gm. of water at 10°C . contained in a copper beaker whose mass is 15 gm. and specific heat 0.095. The resulting temperature of the water and the iron is $12^{\circ}.5\text{C}$.; find the specific heat of iron.

The water equivalent of the beaker is $15 \times 0.095 = 1.425$ gm. The heat lost by the iron is $20 \times s (98 - 12.5)$; that gained by the water and copper vessel is $(75 + 1.425) \times (12.5 - 10)$. Placing these two quantities equal to each other and solving for s , the specific heat of iron, we have $s = 0.112$.

Further illustration: 20 gm. of iron, specific heat 0.112, at 98°C . are placed in 75 gm. of water at 10°C . contained in a copper beaker, whose mass is 15 gm. and specific heat 0.095. What is the resulting temperature? The water equivalent of the beaker is $15 \times 0.095 = 1.425$ gm. The number of calories lost by the iron is

$\times 0.112 (98 - t)$; the number gained by the water and beaker is $(75 + 1.425) \times (t - 10)$; in both expressions t is the temperature sought. Placing the heat lost by the beaker equal to the heat gained by the water and beaker, and solving for t , we have $t = 12^\circ.5 \text{ C.}$

Problems.

1. 125 gm. of a substance at a temperature of 78° C. , when immersed in 250 gm. of water at 12° C. , gave a resulting temperature of 18° C. What is the specific heat of the substance? *0.2*
2. A piece of silver weighing 50 gm. is heated to 83° C. and then dropped into 200 gm. of water at 10° C. The resulting temperature is 14° C. Find the specific heat of silver. *0.0556*
3. An aluminium beaker weighs 54 gm. Find its thermal capacity, the specific heat of aluminium being 0.212. What is its water equivalent? *11.45 gals TC*
4. A lead ball weighs 250 gm. How much heat would it require to raise it from 10° C. to 100° C. , the specific heat of lead being 0.0314? *716.5*
5. A ball of iron (specific heat, 0.112) at 90° C. , weighing 200 gm., is dropped into 100 gm. of water at 20° C. , contained in a copper dish (specific heat, 0.095) weighing 100 gm. Calculate the resulting temperature. *31.9°C*
[The water equivalent of the copper dish must be calculated and added to the quantity of water.]
6. A mass of 250 gm. of copper (specific heat, 0.095) is heated to 100° C. and placed in 100 gm. of alcohol at 10° C. , contained in a copper calorimeter whose water equivalent is 20 gm.; the temperature rises to 20° C. Find the specific heat of alcohol. *0.631*
7. To what temperature must a block of lead (specific heat, 0.0314) weighing 500 gm. be heated in order that it may, when dropped into 100 gm. of water at 15° C. including water equivalent of the calorimeter, raise the water to a temperature of 35° C. ? *162.4°C*
8. The specific heat of antimony is 0.0507. What mass of water can be raised from 0° to 15° C. by plunging into it 1 kgm. of antimony at 90° C. ? *253.5 gms*
9. A piece of platinum weighing 100 gm. (specific heat, 0.0323) is taken from a furnace and at once dropped into 90.5 gm. of water at

10° C. contained in a copper beaker (specific heat, 0.095) weighing 100 gm. The final temperature is 40° C.; find the temperature of the furnace. *96.8°C*

10. Determine the specific heat of a brass ball from the following data: — *.083*

Water equivalent of the calorimeter	23.04 gm.
Weight of water in the calorimeter	500 gm.
Initial temperature of the water in the calorimeter	19° C.
Weight of the brass ball	150.3 gm.
Temperature of the brass ball	62° C.
Final temperature of the water	20° C.

V. CHANGE OF STATE.

329. The Melting Point. — When a body changes from the solid to the liquid state by the application of heat, it is said to *melt*, or *fuse*, and the change is called *melting*, *fusion*, or *liquefaction*. The temperature at which fusion takes place is called the *melting point*. Solidification or freezing is the converse of fusion, and the temperature of solidification is usually the same as the melting point of the same substance. Water, if undisturbed, may be cooled a number of degrees below 0° C., but if it is disturbed it usually freezes at once, and its temperature rises to the freezing point.

The melting point of crystalline bodies is well marked. A mixture of ice and water will remain without change if the temperature is 0° C.; but if the temperature is above zero, some of the ice will melt; if it is below zero, some of the water will freeze. Some substances, like wax, glass, and wrought iron, have no sharply defined melting point. They first soften and then pass more or less slowly into the condition of a viscous liquid. It is this property which permits of the bending and moulding of glass, and the welding and forging of iron.

1 gm ice - 10°C
" " - 0°C
water 0°C

330. Change in Volume accompanying Fusion. — Most substances occupy a larger volume in the liquid state than in the solid; that is, they expand on liquefying. A few substances, like water and bismuth, expand on solidifying. When water freezes, its volume increases nine per cent. If this expansion is resisted, water in freezing is capable of exerting an enormous force.

Experiment. — Fit to a small bottle a perforated stopper through which passes a fine glass tube. Fill with water freed from air by boiling, the water extending halfway up the tube, and then pack in a mixture of salt and finely broken ice. The water column at first will fall slowly, but in a few minutes it will begin to rise, and will continue to do so till water flows out of the top of the tube. The water in the bottle freezes, and the attending expansion causes the overflow.

331. Laws of Fusion. — The following laws have been established by experiment: —

I. *Every crystalline substance begins to melt at a definite temperature, which is invariable for each substance if the pressure is constant.*

II. *The temperature of a body, when slowly melting, remains constant till the whole mass is melted.*

III. *Substances that expand on solidifying have their melting points lowered by pressure, and vice versa.*

The following interesting experiment illustrates the last law: —

Experiment. — Support a rectangular block or prism of ice on a stout bar of wood. Pass a small iron wire around the ice and the bar of wood, and suspend on it a weight of about 25 kgm. The pressure of the wire lowers the melting point of the ice, and the ice melts; the water, after passing around the wire, where it is relieved of pressure, again freezes. In this way the wire passes slowly through the ice, leaving the block solidly frozen.

332. Latent Heat. — When a body passes slowly from one state to another, as from the solid to the liquid, there is no rise of temperature, notwithstanding the constant application of heat. When this fact was first observed, it was generally believed that heat was a kind of matter, called *caloric*. This view led to the introduction of two terms, *sensible heat* and *latent heat*; the former denoting heat which changes the temperature of a body, and the latter heat which changes its state without affecting its temperature. The advocates of the caloric theory of heat thought that heat became hidden or concealed in the process of fusion, and they therefore called it "latent heat." We now know that this view is incorrect, and that the heat which disappears during a change of state ceases to be heat, and is energy converted into the potential form in the work of giving mobility to the molecules. The term *latent* should therefore no longer be applied to heat.

333. Heat of Fusion. — When a solid fuses, a quantity of heat disappears; and, conversely, when a liquid solidifies, the amount of heat generated is the same as disappears during liquefaction. The *heat of fusion* of a substance is the number of calories required to melt a gramme of it without change of temperature. The heat of fusion of ice is 80 calories. The manner of measuring it is illustrated by the following example: — Place 200 gm. of clean ice in 500 gm. of water at 60° C. The ice melts and reduces the temperature of the whole to 20° C. Then the heat lost by the 500 gm. of water equals the heat required to melt the ice plus the quantity required to raise the water formed from the ice from 0° C. to 20° C., or

$$500(60 - 20) = 200 \times L + 200 \times 20.$$

Whence the heat of fusion L equals 80.

334. Heat Lost in Solution. — **Experiment.** — Fill a test-glass part full of water at the temperature of the room, and add some finely divided ammonium nitrate. A thermometer will show a sensible fall of temperature.

Experiment. — Make a saturated solution of sodium hyposulphite at a temperature of 30°C . in a small flask. • Pass a thermometer through a stopper so that its bulb is in the solution. Cool slowly without disturbing the solution to about 20°C . The solution is then undercooled, but no crystals should form. Now remove the thermometer very carefully and allow the liquid on the bulb to evaporate till some crystals of the sodium hyposulphite have formed. Replace the thermometer in the solution. Rapid crystallization will set in and extend through the whole solution. At the same time the temperature will rise to about 30°C .

The first experiment illustrates the fact that heat is absorbed when a body passes from the solid to the liquid state even by solution. It sometimes happens that this absorption of heat is masked by the heat evolved by chemical action between the dissolved body and the solvent. Freezing mixtures are based on the principle of the absorption of heat during the passage of bodies from the solid to the liquid state. When salt and pounded ice are mixed, both solids become liquid and absorb heat in the transition from the one state to the other.

The second experiment is the converse of the first, and shows that heat is evolved when a substance becomes a solid by crystallization from solution.

335. Vaporization. — **Experiment.** — Pour a few drops of ether into a beaker and cover loosely with a plate of glass. After a few seconds bring a lighted taper to the mouth of the beaker. A sudden flash will show that the vapor of ether was mixed with the air.

Experiment. — Support on an iron stand a beaker two-thirds full of water and apply heat. In a short time bubbles of steam will form at the bottom of the beaker, rise through the water, and burst at the top, producing violent agitation throughout the mass.

Vaporization is the conversion of a substance into the gaseous form. If the change takes place slowly, as in the first experiment, and from the surface of a liquid, it is called *evaporation*; but if the liquid is visibly agitated by rapid internal evaporation, the process is called *ebullition* or *boiling*.

There are two other varieties of vaporization, namely, the *spheroidal state* and *sublimation*. When a small quantity of liquid is placed on hot metal, as water on a red-hot stove, it assumes a globular or spheroidal form, and evaporates at a rate between ordinary evaporation and boiling. The vapor acts like a cushion and prevents actual contact between the liquid and the metal. The globular form is due to surface tension. Liquid oxygen at a very low temperature assumes the spheroidal form when placed on water. The temperature of the water is relatively high compared with that of the liquid oxygen. When a substance passes directly from the solid to the gaseous form without passing through the intermediate state of a liquid, it is said to *sublime*. Arsenic, camphor, and iodine sublime at atmospheric pressure, but if the pressure be sufficiently increased, they may be fused. Ice also evaporates slowly at a temperature below freezing.

336. Laws of Evaporation.—The laws of evaporation established by experiment are as follows:—

I. *The rate of evaporation increases with rise of temperature.*

II. *The rate of evaporation increases with the free surface of the liquid.*

This principle is utilized in the manufacture of salt by using large shallow pans for the brine, or by allowing the brine to trickle over bundles of twigs.

III. *The rate of evaporation is increased by a continual change of air in contact with the liquid.*

If the surrounding air is at rest, it soon becomes saturated with vapor from the liquid, and the rate of evaporation is checked. The drying action of the wind on roads after a rain and on wet cloth hanging in the air are illustrations in point.

IV. *The rate of evaporation is increased by diminishing the vapor pressure.*

In order that syrups may be concentrated at a low temperature to avoid burning, the operation is carried on in large covered pans from which the air and vapor are exhausted by air-pumps.

337. Laws of Ebullition.—The following laws express the results of experiment:—

I. *Each liquid has its own boiling point, which is invariable for that liquid under the same conditions.*

II. *The boiling point is dependent upon the character of the inner surface of the containing vessel.*

The temperature of boiling water is slightly higher if the inner surface of the containing vessel is smooth than if it is rough. But the temperature of the vapor given off is independent of the nature of the vessel. Hence, in fixing the boiling point on a thermometer, the thermometer is immersed in the steam and not in the water itself.

III. *The boiling point is raised by salts and lowered by gases dissolved in the liquid.*

When the air has been boiled out of water, the temperature may rise several degrees before ebullition sets in; and in case the inner surface of the vessel is very smooth,

the boiling proceeds intermittently and explosively. The phenomenon is called "*bumping*."

IV. *The boiling point rises with increase of pressure and falls with decrease of pressure.*

The effect of pressure on the boiling point is seen in the low temperature of boiling water at high elevations, and in the high temperature of the water under pressure in digesters used for extracting gelatine from bones. The change in the boiling point of water near 100°C is

$0^{\circ}.1$ for an increase of pressure of 2.71 mm. of mercury. The following experiments show the effect of reduced pressure:—



Fig. 197.

1. Place a flask of warm water under the receiver of an air-pump. It will boil violently when the receiver is exhausted.

2. Fill a round-bottomed Florence flask half full of water and heat till it boils vigorously. Cork the flask, invert, and support it on a ring stand (Fig. 197). The boiling ceases, but is renewed by applying cold water to the flask. The cold water

condenses the vapor, and reduces the pressure within the flask so that the boiling begins again.

338. Relation of Altitude to the Boiling Point.—It has already been stated that, since atmospheric pressure decreases with the elevation, the boiling point of a liquid also decreases. Hence, the boiling point of water may be used as an indicator of the height of a place above the level of the sea. A change of elevation of about 295 m. makes a difference of 1°C . in the boiling point. Thus, at Quito, the highest city in the world, the average boiling point is $90^{\circ}.1\text{C}$. Hence, the height above sea

level is $295 \times (100 - 90.1) = 2920.5$ metres, a quantity greater than the true height by 34.4 metres.

339. Cold by Evaporation. — Experiment. — Put a few drops of ether on the bulb of an air thermometer (§ 315). The index at once begins to rise, showing that the bulb has been cooled.

In the evaporation of the ether, some of the heat of the thermometer bulb has been used to do work on the liquid. The rapid evaporation of liquid ammonia is utilized in the artificial production of ice. Sprinkling the floor of a room cools the air, because of the heat expended in evaporating the water. Porous water vessels keep the water cool by the evaporation of the water from the outside surface. Liquid carbon dioxide is readily frozen by its own rapid evaporation. Dewar liquefied oxygen by means of the low temperature obtained through the successive evaporation of liquid nitrous oxide and ethylene. In like manner, by the evaporation of liquid air he has liquefied hydrogen. The evaporation of liquid hydrogen under reduced pressure has enabled him to maintain a temperature within less than 16° of the absolute zero (§ 321).

340. Condensation and Distillation. — When a vapor is liquefied, all the heat that has disappeared during vaporization is generated again. This fact is applied in steam heating. Some gases may be made to assume a liquid form through their affinity for a liquid. Thus, for instance, when ammonia gas is brought in contact with water, it is rapidly absorbed with a marked rise of temperature.

Distillation involves both evaporation and condensation. Pure water, free from foreign substances such as vegetable and mineral matter, is obtained by distillation. If two liquids are mixed together, the more volatile will be

vaporized by heat first, and it may be condensed and collected by itself. In this way alcohol is separated

from fermented liquors. The apparatus used for evaporating the liquid is called the *still*, and that for liquefying, the *condenser*. The latter is usually a coiled tube, called the *worm*, surrounded by water. Fig. 198 illustrates one of the forms used in laboratories.

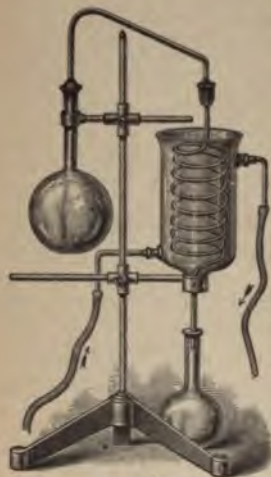


Fig. 198.

341. The Dew Point. — Experiment.

In a polished nickel-plated copper beaker pour some tepid water. Stir gently with a thermometer, reducing the temperature by the introduction from time to time of small pieces of ice. In time a mist will form on the outside of the vessel. Note the temperature of the water on the first appearance of this mist.

The dew point is the temperature at which the aqueous vapor of the atmosphere begins to condense. It may be determined as outlined in the preceding experiment. In the formation of clouds, the precipitation of dew, and in the "sweating" of pitchers of ice-water, we have evidence of the existence of water vapor in the atmosphere. The amount of moisture that the air can retain depends on the temperature. The terms *dryness* and *moistness*, applied to the air, are purely relative, and indicate the proportion of vapor actually present, in comparison with what the air could contain at the existing temperature. At the dew point the air is saturated. *Relative humidity*, or wetness, is expressed by the number of parts by weight of aqueous vapor contained by the air to every 100 that

ld contain. Saturation is represented by 100, and
ate dryness by 0. A humidity of 60 to 70 is de-
ed for health.

. **The Heat of Vaporization** is the number of calories
ed to change one gramme of a liquid at its boiling
into vapor at the same temperature. Water has the
st heat of vaporization of all liquids. The following
ss of obtaining the heat of vaporization of water
ake clear the principles that underlie the general
m : —

up apparatus like that shown in Fig. 199. The
from the boiling water is conveyed into a beaker
ning a known quantity
ter at a known tempera-

The increase in the mass
water gives the amount
eam condensed. The
" in the delivery-tube
es the water that cons-
s before it reaches the
r. Suppose that the ex-
ent gave the following

Amount of water in
eaker, 400 gm. at the be-
ng, 414.1 gm. at the end ;
rature at the beginning,

, and at the end, 41° C.; observed boiling point,
; there were 14.1 gm. of steam condensed. Now,
e principle that the heat lost or given off by the
equals that gained by the water, we have

$$400 \times (41 - 20) = 14.1 \times l + 14.1 \times (99 - 41);$$



Fig. 199.

whence $l = 537.7$ cal. The most carefully conducted experiments show that the heat of vaporization of water is 535.9.

Questions and Problems.

1. How can pure water be obtained from sea water?
2. How is it possible to heat water above the ordinary boiling point?
3. With a good air pump it can be shown that a fog forms in the glass bell jar after a few vigorous strokes of the pump. Explain.
4. Why does a current of air cause one to feel cold?
5. Why is an iceberg frequently enveloped in fog?
6. Why must the barometric pressure be considered in fixing the boiling point on a thermometer scale?
7. Why does warming a room make it drier?
8. Why are morning mists dissipated by the rising sun?
9. Why is the ice pitcher covered with moisture?
10. Water boils at 86°C . on the top of Mt. Blanc. Calculate its height.
11. Calculate the boiling point of water on the top of Pike's Peak, height 4312 m.
12. What must be the elevation of a place in order that the boiling point of water may be 90°C .?
13. What mass of water at 60°C . will melt 6 kgm. of ice at 0°C .?
14. How much ice at 0°C . will 10 kgm. of water at 40°C . melt?
15. How much heat will it require to convert 50 gm. of ice into steam? *3560 cal*
[In solving the problems, use 536 calories as the heat of vaporization of water.]
16. What will be the temperature resulting from mixing 5 kgm. of water at 90°C . with 5 kgm. of ice at 0°C .?
17. 4 kgm. of ice at 0°C . are put in 6 kgm. of water at 40°C . Determine the result.

[Calculate how much of the ice will melt.]

18. How much steam at 100°C. will raise 600 gm. of water from 16°C. to 36°C. ? *20 gm*

19. How much steam at 100°C. will it take to melt 310 gm. of ice at 0°C. and raise the temperature of the water to 16°C. ? *48 gm*

20. If 400 gm. of water at 50°C. mixed with 180 gm. of ice at 0°C. ✓
yield 580 gm. of water at 10°C. , what is the heat of fusion of ice ?

21. 10 gm. of steam at 100°C. are blown into 100 gm. of a mixture of ice and water at 0°C. The final temperature of the mixture is 5°C. Find the quantity of ice.

[Represent the quantity of ice by x , then the heat lost by the steam equals that gained by the water and ice.]

22. What is the heat of vaporization of water derived from the following data: 10 gm. of steam at 100°C. condensed in 610 gm. of ✓
water at 15°C. raised its temperature to 25°C. ?

23. Calculate the heat of vaporization of water from the following data:

Water equivalent of calorimeter	8.6 gm.
Mass of water in calorimeter	200 gm.
Temperature of water before introducing steam	20°C.
Temperature of water after introducing steam	40°C.
Amount of steam condensed	7 gm.
Temperature of steam	100°C.

24. 50 gm. of steam at 100°C. are passed into a mixture of 100 gm. of ice and 185 gm. of water at 0°C. Find the rise of temperature produced. The water equivalent of the vessel containing the mixture is 15 gm. *580*

VI. TRANSMISSION OF HEAT.

343. Three Modes of Transmitting Heat. — Experiment. — Place one end of a metal rod in a Bunsen flame and the other in melting ice. It will be found that heat passes along the rod and melts the ice. Hold the hand high above the flame; it will be warmed by a rising current of hot air. Hold the hand by the side of the flame; again a sensation of heat will be perceived.

This simple experiment illustrates the three ways in which heat may be transmitted from one point to another. They are : —

1. *Conduction*, in which heat is conveyed by matter without any visible motion of the matter itself. It is passed on from the hotter to the colder particles by some invisible molecular motion.

2. *Convection*, in which heat is transferred by the visible motion of heated matter, as by a current of hot air or the flow of hot water through pipes.

3. *Radiation*, in which heat is propagated like light, by a wave motion in the ether, without the aid of matter. It is by this method that radiant energy (heat and light) reaches us from the sun.

344. Conduction.—Experiment.—Twist together two stout wires, iron and copper, of the same diameter, forming a fork with long paral-



Fig. 200.

lel prongs and a short stem. Support them on a wire stand (Fig. 200), and heat the twisted ends. After several minutes find the point on each wire, farthest from the flame, where a sulphur match ignites when held against the wire. This point will be found farther along on the copper than on the iron, showing that the former has led the heat farther from its source.



Fig. 201.

Experiment.—Prepare a cylinder of uniform diameter, half of which is made of brass and half of wood. Hold a piece of writing paper firmly around the junction like a loop (Fig. 201). By apply-

Bunsen flame the paper in contact with the wood is soon charred, while the part in contact with the brass is scarcely injured. Metal conducts the heat away and keeps the temperature of the wood below the point of ignition.

These experiments show that solids differ in their conductivity for heat. The metals are the best conductors; wood, leather, flannel, and organic substances in general are poor conductors; so also are all bodies in a powdered state, doubtless to a lack of continuity in the material.

It is a common mistake to assume that the rate at which the temperature rises is a measure of conductivity. For example, if equal bars of iron and lead are arranged so that one end of each is heated alike, pieces of phosphorus at the same distance from the heated ends will be the first to take fire on the lead, although iron is the better conductor. This is due to the fact that iron has four times the specific heat of lead, and hence requires four times as much heat to produce the same change of temperature. The phosphorus therefore acquires the necessary temperature to ignite the phosphorus much longer before the iron.

Conductivity of Liquids.—Experiment.—Pass the tube of a simple air thermometer through a cork fitted to the neck of a large funnel. Support the apparatus as shown in Fig. 202. Fill the funnel with water, covering the bulb of the thermometer to a depth of about one centimetre. Pour a small amount of ether on the water and set it on fire. The steadiness of the index shows that little if any of the heat due to the burning ether is conducted to the water.



Fig. 202.

It appears from this experiment that water is a poor conductor of heat. This is equally true of all liquids except molten metals.

346. Conductivity of Gases. — The conductivity of gases is very small, and its determination is very difficult because of radiation and convection. The conductivity of hydrogen is about 7.1 times that of air, while the conductivity of water is 25 times as great.

347. Applications. — Some articles in a room feel cold to the touch while others feel warm. An explanation will be found in the fact that those which feel cold are good conductors of heat, and those which feel warm are bad conductors. The former conduct away the heat from the hand faster than the body supplies it, causing the sensation of cold; the latter do not carry off the heat, and consequently they do not feel cold.

The handles on metal instruments that are to be heated are usually made of some poor conductor, as wood, bone, etc.; or else they are insulated by the insertion of some non-conductor, as in the case of the handles to silver tea-pots, where pieces of ivory are inserted to keep them from becoming too hot.

The non-conducting character of air is utilized in houses with hollow walls, in double doors and double windows, and in clothing of loose texture. The warmth of woollen articles and of fur is due mainly to the fact that much air is enclosed within them on account of their loose structure.

348. Convection. — **Experiment.** — Remove the bottom from a wide-mouthed bottle. Fit a double-perforated stopper to the mouth

and pass through it two glass tubes; one of these *CD* should be straight, and the other *EF* should have some such form as shown in Fig. 203. The other ends of these tubes pass through a stopper fitted to the Florence flask, *B*. The tube *CD* should extend from the top of the flask nearly to the top of the open vessel; and the tube *EF* should reach from the bottom of the flask to the bottom of the open vessel. Support the apparatus on a heavy ring stand. Fill the flask with water colored with red aniline, and the open vessel with water colored with blue aniline. Blow through the straight tube till all air-bubbles are removed. Now place a Bunsen burner beneath the flask. In a short time the red liquid will be seen gathering on the top of vessel *A*, and the blue liquid at the bottom of vessel *B*.

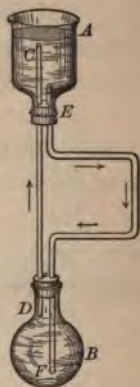


Fig. 203.

The water in *B*, on being heated, is expanded and rendered less dense. Hence, it is forced to rise by the downward pressure of the colder water in *A*. This circulation down *EF* and up *DC* will continue so long as *A* is colder than *B*.

Experiment.—Fill a large glass beaker about three-fourths full of cold water, and pour on it carefully enough warm water, colored with an aniline dye, to form a layer about 2 cm. in thickness. Fill a large test-tube full of a freezing mixture of salt and ice, and hold it in the colored water. Streams of the colored water will soon begin to descend through the uncolored part. The freezing mixture reduces the temperature of the colored water below that of the uncolored, making it heavier.

These experiments show that by raising the temperature of the lower part of a liquid in a vessel, or by lowering the temperature of the upper part, currents can be set up within it. The same is true of gases, as may be seen in the rising currents of air above a hot stove. The currents set up in fluids through differences of temperature are called

convection currents. The heating of buildings by hot water circulating through pipes, or by hot-air furnaces, is a familiar application of convection currents. Land and sea breezes, the trade winds, and, in fact, winds in general, are convection currents on a gigantic scale.

349. Ventilation. — Experiment. — Support in a shallow dish a short piece of candle, and place over it a lamp chimney. Pour enough

water into the dish to close the lower end of the chimney. The flame is soon extinguished. Why? Relight the candle, and insert a cardboard partition in the chimney, as in Fig. 204. The candle will now burn, and if a piece of lighted touch-paper¹ is held over the top of the chimney, it will show that there is a current of air down one side and up the other.



Fig. 204.

The office of a lamp chimney is to increase the supply of oxygen to the flame. The air within it is heated by the flame and rises, and cold air flows in through

the bottom to restore the equilibrium, becomes heated in passing over the flame, and thus keeps up the high temperature of the chimney. This principle underlies the ordinary methods of ventilating rooms. A flue carries off the impure air, and fresh air flows in to take its place, after passing over heated pipes or through a heated furnace.

That the existence of flues opening into a room does

¹ Made by soaking porous paper in a strong solution of saltpetre, and drying. It burns without flame and gives off smoke.

the ventilation, unless means are adopted to make the upward movement of the air in such flues, is done in the following experiment : —

Experiment. — Fit to a wide-mouthed bottle of about 2 l. capacity through which pass two glass tubes, each at least 2 cm. in diameter, and 20 cm. in length, the corresponding ends of the tubes being at the same level. A wire also passes through the cork, with a candle at the lower end. The wire is so arranged that the candle can be brought directly under one of the tubes, or can be turned away from both of them. First, set the candle in the last position, and insert the cork with its tubes in the bottle. The flame will soon go out, no air entering through either tube, although both are open. Second, turn the candle, relight the candle, turn the candle so the flame is directly under one of the tubes, and insert the cork in the bottle. The candle will now burn brightly. If lighted touch-paper is held to the top of the tubes in succession, it will be found that there is a downward current in one, and an upward in the other.



Fig. 205.

Radiation. — The heat perceived when one stands near a hot stove is not received by conduction, but is conveyed by the air. The heat energy of a hot stove is constantly passing into space as radiant energy in the form of ether. Radiant energy becomes heat when it is absorbed by bodies upon which it is transmitted in this way is, for convenience, called *radiant heat*, although it is transmitted as energy, and is transformed into heat by absorption. Heat and light are physically identical, but are distinguished through different avenues of sensation. Radiant heat produces sight when received through the eye, and sensation of warmth through the nerves of touch,

or heat a thermometer when incident upon it. The long ether waves do not affect the eye, but they heat a body which absorbs them.

351. Laws of Heat Radiation. — The following laws have been established experimentally: —

I. *Radiation proceeds in straight lines.* This law is illustrated in the use of fire screens and sun shades.

II. *The amount of radiant energy received by a body from any small area varies inversely as the square of its distance from this area as a source.*

III. *Radiant energy is reflected from a polished surface so that the angles of incidence and reflection are equal.* Archimedes is said to have set fire to the Roman ships during the siege of Syracuse in 212 B.C., by concentrating on them the heat of the sun by the aid of a large concave reflector.

IV. *The capacity of a surface to reflect radiant energy depends both on the polish of the surface and the nature of the material.* Polished brass is the best reflector, and lampblack is the poorest.

V. *The rate at which the temperature of a cooling body falls by radiation is proportional to the excess of its temperature over that of the surrounding medium.* This is known as Newton's *Law of Cooling*, and holds approximately for small differences of temperature but fails when the excess is large. According to this law a body at a temperature of 30°C . cools twice as fast as one of 25°C . in air at 20°C ., for the excess 10° , in the first case is twice 5° , the excess in the second.

352. Absorption of Heat. — **Experiment.** — In slots cut 10 cm. apart in a narrow board, support two pieces of bright tin plate, each

10 cm. square. Coat the inner face of one of these squares of tin with lampblack. Stick balls of equal size, one at the centre of each outside face, with shoemaker's wax, using as little as possible. Hold a heated plate of iron midway between the two tin squares. The ball will soon fall from the blackened plate, showing that lampblack is a ready absorber of heat.

By using plates coated with different substances, it will be found that these substances differ in their capacity of absorbing heat. Leslie discovered that the best absorbers, as lampblack, ashes, and rough surfaces, are bad reflectors; while good reflectors, as polished metals, are bad absorbers.

353. The Radiometer. — This instrument was invented by Sir William Crookes in 1873 while investigating the properties of highly attenuated gases. It consists of a glass bulb from which the air has been exhausted till the pressure does not exceed 7 mm. of mercury (Fig. 206). Within the bulb is a light cross of aluminum wire carrying small diamond-shaped vanes of mica, one face of each being coated with lampblack; the whole is mounted to revolve on a vertical pivot. When the instrument is placed in the sunshine or in the radiation from any heated body, the cross revolves with the blackened faces of the vanes moving away from the source of heat.



Fig. 206.

The explanation of this interesting phenomenon is to be found in the kinetic theory that the mean free path of the molecules between collisions with other molecules becomes,

at this low pressure, at least equal to the distance between the vanes and the wall of the bulb. The infrequent collisions among the molecules in such a vacuum prevents the equalization of pressure throughout the tube. Now the blackened sides of the vanes absorb more heat than the bright ones, and the gas molecules rebound from the warmer surfaces with a greater velocity than from the others, thus giving the vanes an impulse in the opposite direction. This impulse is the equivalent of a pressure, and the residual gas has lost the power of rapid adjustment of pressure throughout its mass. When the vacuum is not so good, no difference of pressure on the two sides of the vanes can exist, and there is no motion of the vanes.

354. Selective Absorption. — Experiment. — Fill a large flat bottle with clear water, and place it between a lamp and the radiometer. The rate of rotation of the vanes will be much reduced. Repeat the experiment, using a similar bottle filled with a solution of iodine in carbon disulphide. There will be no perceptible effect on the rate of rotation.

These experiments show that water cuts off the greater part of the radiant heat, while the solution of iodine does not perceptibly affect the intensity. Substances which transmit radiant heat are called *diathermanous*, and those which do not, *athermanous*. Rock salt is the most highly diathermanous substance known. On the other hand, alum, sugar, glass, water, and ice are extremely athermanous. The diathermanous character of a substance varies with the temperature of the radiant. Such substances as alum, water, etc., transmit little or none of the radiation from a surface of low temperature. The radiant energy from the sun passes readily through the atmosphere to the earth, warming its surface; but the radia-

tions from the earth are stopped by the enveloping atmosphere. So also the radiant heat from the sun passes readily through the glass of the greenhouse, but that from within is unable to pass outward.

Questions.

1. When snow and ice melt why do they not liquefy all at once?
2. Why does a metal liquefy so rapidly when it begins to melt?
3. Why does snow protect from cold?
4. Why will a current of air extinguish a candle?
5. Why will ice packed in sawdust or straw keep from melting?
6. Why do men working about smelting-furnaces wear flannel clothing?
7. Why is paper so effective in protecting plants from frost?
8. Why is it difficult to boil water in a "furred" kettle?
9. Why is the direct radiation of the sun on the top of a mountain more intense than at the base?
10. Why is there little or no dew on a windy night?
11. Should the surfaces of stoves and heat radiators be rough or polished?
12. If a pond is freezing over, what is the temperature of the water at the bottom?
13. Why does increasing the height of a chimney increase the draught?

VII. HEAT AND WORK.

355. Heat and Mechanical Action. — **Experiment.** — Strike the edge of a piece of flint a glancing blow with a piece of hardened steel. Sparks will fly at each blow.

Experiment. — Pound a bar of lead vigorously with a hammer. The temperature of the bar will rise.

Experiment. — Place a small piece of tinder, such as is employed in cigar lighters, in the cavity at the end of the piston of a fire syringe (Fig. 207). Force the piston quickly into the barrel. If the piston is quickly withdrawn the tinder will probably be on fire.

These experiments illustrate the transformation of mechanical energy into heat. Some of the energy of the descending flint, the hammer, and the piston have in each case been transferred to the molecules of the bodies themselves, increasing their kinetic energy; that is, raising their temperature. Savages kindle fire by rapidly twirling a dry stick, one end of which rests in a notch cut in a second dry piece. The axles of carriages and the bearings in machinery are heated to a high temperature when not properly lubricated. The heating of drills and bits in boring, the heating of saws in cutting timber, the burning of the hands by a rope slipping rapidly through them, the stream of sparks flying from an emery wheel, are instances of the same kind of transformation; the work done against friction produces kinetic energy in the form of heat.



Fig. 207.

356. Numerical Relation between Heat and Work.—In 1840 Joule of Manchester began a series of experiments to determine the numerical relation between the unit of heat and the foot-pound. Joule's experiments by a number of methods extended over a period of nearly forty years. His most successful method consisted in determining the heat produced when a known amount of work was expended in heating water by stirring it with paddles driven by weights falling through a known height. He concluded that 772 foot-pounds of work, when converted into heat, will raise the temperature of one pound of water 1° F. The equivalent for 1° C. is 1390 foot-pounds.

The investigations of Rowland in 1879, and of Griffiths in 1893 have shown that 778 foot-pounds for 1° F., or 427 kilogramme-metres for 1° C., are the nearest whole num-

er values; that is, 778 foot-pounds of work when converted into heat will raise the temperature of one pound of water 1° F., or 427 kilogramme-metres of work when converted into heat will raise the temperature of one kilogramme of water 1° C. This numerical relation between heat and work is known as the *mechanical equivalent of heat*. Its value expressed in absolute units is 1.9×10^7 ergs. This is the energy value of one calorie.

The following problem illustrates one of the uses which may be made of this relation: A mass of iron weighing 10 kgm. (specific heat 0.112) falls through a height of 100 m. Find the heat generated when it strikes the ground.

The work done by gravity is $10 \times 100 = 1000$ kgm.-m.; $1000 \div 427 = 2.342$ kgm.-degrees of heat, that is, the heat that would raise 2.342 kgm. of water through 1° C. Then $2.342 \div 0.112 = 20.9$, the number of kgm. of iron that 2.342 kgm.-degrees of heat will raise through 1° C. $20.9 \div 10 = 2.09$ degrees. If all the heat were confined to the iron, its temperature would rise 2.09° C.

357. The Steam Engine, in its most essential features, is the invention of James Watt. It is a device for transforming the energy stored in steam into that of mechanical motion. In the more common of its many modern forms it consists of a strong cylinder in which a piston is made to move to and fro by applying the pressure of steam to its two faces alternately.

Figure 208 shows a longitudinal section of a simple engine divested of many of the more complicated accessories designed to improve its efficiency. The piston *M* moves to and fro in the cylinder *D* by virtue of the pressure of the steam supplied by the boiler through the tube

$$1 \text{ Cal} = 427 \text{ kg met (large calorie)}$$

$$1 \text{ cal} = .427 \text{ kg met (small calorie)}$$

F. In the steam chest *E* works the slide valve *R*, which admits the steam alternately to the ends of the cylinder

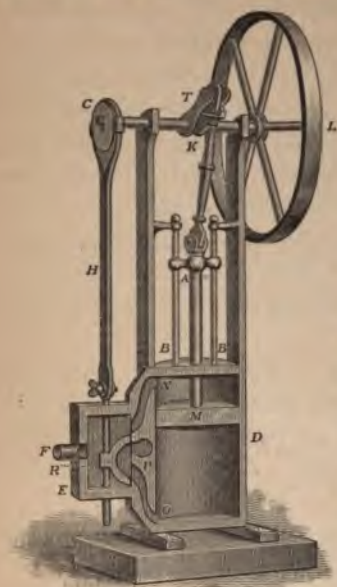


Fig. 208.

through *N* and *O*. When the valve is situated as shown, the steam passes into the upper end of the cylinder and drives the piston down. At the same time the other end is connected with an exhaust pipe, shown at *P*, through which the steam either escapes into the air, as in *high pressure* or *non-condensing engines*, or into a large chamber, as in *low pressure* or *condensing engines*, where it is condensed to water, reducing the pressure on that face of the piston. The slide valve is moved by the rod *H*, connected to an eccentric *C*, a wheel pivoted a little to one side of its

centre, on the horizontal shaft *K*. This shaft receives its motion from the piston by means of the jointed rod *A* and the crank *T*. The flywheel *L* serves the double office of belt pulley and reservoir of energy. It is made with a heavy rim in order that when the piston is at the end of the cylinder, and the direction of motion must change, the energy stored in it may be sufficient to carry the shaft beyond these *dead points* to a position where the piston can again turn the shaft. It also serves to give uniformity of motion to the shaft, which would otherwise vary because the effective part of the force exerted on the

crank is not constant, being greatest when the crank is at right angles to the connecting rod, and diminishing to nothing when parallel to it.

In order that the piston rod may always move in a straight line, and the piston maintain a steam-tight fit in the cylinder, the former is attached to a transverse bar, or cross head *A*, which slides on two guide-bars *B B*, firmly bolted to the framework of the engine, and adjusted accurately parallel to each other and to the piston rod. In many large engines the cylinder is given a horizontal position.

357 a. The Gas Engine. — The gas engine is a type of internal combustion engine, which includes motors using gas, gasoline, kerosene, or alcohol as fuel. The fuel is introduced into the cylinder of the engine either as a gas or a vapor, mixed with the proper quantity of air to produce a good explosive mixture, and this mixture is exploded at the right instant by means of an electric spark. The force of the explosion drives the piston forward in the cylinder.

In the "four-cycle" type of gasoline engine an explosive mixture is admitted and ignited only every other revolution of the engine, while in the "two-cycle" type an explosion occurs every revolution. The former is used in nearly all motor car engines.

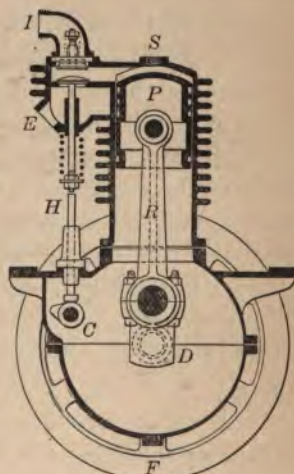


Fig. 208 a.

The operation of a "four-cycle" engine may be understood from a description of the four steps in the series, as illustrated by the simple engine shown in Fig. 208 *a*. This figure is a section of an air-cooled engine (as indicated by the ribs for cooling). The upper valve, closing the pipe *I*, is the inlet valve; the lower one, closing the pipe *E*, is the exhaust valve for the disposal of the spent gases. The piston *P*, which works up and down in the cylinder, is surrounded with packing rings (four in the figure), and the connecting rod *R* leads from the piston to the crank-shaft *D*. The piston is shown in its highest position with both valves closed.

If the engine is running, the motion of the flywheel *F* carries the piston down, the partial vacuum produced behind it opens the inlet valve in *I*, and the explosive mixture fills the cylinder. The motion continuing, the piston rises and compresses the explosive mixture, the exhaust valve still being closed, and when the piston is again at its highest point, a spark from the spark plug screwed into the opening at *S* ignites the mixture. The force of the explosion drives the piston down. When it rises again, the exhaust valve is opened mechanically by means of the cam *C* and the vertical rod *H*; the spent gases, still very hot, escape through the exhaust pipe *E*. The piston has now traversed the cylinder four times, twice in each direction, and the series of operations begins again.

Both valves are kept normally closed by the springs shown surrounding the valve stems. The small shaft to which the cam *C* is fixed is actuated by spur wheels (not shown) from the shaft of the engine, and it rotates only half as often as the main shaft. Hence the exhaust valve opens only every other revolution of the engine.

In most modern engines the inlet valve is also operated mechanically from a cam shaft like the one shown at *C* and in the same manner as the exhaust valve. The two valves are then usually on opposite sides of the cylinder.

If the cylinder is water-cooled, it is surrounded by a jacket through which water is kept circulating, usually by means of a pump. The water goes from the hot jacket to the radiator and thence back to the cylinder jacket.

The inlet pipe *I* in a gasoline motor leads from the carburetor, into which the gasoline comes as a spray, is vaporized and mixed with air.

Problems.

1. A mass of 100 gm. moving with a velocity of 50 m. per second is suddenly stopped. If all its energy were converted into heat, how much would it be?

[Kinetic energy = $\frac{1}{2}mv^2$, a calorie = 4.19×10^7 ergs.]

2. If 10 kgm. of water fall a distance of 1000 m. and all the energy goes to heating the water, to what temperature will the water be raised? ✓

[Calculate the energy in kilogramme-degrees (§ 356). Then 10 kgm. will be warmed one-tenth as many degrees as 1 kgm.]

3. An iron bullet (specific heat 0.112) weighing 50 gm. strikes a target with a velocity of 400 m. per second. Assuming 20 per cent of the kinetic energy of the bullet to remain in the bullet as heat, find how much its temperature will be raised.

[As in problem 1, calculate the energy in calories. Find how many calories will raise 50 gm. of iron 1° C. (§ 326).]

4. A hammer weighing 10 kgm. falls from a height of 10 m. and strikes an anvil. If half of the energy is used in heating the hammer, how many degrees will its temperature be raised, the specific heat of the material of the hammer being 0.1? ✓

5. How much heat is produced in stopping a train of 100,000 kgm. mass, running at 36 km. an hour?

CHAPTER VII.

MAGNETISM AND ELECTRICITY.

I. MAGNETS AND MAGNETIC ACTION.

358. The Natural Magnet or Lodestone. — Certain ores, consisting of iron and oxygen, sometimes possess the property of attracting and holding small particles of iron. This property was known to the ancients and was exhibited in a marked degree by iron ores from Magnesia in Asia Minor; they were therefore called *magnetic stones*. They are now known as *natural magnets*, and the properties exhibited by them are called *magnetic properties*.

Experiment. — Sprinkle iron filings over a piece of natural magnet. The filings will adhere to it in tufts, not uniformly over the surface, but chiefly at the ends and on projecting edges.



Fig. 209.

Experiment. — Make a stirrup out of wire, place in it the piece of natural magnet, and suspend it by an untwisted thread (Fig. 209). Carefully exclude all air-currents and allow the magnet to come to rest. Note its position, then disturb it slightly, and again let it come to rest. It will be found that it invariably returns to the same position, the line connecting the two ends to which the filings chiefly adhered in the preceding experiment lying north and south.

This property of the natural magnet was early turned to account in navigation, and secured for it the name of *lodestone* (leading-stone).

359. Artificial Magnets.—**Experiment.**—Stroke a large darning-needle from end to end, and always in the same direction, with one end of the lodestone. Roll it in iron filings and they will cling to its ends as they did to the lodestone. The needle has become a magnet.

Experiment.—Use the needle of the last experiment to stroke another needle. This second needle also acquires magnetic properties, and the first one has suffered no loss.

Bars of hard steel, that have been made magnetic by the application of a lodestone or of some other magnetizing force, are called *artificial magnets*. The form of artificial magnets, or simply magnets, most commonly met with are the *bar* and the *horseshoe* (Fig. 210), so called from their shape.



Fig. 210.

360. Polarity.—**Experiment.**—Roll a bar magnet in iron filings. It will become thickly covered with the filings near its ends. Few, if any, will adhere at the middle (Fig. 211).



Fig. 211.

The experiment shows that the greater part of the magnetic attraction is concentrated at the ends of the magnet. These are called its *poles*, and the magnet is said to have *polarity*. The line joining the poles of a long slender magnet is its *magnetic axis*.

361. Experiment.—Straighten a piece of steel watch spring and float it on a piece of cork in a glass vessel filled with water. Note its position after it



Fig. 212.

comes to rest (Fig. 212). Change its position several times. It will be found that there is no uniformity in the direction it takes when at rest. Now stroke the spring from end to end with one pole of a magnet and repeat the tests. The floating spring will now always come to rest in a north-and-south line and with the same end to the north.

The end pointing toward the north is called the *north-seeking pole*, and the other the *south-seeking pole* of the magnet. They are commonly called simply the *north pole* and the *south pole*.

362. Consequent Poles. — **Experiment.** — Draw the temper of a knitting-needle slightly at two or three points, and then stroke it from end to end with one pole of a strong magnet. Dip it in iron filings. They will adhere in tufts at the points where the temper was drawn as well as at the ends, showing that there are several poles.

Magnetic poles intermediate between those at the ends are called *consequent poles*.

363. A Magnetic Substance is one which is attracted by a magnet, or which can be magnetized. Faraday showed that most substances are influenced by magnetism. Ordinary magnets, however, produce a noticeable effect on but few substances besides iron and its compounds. Cobalt and nickel stand next to iron in respect to magnetism. Some substances, like antimony and bismuth, are slightly repelled by powerful magnets. They are said to be *diamagnetic*.

364. Magnetic Transparency. — **Experiment.** — Cover the pole of a strong bar magnet with a thin plate of glass. Bring the face of the plate opposite the pole in contact with a pile of iron tacks. A number will be found to adhere, showing that the attraction takes place through glass. In like manner, try thin plates of mica, wood,

paper, zinc, copper, and iron. No perceptible difference will be seen except in the case of the iron, where the number of tacks lifted will be much less.

Magnetic force acts freely through all substances except those classed as *magnetic*. Soft iron serves as a more or less perfect screen to magnetism. Watches may be protected from magnetic force that is not too strong by means of an inside case of soft sheet iron.

365. Magnetic Needle. — A slender magnetized bar, suspended by an untwisted fibre or pivoted on a point, like a compass needle, is a *magnetic needle*. The direction in which it comes to rest without torsion or friction is the *magnetic meridian*.

Experiment. — Magnetize a piece of watch spring about 2 cm. long. Fasten a fibre of unspun silk to the bit of magnetized steel so that it will hang horizontally. Suspend it inside a wide-mouthed bottle by attaching the fibre to a cork fitting the mouth of the bottle. The little magnetic needle will then be protected from currents of air. It may be made visible at a distance by sticking fast to it a piece of thin white paper.

366. Mutual Action between Magnets. — **Experiment.** — Magnetize a piece of large knitting-needle, about four inches long, by stroking it from the middle to one end with the north pole of a bar magnet, and then from the middle to the other end with the south pole. Repeat the operation several times. Suspend the needle in a small stirrup like that of Fig. 209 and mark the north pole with red paint.

Present the north pole of the magnetized knitting-needle to the north pole of the needle suspended in the bottle. The latter will be repelled. Present the same pole to the south pole of the little magnetic needle; it will be attracted. Repeat with the south pole of the knitting-needle and note the deflections.

The results may be expressed by the following law of magnetic attraction and repulsion:—

Like poles repel and unlike poles attract each other.

The suspended magnet affords a ready means of ascertaining which pole of another magnet is the north pole, for the north pole of one will repel the north pole of the other. Repulsion is always a more reliable indication of polarity than attraction. The reason will be obvious from the experiments which follow.

367. Induced Magnetism.—**Experiment.**—Hold one end of a short rod of soft iron near one pole of a strong bar magnet, and while in this position dip the other end into iron filings. They adhere to it as to a magnet, but fall off when the magnet is removed.

Magnetism produced in magnetic substances by the influence of a magnet is said to be *induced*.

368. Polarity of the Iron Bar.—**Experiment.**—Support a strong horseshoe magnet in a vertical plane, with its poles uppermost, and the line joining them horizontal (Fig. 213). Suspend by a thread a short rod of soft iron so that it hangs horizontally above and near the poles of the magnet. Now bring near one end of this rod a bar magnet, so that its pole is opposite in name to that of the vertical magnet. The repulsion of the rod indicates that its polarity is the same as that of the bar magnet, and hence the reverse of that of the horseshoe magnet.



Fig. 213.

It appears, therefore, that when a magnet is brought near a piece of iron it magnetizes it by induction, and that the attraction is between unlike poles. The inductive action can take place through a series of iron rods, as exemplified by the attraction of a bunch of filings or tacks.

59. Permanent and Temporary Magnetism.—In the last experiments the soft iron ceases to be a magnet when moved to a distance from the bar or horseshoe magnet. When a piece of hardened steel is brought near a magnet, it acquires magnetism as the piece of soft iron does under the same conditions; but the steel retains its magnetism when the magnetizing force is withdrawn, while the soft iron does not. In addition, therefore, to the *permanent magnetism* exhibited by the magnetized steel, we have *temporary magnetism* induced in a bar of soft iron when it is brought near a magnet or in contact with it.

II. NATURE OF MAGNETISM.

70. Magnetism a Molecular Phenomenon.—**Experiment.**—Magnetize a piece of watch spring, then heat it red hot and test it for magnetism. It will be found to have lost the power of attracting filings.

Experiment.—Magnetize a knitting-needle and find by averaging several trials how many tacks can be lifted by it. Now hold one end of the needle against the edge of the table or in a vise and, by plucking the other end, cause the needle to vibrate vigorously for a few seconds. The power of the magnet to pick up tacks will be found to be appreciably lessened by the vibration.

Experiment.—Take a piece of iron wire, about 30 cm. long and 1 mm. diameter, and carefully anneal it. Bend it to the form shown in Fig. 214. Stroke it carefully several times with a strong magnet. It will be a weak magnet. (How shown?) Now hold it by the turned-up ends and give the wire a sudden twist. If the wire is again tested for magnetism it will be found to have lost nearly all.



Fig. 214.

In each of the preceding experiments the molecular arrangement has been disturbed; and it is interesting to note that in each the magnetism has been weakened.

The conclusion is that magnetism is connected in some way with the molecular arrangement of the iron or steel.

371. Further Evidence. — Experiment. — Magnetize a piece of straightened watch spring; notice that it has two poles, one at each end, the centre being neutral. Break it at the neutral point; each piece will be found to have two poles, two new ones having been formed at the point that was formerly neutral. If these pieces in turn be broken, their parts will be magnets with two poles like the original.

There is no apparent limit to the extent to which this process may be carried, indicating that possibly if carried as far as the molecules, they too would prove to be magnets.

Experiment. — Fill a slender glass tube nearly full of steel filings, closing the ends with cork. Stroke the tube from end to end with one pole of a strong magnet; the filings acquire magnetic properties. Shake up the filings thoroughly; all polarity is lost.

A minute examination of each steel particle will show that it is a magnet. The loss of polarity is evidently due to the neutralization of the actions of many little magnets by disturbing their arrangement. Undoubtedly the polarity would be restored if the particles could be restored to their original positions. The experiment strongly supports the theory that each molecule of a magnetic substance is a magnet.

It is worthy of notice that magnetization is facilitated by jarring the substance, or by heating it and then cooling it, while under the magnetizing influence.

372. Retentivity. — Experiment. — Prepare three bars of the same size, one each of soft iron, soft steel, and hard steel. Dip one end of each in succession into iron filings, and bring a strong magnet in contact with the other end. When the bars are withdrawn and the magnet is removed, most of the filings drop from the iron, the hard steel retaining the largest number.

The difference exhibited by these substances is due to what is called *retentivity*, or the ability to retain magnetism.

373. Theory of Magnetism.—Experiments like the preceding ones lead to the conclusion that magnetism is a molecular phenomenon. The most probable theory is that the individual molecules of a magnetic substance are always magnetized. In an unmagnetized bar the poles of these molecular magnets are turned in all directions, or else the little magnets form stable combinations or closed chains, so that no magnetism external to the bar is exhibited. When they are turned by an external magnetizing force, so that a certain portion of the molecules have their poles pointing in the same direction, then the bar is magnetized. The larger the proportion of the molecules which have their molecular axes turned in the same direction, the stronger the magnet.

The molecules of soft iron are readily turned by a magnetizing force ; but when this force is withdrawn, they revert to the unmagnetized state. Hardened steel, on the other hand, requires a greater magnetizing force to shift the magnetic axes of the molecules, but once shifted they remain in the new position, and the bar is permanently magnetized.

III. THE MAGNETIC FIELD.

374. Lines of Magnetic Force.—**Experiment.**—Place a sheet of paper or glass over a small bar magnet and sift iron filings evenly over it from a muslin bag, tapping the paper or glass gently to aid the filings in arranging themselves under the influence of the magnet. They will cling together in curved lines, which diverge from one pole of the magnet and meet again at the opposite pole.

These lines are called *lines of magnetic force* or of *magnetic induction*. Each particle of iron becomes a magnet by induction; hence the lines mapped out by the filings are the lines along which magnetic induction takes place.



Fig. 215.

Figure 215 was made from a photograph which was taken by sifting iron filings on the sensitized side of a photographic plate with a piece of magnetized watch spring under it. The plate was then exposed for about a second to the light of an incandescent lamp, and was developed in the usual way. These lines of force spring from the north pole, curve round through the air to the south pole, and complete their circuit through the magnet itself.

Figure 216 was made from two magnets with their unlike poles turned toward each other. The lines of force from

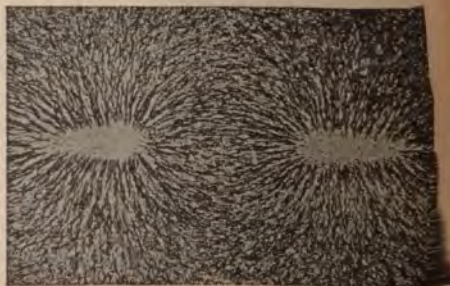


Fig. 216.

the north pole of one extend across to the south pole of the other. Lines of magnetic force are always to be considered as under tension and as possessing elasticity. Figure 216 is therefore a picture of attraction.

11/17/72 m.m.
from

opp poles attract
+ pole

Figure 217 was made from two magnets with their like poles turned toward each other. None of the lines springing from one of them enter the other. This figure is a picture of magnetic repulsion.

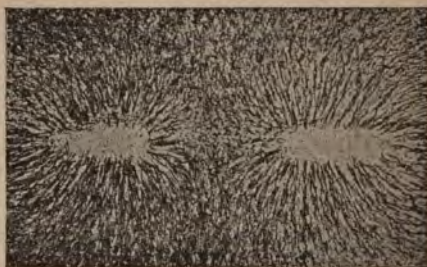


Fig. 217.

375. Magnetic Field.

— The region around a magnet, or a space within which there are lines of magnetic force, is called a *magnetic field of force*. Faraday introduced the method of studying magnetic fields by means of the lines of force illustrated above.

376. Direction of Lines of Force. — The direction of a line of force at any point is that of a line drawn tangent to the curve at the point; the direction along it is the same as that in which a north pole is urged. The north pole of a magnetic needle is repelled from the north pole of a bar magnet. Hence, if an observer stands with his back to the north pole of a magnet, he is looking in the direction of the lines of force coming from that pole.

377. Permeability. — Experiment shows that when iron is placed in a magnetic field, the lines of force are concentrated by it. This property possessed by iron, when placed in a magnetic field, of concentrating the lines of force and increasing their number is known as *permeability*. The superior permeability of soft iron explains the action of magnetic screens (§ 364). In the case of the

iron is easily magnetized

watch shield, the lines of force pass through the iron and not across it; the watch is thus protected from magnetism because the lines of force do not enter it.

IV. TERRESTRIAL MAGNETISM.

378. The Earth a Magnet. — Experiment. — Place a long bar magnet on the table and suspend over it a magnetic needle mounted so as to turn readily about a horizontal axis. When over the north-seeking pole, the needle will be vertical, with its south-seeking pole down; when over the middle, it is horizontal; and when over the south-seeking pole, the needle is again vertical, but with its north pole pointing downward.

The earth acts in a similar manner toward such a needle when moved over its surface from pole to pole. At a point in Boothia Felix, west of Baffin's Bay, the needle is nearly vertical, with its north pole down; at points successively farther south, it dips less and less toward the vertical, becoming horizontal near the earth's equator, and again gradually inclining toward the vertical, with its south pole down as it nears the south magnetic pole of the earth. If a bar magnet, about half the length of the earth's diameter, were thrust through the earth's centre, making an angle of about 20° with its axis, it would account for many of the phenomena of terrestrial magnetism.

379. Earth's Induction. — Experiment. — Procure a thoroughly annealed iron bar about 75 cm. long, showing little or no polarity when tested with a magnetic needle while the bar is supported horizontally in an east-and-west line. Hold this bar in a meridian plane, but with its north end dipping down some 70° below the horizontal. Tap the end of the bar with a hammer and then test for polarity. The lower end will be found strongly north-seeking, and the upper end south-seeking. If the bar is turned end for end, and again tapped with a hammer, the lower end again becomes north-seeking.

The experiment illustrates the inductive action of the earth. If we examine any iron object which has remained undisturbed for some time, as a stove or a supporting column of a building, we shall find that it is polarized like the bar of the above experiment. The inductive action of the earth probably accounts for the existence of natural magnets.

180. Magnetic Dip. — Experiment. — Thrust through a cork an iron magnetized knitting-needle, and at right angles to this two short pieces of wire (Fig. 218). Support the apparatus between the edges of two wine glasses, with the axis in an east-and-west line, and the needle adjusted so as to rest horizontally. Now magnetize the needle, being careful not to displace the cork. It will no longer remain a horizontal position, the north end dipping down as if it had become heavier.



Fig. 218.

The angle made by this needle with a horizontal plane is called the *inclination* or *dip* of the needle. A magnetic needle mounted so as to move freely in a vertical plane, and provided with a graduated arc for measuring the inclination, is called a *dipping needle* (Fig. 219).



Fig. 219.

The magnetic poles of the earth are points where the dip is 90° ; the dip at the magnetic equator is 0° . Lines on the earth's surface, pass-

ing through points of equal dip, are called *isoclinic lines*; they are irregular in direction, though resembling somewhat parallels of latitude.

381. Declination. — The magnetic poles do not coincide with the geographic poles, and consequently in most places the direction of the magnetic needle is not that of the meridian of the place. The direction of the magnetic needle at any place is that of the magnetic meridian of the place. The *declination* of the needle is the angle between the magnetic and the geographical meridian.

382. The Line of no Declination passes through those places where the needle points true north. Such a line, in 1900, ran from the north magnetic pole across the eastern end of Lake Superior, through Lansing, Mich., Columbus, Ohio, through West Virginia and South Carolina, and left the mainland at Charleston, on its way to the south magnetic pole. The returning line through the eastern hemisphere is quite irregular in direction. At places east of this line the needle points west of north, and west of the line it points east of north. Lines passing through points of equal declination are called *isogonic lines*.

V. ELECTRIFICATION.

383. Electrical Attraction. — **Experiment.** — Cut a number of small balls out of the pith of common elder. Place them in a pile on the table and touch them with a rod of sealing-wax. Notice that the rod does not affect the balls in the least. Now rub the rod with dry flannel and again bring it up to the pile of balls. They will be alternately attracted and repelled.

Rods of glass, shellac, sulphur, very dry wood, ebonite, etc., may be substituted for the sealing-wax, and a collec-

any light objects, bits of tissue paper for example (Fig. 220), for the pith-

balls which exhibit the power of attracting small bodies after being electrified. They are said to be electrified. Electrification may be brought about in a variety of ways in addition to those mentioned here, as will appear in the course of this chapter.



Fig. 220.



Fig. 221.

384. Attraction Mutual. — Experiment. — Prepare a glass tube about 2 cm. in diameter and 40 cm. long. Remove all sharp corners by fusion in the flame of a blowpipe. Electrify the tube by friction with a piece of silk, and hold it near the end of a long wooden rod resting in a wire stirrup suspended by a silk thread. The suspended rod is attracted. Now, replace the rod by the electrified tube (Fig. 221). When the

electrified tube is held near the rubbed end of the wooden rod, the latter moves as if attracted by the former.

This experiment teaches that electricity attracts the other; that *the action is mutual*.

Electrical Repulsion. — Experiment. — Suspend several pith-balls from a glass rod, by linen threads from a glass rod, and rub them with an electrified glass rod (Fig. 222). At first they are



Fig. 222.

attracted, but they soon fly away from the tube and from one another. When the tube is removed to a distance, the balls no longer hang side by side, but keep apart for some little time. If we bring the hand near the balls they will move toward it as if attracted, showing that the balls are electrified.

It thus appears that bodies become electrified by coming in contact with electrified bodies, and that electrification may show itself by repulsion as well as by attraction.

386. Two Kinds of Electrification. — **Experiment.** — Rub a glass tube with silk and suspend it as in Fig. 221. Excite a second glass tube and hold it near one end of the suspended one. The suspended tube will be repelled. Bring near the suspended tube a rod of sealing-wax excited by friction with flannel. The suspended tube is now attracted. Repeat these tests with an electrified rod of sealing-wax in the stirrup instead of the glass tube. The electrified sealing-wax will repel the electrified sealing-wax, but there will be attraction between the sealing-wax and the glass tube.

The experiment shows that there are *two kinds of electrification*: one developed by rubbing glass with silk, and the other by rubbing sealing-wax with flannel. In the former case the body is said to be *positively* electrified; in the latter case *negatively* electrified.

387. First Law of Electrostatic Action. — It was seen in the last experiment that there was repulsion between the electrified glass tubes, and that the electrified sealing-wax attracted the electrified glass. These facts are expressed by the following law: —

Electrical charges of like sign repel each other; electrical charges of unlike sign attract.

388. The Electroscope, as the name implies, is an instrument for detecting electrical charges. The most common

$q = q_1 q_2$ Coulombs have

rm is the *gold-leaf electroscope*. Through the insulating stopper of a glass flask passes a brass rod terminating in a ball on the outside (Fig. 223), and two strips of thin gold or aluminum foil on the inside, hanging parallel and close together. If an electrified object is brought in contact with the rod, the metal strips become similarly charged, and are repelled from each other.



Fig. 223.

389. Use of the Electroscope. — In order to determine the kind of electrification on a body, a *proof-plane* may be used. It is a small metal disk cemented to one end of an ebonite or shellac rod (Fig. 224). Charge the electroscope by touching its knob with a glass rod excited by friction with silk. Slide the metal disk of the proof-plane along the surface to be tested, and then bring it near the ball of the electroscope. If the leaves diverge farther, the body in question is positively charged (§ 394); if they diverge less, the charge is probably negative. An increase of divergence is a more reliable indication



Fig. 224.

than a decrease, because the divergence will decrease when the proof-plane is not charged (§ 406). The test may be varied by touching the charged proof-plane to the knob of the electroscope. In either case, if there is a decrease in the divergence of the leaves, repeat the test by charging the electroscope by means of a stick of sealing-wax rubbed with flannel. If the leaves now diverge, the body in question is negatively electrified.

390. Simultaneous Development of the Two Electrifications.—

Experiment.—Fit to the end of a rod of sealing-wax a cap of flannel, three or four inches long, with a silk cord attached to draw it off (Fig. 225). Electrify the rod by turning it around inside the cap, and then hold it near the knob of the electroscope. No divergence will be observed. By the aid of the cord remove the flannel cap, and present it to the positively charged electroscope. The increased divergence of the leaves shows that the cap is positively charged. If we test the rod of sealing-wax in the same way, it will be found to be negatively charged.



Fig. 225.

The experiment shows (1) *that one kind of electrification is not developed without the other; and (2) that the two kinds of electrification are produced in equal quantities*, as demonstrated by the fact that the quantity in the rubber exactly neutralizes that on the rod when the two are in contact. The two charges behave like equal positive and negative quantities.

391. Conductors and Non-conductors. — Experiment.—Fasten a smooth metallic button to a rod of sealing-wax. Connect it with the knob of the electroscope by a fine copper wire, 50 to 100 cm. long. Hold the sealing-wax in the hand and touch the button with an electrified glass tube. The divergence of the leaves indicates that they are electrified. If we repeat the experiment, using a silk thread in place of the wire, no effect will be produced on the leaves.

All substances may be roughly classed under two heads, *conductors* and *non-conductors*. In the former if one point of the body is electrified by any means, the electrification spreads over the whole body, but in a non-conductor the electrification is confined to the vicinity of the point where it is excited. Non-conductors are commonly called *insulators*. Substances differ greatly in their conductivity, so

that it is not possible to divide them sharply into two classes. Metals, carbon, and the solution of some acids and salts are the best conductors. Among the best insulators are paraffin, turpentine, silk, sealing-wax, india-rubber, gutta-percha, dry glass, porcelain, mica, shellac, spun quartz fibres, and liquid oxygen. Some insulators, like glass, become good conductors when heated to a semi-fluid condition.

392. Probable Nature of Electrification.—It was suggested by Faraday, and a multitude of facts tend to confirm his view, that the electrification of a body is a strained condition of the ether which surrounds it and pervades it. Conductors differ from insulators in this: in the former, the molecular mobility is such that this state of strain is continually giving way, while in the latter considerable distortion is possible before the molecular structure yields to the strain. The phenomena of attraction and repulsion exhibited by electrified bodies are due to the attempt of the strained ether in and around the bodies to return to its normal condition. In producing electrification, work is done in distorting the medium; hence electrification is a form of potential energy.

VI. ELECTROSTATIC INDUCTION.

393. Electrification by Induction.—**Experiment.**—Excite a glass tube by friction with silk. Bring it gradually near the ball of an electroscope. The leaves begin to diverge when the tube is some distance from the knob, and the amount of divergence increases as the tube is brought nearer. When the tube is removed the leaves collapse.

It is evident, since the leaves do not remain apart, that there has been no transfer of electrification from the tube to the electroscope. The electrified condition, produced in

the electroscope when the electrified body is brought near it, is due to what is called *electrostatic induction*. Why such an effect should occur is easily understood when we recall Faraday's views of electrification, that it is a distortion of the ether about the body. Evidently, then, any body placed within this *electrical field* would be electrified.

394. Charging a Body by Induction. — Experiment. — Support a smooth metallic ball on a dry plate of glass. Connect it with the knob of the electroscope by means of a metallic wire, the ends of which are bent into a loop and smoothly soldered. The ball and the electroscope now form one continuous conductor. Bring near the ball an electrified glass tube; the leaves of the electroscope diverge. Before removing the excited tube, remove the wire, handling it with some non-conductor. The electroscope remains charged, and it will be found to be positive. A similar test made of the ball will show that it is negatively charged. Repeat the experiment without removing the connecting wire. There are no signs of electrification after removing the excited tube.

Hence, we learn that *when an electrified body is brought near an object it induces the opposite kind of electrification on the side next it and the same kind on the remote side.*

395. The Inducing Charge equal to the Induced Charge. — Experiment. — Support a metallic vessel, like the one shown in Fig. 226, on a glass plate and connect it with the knob of an electroscope by a fine wire. Attach a silk thread to a metallic ball about an inch in diameter, and charge the ball, holding it by the silk thread. Lower the charged ball into the insulated vessel and observe that the leaves of the electroscope diverge as the ball enters the vessel. The divergence increases till the ball has been lowered perhaps two inches below the top, and then remains the same, even when the ball touches the bottom and communicates its charge to the insulated vessel. Suppose the ball charged positively; it induces a negative charge on the interior of the vessel and repels a positive charge to the outside. This positive charge is equal to the charge on the ball, for the divergence of the leaves does not change when the ball gives up its charge to

the vessel. The charge on the ball neutralizes the equal negative charge on the interior, leaving the equal positive charge on the exterior.

Discharge the electroscope and charge the ball a second time. After it has been lowered into the insulated vessel without touching it, place the finger on the ball of the electroscope; the leaves will collapse. Remove the finger and lift the ball by the silk thread; the leaves will again diverge. Lower the ball again till it touches the vessel, and the leaves will again collapse. The charge induced on the inside is exactly neutralized by the inducing charge on the ball.

Hence, *the induced and the inducing charges are equal to each other.*

396. Charging an Electroscope by Induction.—Experiment.—Hold one finger on the ball of the electroscope and bring near it an electrified glass tube. Remove the finger before taking away the tube and the electroscope will be charged. Explain. What kind of electrification will then be on the electroscope? How can you modify the intensity of the charge?

VII. ELECTRICAL DISTRIBUTION.

397. The Charge on the Outside of a Conductor.—

Experiment.—Place a cylindrical metallic vessel of about one litre capacity on an insulated support (Fig. 226). A vessel free from sharp edges should be selected. Electrify strongly and test in succession both the inner and the outer surface, using a proof-plane to convey the charge to the electroscope. It will be found that the inner surface gives no sign of electrification.



Fig. 226.

Hence, it appears that *the electrical charge of a conductor is confined to its outer surface.*

398. Effect of Shape.—**Experiment.**—Charge electrically an insulated egg-shaped conductor (Fig. 227). Touch the proof-plane to the large end, and convey the charge to the electroscope. Notice the amount of separation of the leaves. Test the small end of the conductor in the same way. A greater divergence of the leaves will be observed in the latter case.



Fig. 227.

The distribution of the charge is, therefore, affected by the shape of the conductor, the surface density being greater the greater the curvature. By *surface density* is meant the quantity of electrification on a unit area of the surface of the conductor. The experiment shows that the surface density is greatest at the small end of the conductor.

399. Effect of Area.—**Experiment.**—Employing an electroscope provided with a disk instead of a ball, or with an insulated disk connected with the ball by a conductor, place on the disk a chain, and charge the electroscope by induction so that the leaves diverge widely. Now lift the chain by a dry glass rod, so as to increase the surface of the conductor. The leaves of the electroscope will slowly collapse. When the chain is lowered they will again diverge.

Hence, *with a given charge, the larger the surface the smaller the surface density.*

400. Action of Points.—**Experiment.**—Cement the middle of a sewing-needle to a stick of sealing-wax, as an insulator, so that the needle and the wax form a T. Place the eye end of the needle against the ball of the electroscope, and hold over and near its point an excited glass tube. When both are removed to a distance, the leaves of the electroscope remain separated. The approach of the excited tube will increase their divergence, showing that they are positively charged.

Discharge the electroscope and hold the eye end of the needle against the brass lower down where it joins the glass of the instrument. Now bring the excited glass tube near the top of the ball. When the tube is withdrawn the leaves will again diverge. The divergence decreases upon the approach of the excited tube, showing that the electroscope is now charged negatively. In the first case the attracted negative passed off from the electroscope by the point of the needle, leaving the leaves with a positive charge. In the second case the repelled positive was discharged into the air by the needle point, leaving the electroscope with a negative charge.

From these experiments it appears that electrification is discharged by pointed conductors. This conclusion might have been drawn from § 398. When the curvature becomes very great and the surface assumes the shape of a point, the surface density becomes very great. The particles of air near the point become highly charged and are then repelled. The charge is thus conveyed away, and the current produced is called an *electric wind* (§ 417).

VIII. ELECTRIC POTENTIAL AND CAPACITY.

401. The Unit of Electrification or Charge.—The electrification of a conductor is capable of exact measurement, though it is known only by the phenomena it presents. The unit of electric quantity may be readily defined. Imagine two minute bodies similarly charged with equal quantities. They will repel each other. If the two equal and similar charges are one centimetre apart in air, and if they repel each other with a force of one dyne, then the charges are both unity. *The electrostatic unit of charge is that quantity which will repel an equal and similar quantity at a distance of one centimetre in air with a force of one dyne.* It is necessary to say "in air" because, as will be seen later, the force between two charged bodies depends on the nature of the medium between them (§ 407).

402. Coulomb's Law. — Coulomb demonstrated that the force exerted on each other by two charged conductors, the size of which is very small in comparison with the distance between them, is directly proportional to the product of the charges, and inversely proportional to the square of the distance between them.

Let q and q' be the number of units of charge on the two bodies respectively, and let d be the distance between them in centimetres. Then the force which they exert on each other in dynes is given by the equation

$$F = \frac{qq'}{d^2}. \quad (28)$$

If the charges are of the same sign, the force will be a repulsion; if they are of opposite sign, it will be an attraction.

403. Difference of Potential. — Imagine two conductors similarly charged. Work will have to be done to bring one of them nearer the other. The force worked against is the force of repulsion between like charges. *The difference of potential between the two conductors is the work required to transfer a unit quantity of electrification from one of the conductors to the other.*

If two similar conductors, which are unequally charged, are brought into contact, or are connected by a wire, a part of the charge on the conductor with the higher charge will pass to the one with lower charge till equilibrium is established. The conductor which parts with some of its charge is said to be of the higher potential. Difference of electric potential is, therefore, like difference of temperature for heat, or difference of level for water. As heat flows from bodies of higher to bodies of lower tempera-

ture, and as water flows from a higher to a lower level, so positive electrical charges flow from conductors of higher to conductors of lower potential.

404. Zero Potential. — In measuring the potential difference between a conductor and the earth, the potential of the earth is assumed to be zero. The potential difference is then numerically the *potential of the conductor*. If a conductor of positive potential be connected with the earth by an electric conductor, the positive charge will flow to the earth. If the conductor has a negative potential, the flow of the positive quantity will be in the other direction.

405. Electrostatic Capacity. — **Experiment.** — Suspend a small, smooth metallic ball by a silk thread. Charge a gold-leaf electroscope till the leaves diverge widely. Bring the small ball in contact with the knob of the electroscope; the leaves will partly collapse, showing that the potential has been lowered.

If the charge on an insulated conductor is doubled, the force on a unit charge, anywhere in the neighborhood of the conductor, will also be doubled. The work required to bring a unit charge from the earth, whose potential is zero, to the conductor will then be doubled also. This means that the potential of the conductor is doubled by doubling the charge. There is, therefore, a constant ratio between the charge on a conductor and its potential. This ratio is its *electrostatic capacity*. In symbols, if a charge Q raises a conductor to the potential V , its capacity is

$$C = Q/V. \quad (29)$$

An equivalent definition of *capacity* is the charge required to raise the potential of a conductor from zero to unity. From (29) $Q = CV$, and $V = Q/C$.

Electric potential is analogous to pressure in a tank containing a gas. The capacity of the tank is the quantity of gas which it holds when the pressure is one atmosphere. The whole quantity of gas is proportional to the pressure, or $Q = CP$, and $C = Q/P$, relations precisely like those expressed by equation (29).

The experiment shows that the capacity of a conductor is increased by increasing its surface, since the potential decreased when the surface was augmented, the charge remaining the same. Conversely, with a larger surface a greater charge is required to raise the conductor to the same potential.

406. Condensers. — Experiment. — Support a metal plate in a vertical position by means of two silk cords. Connect it with the knob of an electroscope by a fine copper wire. Charge the plate till the leaves of the electroscope show a wide divergence. Now bring an uninsulated conducting plate near the charged one and parallel to it. The divergence of the leaves will decrease; remove the uninsulated plate, and the divergence will increase again.

The capacity of an insulated conductor is not dependent on its dimensions alone, but it is increased by the presence of another conductor connected with the earth. The effect of this latter conductor is to decrease the potential to which a given charge will raise the insulated one (§ 405). Such an arrangement of parallel conductors separated by an insulator or *dielectric* is called a *condenser*.

A condenser is a device which greatly increases the charge on a conductor without increasing its potential. In other words, the plate connected with the earth greatly increases the capacity of the conductor.

407. Influence of the Dielectric. — Experiment. — With the apparatus of the last experiment, and with the uninsulated plate at

nient distance from the charged plate and parallel to it, suddenly between the two a cake of clean paraffin as large as the plates or larger, and from 2 to 4 cm. thick. Note that the leaf of the electroscope collapse slightly. Remove the paraffin and the divergence will increase again. A cake of sulphur about 10 cm. thick will produce a marked effect on the divergence of the leaves.

The presence of the paraffin or the sulphur increases the capacity of the condenser and, hence, decreases its potential, the charge remaining the same. Paraffin and sulphur, as examples of dielectrics, are said to have a greater *dielectric capacity* or *dielectric constant* than air. Paraffin has a dielectric capacity from four to five times greater than air.

The Leyden Jar is a common and convenient form of condenser. It consists of a glass jar coated part way up, inside and outside, with tin-foil (Fig. 228). Through the wooden or rubber stopper passes a brass rod, terminating on the outside in a ball and on the inside in a metallic chain which reaches to the bottom of the jar. The tin-foil represents the collecting surface, and the glass represents the electric separating the two conductors.



Fig. 228.

Charging and Discharging Jars.—To charge a Leyden jar connect the outer surface by a metallic conductor, such as a chain, with one pole of an electric machine (§ 414). Place the ball in contact with the other pole, at the same time turning the machine for a few minutes. To discharge a Leyden jar connect a wire into the form of the letter V. With one end

of the wire touching the *outer* surface of the jar (Fig. 229), bring the other around near the ball, and the discharge will take place.



Fig. 229.

410. Action of Jar Explained.

— When a positive charge is communicated to the inner surface of a Leyden jar it acts inductively through the glass, attracting to the outer surface an equal negative charge and repelling positive to the earth. These two charges act inductively on each other through the glass and are said to be “bound,” in distinction from the condition where an electrified conductor is at some distance from any other conductor, in which case the whole charge is “free.” When the outer surface of the jar is connected with the earth, the electrical capacity of the inner coating of the jar is largely increased. (Why?)

411. Seat of Charge. — Experiment. — Charge a Leyden jar made with movable metallic coatings (Fig. 230) and set it on an insulating stand. Lift out the inner coating, and then, taking the top of the glass vessel in one hand, remove the outer coating with the other. The coatings now exhibit no sign of electrification. Bring the glass vessel near a pile of pith-balls; they will be attracted to it, showing that the glass is electrified. Reach over the rim with the thumb and forefinger and touch the glass. A slight discharge may be heard. Now build up the jar by putting the parts together; the jar will still be highly electrified and may be discharged in the usual way.



Fig. 230.

This experiment, due to Franklin, shows that the electrification is a phenomenon of the glass. Faraday proved that during the act of charging the jar the glass is strained, the office of the conductors being to facilitate the release from strain. This view is supported by the facts that thin jars can be broken by overcharging; that a jar enlarges on charging; that on heating a jar its charge disappears; and that on charging a jar heavily and then discharging in the usual way, a second charge accumulates after a few minutes, the time being lessened by tapping the jar. The glass acts as if it were strained or distorted to so great a degree that, like a twisted glass fibre, it does not return at once to its normal state when released. This second charge is called the *residual charge*.

Questions and Problems.

1. Stand a charged Leyden jar on a cake of paraffin. Touch the ball with the finger; it gives a feeble spark, but it is not discharged. Why?

2. Connect the inner surface of an uncharged Leyden jar to an electroscope, insulate the outer surface from the earth, and by means of a proof plane impart a small charge to the inner surface. Notice the violent separation of the leaves of the electroscope. Now discharge the jar and connect the outer surface to the earth. Notice that a similar charge imparted to the inner coating does not affect the electroscope as violently as before. Explain.

3. Given a glass rod and a piece of silk, show how to charge an electroscope either positively or negatively, as may be desired. Explain.

4. When a piece of metal is brought near the ball of a charged electroscope the divergence of the leaves diminishes, but increases to its first value when the metal is removed. Explain.

5. Why do two oppositely charged metallic plates show a lower degree of electrification when close together than when far apart?

IX. ELECTRICAL MACHINES.

412. Friction Machines. — We have thus far refrained from describing any device for obtaining electrification beyond such simple means as rubbing a glass tube with silk or sealing-wax with flannel. The explanation of modern electrical machines involves some knowledge of electrostatic induction; it has therefore been deferred till the subject of induction has been introduced.

The oldest forms of apparatus for producing electrification by mechanical means consisted of a glass cylinder or plate, mounted so as to be capable of rotating, and provided with silk-covered pads pressing against the glass. When the glass was revolved the friction electrified it positively. Opposite the rubbers was an insulated conductor with sharp points just grazing the surface of the glass. The attracted negative charge passed off these points and neutralized the positive on the glass, leaving the insulated conductor positively electrified.

These machines were very unsatisfactory, and they have been long displaced by machines which depend for their action on the inductive influence of an electrified body on an insulated conductor.

413. The Electrophorus. — The simplest induction (or influence) electrical machine is the *electrophorus*, invented by Volta. It is shown in Fig. 231. A cake of resin or disk of vulcanite rests in a metallic base. Another metallic disk or cover is provided with an insulating handle. The resin or vulcanite is electrified by rubbing with dry flannel or striking with a catskin, and the metal disk is then placed on it. Since the cover touches the non-conducting sole in a few points only, the negative charge

due to the friction, is not removed. The two disks with the film of air between them form a condenser of great capacity. Touch the cover momentarily with the finger, and the repelled negative charge passes to the earth, leaving the cover at zero potential. Lift it by the insulating handle, the positive charge becomes free, and a spark may be drawn by presenting the finger. This operation may be repeated an indefinite number of times without reducing the charge on the vulcanite.



Fig. 231.

When the cover under inductive influence from the base is touched, it possesses no potential energy. (Why?) But when it is lifted by the insulating handle, work is done against the electrical attraction between the negative charge on the vulcanite and the positive on the cover. The energy of the charged cover represents this work. The electrophorus is, therefore, a device for the continuous transformation of mechanical work into the energy of electric charges.

414. Influence Electrical Machines.—The influence or induction electrical machine has undergone many modifications since its first introduction. It must suffice here to describe only one machine.

The Holtz machine, as modified by Toepler and Voss,

is illustrated in Fig. 232. There are two glass plates, e' and e , about 5 mm. apart, the former stationary and the latter turning about an insulated axle by means of the handle h and a belt. The stationary plate supports at the back two paper sectors, c and c' , called *armatures*. Underneath them are disks of tin-foil connected by a narrow strip of the same material. The disks are electrically connected with two bent metal arms, a and a' (opposite a), which carry tinsel brushes long enough to rub against low

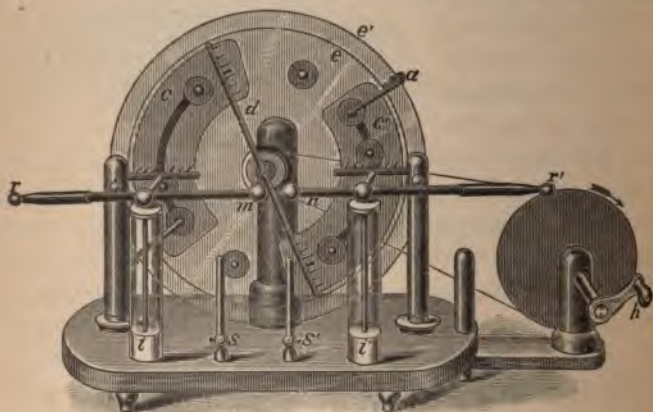


Fig. 232.

brass buttons cemented to small tin-foil disks, called *carriers*, on the front of the revolving plate. Opposite the paper sectors and facing them are two combs with several sharp-pointed teeth set close to the revolving plate, but not touching the metal buttons and carriers. The diagonal neutralizing rod d has tinsel brushes in addition to the combs. The two insulated conductors, terminating in the balls, m and n , have their capacity increased by connection with the inner coating of two

small Leyden jars, i and i' ; the outer coatings are connected under the base of the machine.

415. Action of the Machine. — The operation of all influence machines depends on the employment of a small initial charge to act inductively on the conductors or carriers. The attracted charges pass off by the sharp points of the combs or by the tinsel brushes, and are conveyed away by the carriers and the revolving plate. They go to increase the initial charges and to electrify the insulated conductors.

Suppose the two armatures slightly charged, the one on the left positively. (There is usually enough excitation due to friction, or to the contact of dissimilar substances, to furnish the very slight initial charges.) The brushes on the neutralizing rod d are set so as to connect two carriers at opposite ends of the rod just before they pass beyond the influence of the armatures, c and c' . They thus acquire by induction negative and positive charges respectively, which they carry forward till they are brought into momentary connection with the armatures by means of the brushes and the bent rods a and a' . They then deliver to them their small charges. This action is repeated by each pair of carriers twice during each revolution. In this way the armature c becomes more highly positive and c' more highly negative. When the armatures are highly electrified the carriers do not give up their entire charge to them; the collecting combs attached to the rods m and n receive the residue, in addition to the charges carried on the glass. The positive charge on m and the negative on n increase till a spark passes between the balls, or till further accumulation is prevented by leakage.

X. EXPERIMENTS WITH ELECTRICAL MACHINES.

416. Attraction and Repulsion. — Experiment. — Charge the electrophorus as directed. Before lifting the plate, place a handful of small bits of paper on it. Why do they fly off when the plate is lifted?

Experiment. — Support a metallic plate on a block of wood; on it place the glass cylinder of § 152, and let a second metallic plate rest on this glass. Connect the bottom plate to one conductor of the induction machine and the top one to the other. In the glass vessel put a handful of pith-balls. Work the machine, and account for the dancing of the pith-balls.

Experiment. — Support a small glass funnel, having an aperture of about one-eighth of an inch, in some suitable way, and fill it with fine dry sand. Notice that the sand runs out in a smooth stream. Pass one end of a wire into the sand in the funnel, and connect the other end with one of the conductors of an electrical machine. Set the machine in action, and observe that the sand of the escaping stream scatters. Explain.

417. Discharging Points. — Experiment. — Suspend two pith-balls side by side from one of the conductors of an electrical machine. When the plate is turned the balls separate widely. Why? Now hold a pointed rod near the conductor; the balls drop, showing that the conductor is discharged. Explain.

Experiment. — Fasten a cambric needle to one of the conductors of an electrical machine, so that the point projects. Try to charge the machine. Account for the failure to obtain sparks between the conductors. Hold the flame of a candle near the point. It is driven away as by a wind. Account for this air current.

Experiment. — Connect an *electric tourniquet* (Fig. 233) to one of the conductors of an electrical machine, the other one being grounded. Notice the shape of its arms. Work the machine; the whirl rotates rapidly. Why?

Experiment. — Obtain two smooth round metal plates 10 or 15 cm. in diameter. Lay



Fig. 233.

them on the top of a tumbler, and support the other a few centimetres above the first by means of a stick of sealing-wax cemented to the centre. Connect the

plates by wires or with the positive and negative conductors respectively of an electrophorus. Place a small piece of metal like a collar button on the lower plate. When the machine is worked, the metal will pass across between the elevated metal

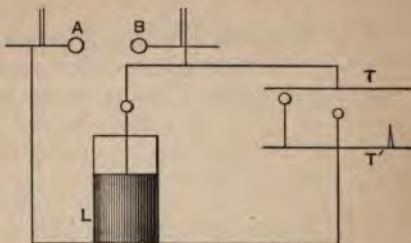


Fig. 234.

upper plate. They will be heavier and brighter if the plates are connected, the one with the outer and the other with the inner of a Leyden jar (Fig. 234).

Run a pin through a small cork so that it will stand on the lower plate with its point up. No sparks now pass across the space between the insulated plates. The sharp pin protects the projecting metal even when it is higher than the pin.

Mechanical Effects of a Discharge. — **Experiment.** — Hold a piece of cardboard between the discharging balls of an electrical machine. It will be perforated by a spark and the hole will be burred on both sides. A thin dry plate of glass, or a thin test-tube with a sharp point, may be perforated by a very heavy discharge.

Heating Effects. — **Experiment.** — Charge a Leyden jar. Heat its outer surface with a gas-burner by a chain or wire. Turn off the gas and bring the ball of the Leyden jar near enough to the flame of the burner for a spark to pass. The gas will be lighted by the charge. Explain.

Experiment. — Ignite ether in an iron spoon by proceeding as in the last experiment.

Experiment. — Make a torpedo as follows: Fit corks to the ends of a glass tube, about 5 cm. long and 1 cm. diameter. Through one of the corks thrust two pieces of copper wire, the ends within the tube not touching. Fill the tube with fine gunpowder, close it, and place

at a safe distance. Discharge a heavily charged Leyden jar through the two copper wires, a piece of wet string being included in the discharge circuit.

420. Magnetic Effect. — **Experiment.** — Make a helix of copper wire insulated with gutta-percha by winding it half a dozen times round a lead pencil. Place inside the helix an unmagnetized sewing-needle, and discharge a large Leyden jar through the helix. The needle will be magnetized. Compare the polarity of the needle with the direction of the discharge round it.

421. Brush Discharge. — When the balls of an electrical machine are separated too far for a spark to pass, the discharge then takes the form of an *electric brush*, accompanied by a characteristic hissing noise. In the dark the brush is seen to consist of innumerable branching streams of pale blue light diverging from a point not far from the metal. The brush appears to tear away metallic particles from the electrode, and to form more readily at the negative pole than at the positive. It is also brighter, smaller, and less finely divided at the negative pole.

Omni
XI. ATMOSPHERIC ELECTRICITY.

✓ **422. Lightning.** — It was demonstrated by Franklin in 1752 that lightning is identical with the electric spark. He sent up a kite during a passing storm, and found that as soon as the hempen string became wet, long sparks could be drawn from a key attached to it, Leyden jars could be charged, and other effects characteristic of static electrification could be produced.

423. Lightning Flashes are discharges between oppositely charged conductors. They occur either between two clouds or between a cloud and the earth. The rise of potential in a cloud causes a charge to accumulate on

earth beneath it. If the stress in the air reaches a limiting value, the air breaks down, or is ruptured, like any other dielectric, and the two opposite charges unite in a long zigzag flash. This occurs when electric tension reaches about 400 dynes per square metre. A lightning flash allows the strained medium to turn to equilibrium.

4. **Atmospheric Electrification.** — No satisfactory explanation of the cause of atmospheric electrification has been given. It has been ascribed to evaporation and action between solid and liquid particles.

The potential of the air in clear weather is generally low, and it is sometimes nearly as high as during a storm, but it shows smaller fluctuations. The observations at the Blue Hill Observatory near Boston show an average potential difference between the earth and the air of 40 volts (§ 465) for an elevation of 138 m., or nearly 0.3 volts per metre. This is equivalent to 0.00013 electrostatic unit (§ 403) per centimetre of elevation. During thunder-storms this potential difference may amount to 400 volts per metre. It is possible to obtain sparks from a cloudless sky when the exploring apparatus is at an elevation not exceeding 500 m.

5. **Thunder.** — A flash of lightning ruptures the air and heats it along the path of the discharge, producing a sudden expansion equivalent to a partial vacuum. Since the atmospheric pressure on the walls of this opening in the air is about 15 pounds per square inch, they come together with a violent crash. At a distance the direct report is mingled with its echoes from the clouds and the earth, producing a long series of reverberations of distant thunder.

426. Oscillatory Discharge.—When a Leyden jar is highly charged, the potential difference between its coatings increases till the dielectric between the discharge terminals suddenly breaks down and a spark passes. This discharge usually consists of several oscillations or to-and-fro discharges, like the vibrations of an elastic system, or the surges of a mass of water after sudden release from pressure. Imagine a tank with a partition across the middle and filled on one side with water. If a small hole be made in the partition near the bottom, the water will slowly reach the same level on both sides without agitation; but if the partition be suddenly removed, the first violent subsidence will be succeeded by a return surge, and the to-and-fro motion of the water will continue with decreasing violence till the energy is all expended.

A series of similar surges occurs when a condenser is suddenly discharged by the breaking down of the dielectric. The oscillatory character of such electric discharges was discovered by Joseph Henry in 1842. Its importance has been recognized only in recent times. Similar electric oscillations probably take place in some lightning flashes.

427. The Aurora.—The *aurora* is due to silent discharges in the upper regions of the atmosphere. Within the arctic circle it occurs almost nightly, and sometimes with indescribable splendor. The illumination of the aurora is due to positive discharges passing from the higher regions of the atmosphere to the earth. In our latitude these silent streamers in the atmosphere are infrequent. When they do occur they are accompanied by great disturbances of the earth's magnetism and by earth currents. Such magnetic disturbances sometimes occur at the same time in widely separated portions of the earth.

XII. ELECTRIC CURRENTS.

428. An Electric Current. — When a condenser is discharged through a wire, there is produced in and around the wire a state called an *electric current*. Electrification is a condition of strain in the dielectric; the electric current rapidly relieves this strain through the discharging inductor. If the state of strain is reproduced by the generator "as fast as it is relieved by the conductor, the result is a continuous current. To accomplish this result work must be done, and therefore an electric current presents energy. The expression, "current of electricity," was introduced when electricity was regarded as a fluid which flowed from higher to lower potential through a wire, just as water flows through a pipe from a higher to a lower level. So far as we know, however, the only thing transferred is energy, and the belief is growing that the energy is not transmitted by the wire at all, but by the ether surrounding the wire. However that may be, a uniform electric current through a conductor requires the maintenance of a constant potential difference between its terminals. One of the simplest means of doing this is the primary cell or battery.

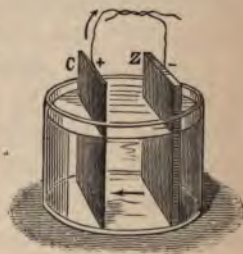


Fig. 235.

429. The Simple Voltaic Cell. — **Experiment.** — Cut a strip of dry sheet zinc and one of sheet copper, each about 10 cm. long and 1 cm. wide. Scour the zinc with emery paper till it is bright. Support these strips side by side in a glass vessel nearly full of dilute sulphuric acid (one part acid to twenty of water). When the strips are brought together, innumerable bubbles of gas will rise from the

copper strip, and some also from the zinc. This gas is hydrogen. Remove the copper strip, or do not allow the two to touch, and the chemical action is much diminished. If a little mercury be now rubbed on the zinc, no gas will be given off by it; but if the upper ends of the two strips be connected by any good conductor (Fig. 235), gas will again come off freely from the copper. This action will cease if the connection be made by any non-conductor. If the action is continued for some time, the zinc will be found to waste away, while the copper is unaffected.

Such a combination of two conductors, immersed in a compound liquid, called an *electrolyte*, which is capable of reacting chemically with one of the conductors, is called a *voltaic cell* or *element*. The name is derived from Volta of Padua, who first described such a cell in 1800.

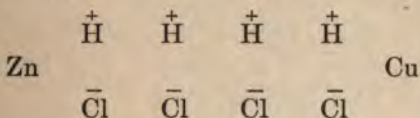
A sufficiently sensitive test shows that the copper strip or plate is positive and the zinc negative. A potential difference is therefore established between the plates by immersion in the acid solution. The copper plate is called the *positive electrode* or the *cathode*, and the zinc the *negative electrode* or the *anode*. A current always leaves the solution by the *cathode* (way down or out).

430. The Circuit of a voltaic cell comprises the entire path traversed by the current, including the electrodes and the liquid in the cell as well as the external conductor. *Closing the circuit* means joining the two electrodes by a conductor; *breaking* or *opening the circuit* is disconnecting them. The flow of current in the external circuit is from the positive electrode (copper) to the negative (zinc), and in the internal part of the circuit from the negative electrode to the positive (Fig. 235).

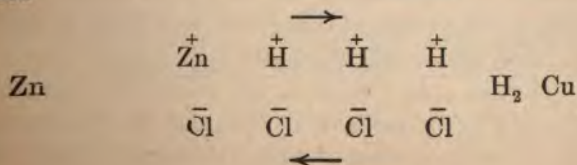
431. Electrochemical Actions in a Voltaic Cell.—The modern theory of dissociation furnishes an explanation

of the manner in which an electric current is conducted through a liquid. It is briefly as follows: When a salt or an acid, such as hydrochloric acid (HCl), for example, is dissolved in water, some of the molecules at least split into two parts (H^+ and Cl^- , for example), one part having a positive electrical charge and the other a negative one. The two parts of the dissociated substance with their electrical charges are called *ions*. An electrolyte is a compound capable of such dissociation into ions. It conducts electricity only by means of the migration of the ions resulting from the splitting in two of the molecules. The separated ions convey their charges with a slow and measurable velocity through the liquid. Electropositive ions, such as zinc and hydrogen, carry positive charges in one direction, electronegative ions, such as chlorine and "sulphion" (SO_4), carry negative charges in the opposite direction, and the sum of the two kinds of charges carried through the liquid per second is the measure of the current.

The active components in a simple voltaic cell set up with hydrochloric acid may be represented as follows:—



Immediately after the circuit has been closed this becomes



Zinc goes into solution as zinc chloride (ZnCl_2), and hydrogen appears as free hydrogen gas at the copper plate. Zinc ions crowd out hydrogen ions, while the positive and negative charges brought to the copper and the zinc plate respectively reunite as a current through the external conductor.

432. Electromotive Force. — A voltaic cell is an electric generator. It is analogous to a rotary pump which produces a difference of pressure between its inlet and its outlet. Such a pump may cause water to circulate through a system of horizontal pipes against friction. In any portion of the pipe system the force producing the flow is the difference of water pressure between those points. But the force is all applied at the pump, and this produces a pressure throughout the whole circuit.

A voltaic cell generates electric pressure called *electromotive force*. It does not generate electricity, but it supplies the electric pressure to set electricity flowing. This electromotive force (E.M.F.) is numerically equal to the work which must be done to transport a unit quantity of electricity entirely round the circuit. Work is required to effect this transfer, because all conductors offer resistance to the passage of a current. The energy thus expended goes to heat the conductor.

The difference of potential between two points on the external conducting circuit is the work done in carrying a unit quantity of electricity from one point to the other. If E denotes this potential difference and Q the quantity conveyed, then the whole work done is the product EQ . But the quantity conveyed by a conductor per second is called the *strength of current*, I . The energy transformed, therefore, when a current I flows through a conductor,

ider an electric pressure or potential difference of E its between its ends, is EI ergs per second.

433. Detection of Current. — **Experiment.** — Solder a copper e to each of the strips of a voltaic cell, and connect the wires h some form of key to close the circuit. Stretch a portion of the e over a mounted gnetic needle (g. 236), holding parallel to it and near as possible thout touching. w close the circit, and observe at the needle is flected; after a

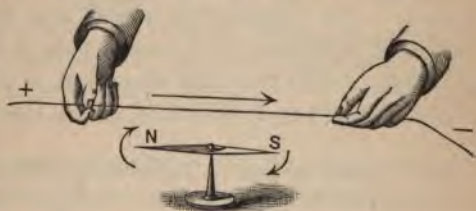


Fig. 236.

w oscillations it comes to rest at an angle with the wire. Next rm a rectangular loop of the wire, and place the needle within it. greater deflection is now obtained. If a loop of several turns is rmed, the deflection is still greater. A magnetic needle employed this way becomes a *galvanoscope*, a detector of electric currents.

This experiment, first performed by Oersted in 1819, ows that the region round the wire has magnetic prop- ties during the flow of electricity through it. In other rds, it is a magnetic field (§ 375). Water flowing rough a pipe produces no disturbance in the region und it, corresponding to the magnetic field round a con- ctor conveying a current of electricity. The analogy tween a current of water and a current of electricity ls therefore in this respect.

434. Relation between the Direction of the Current and the rection of Deflection. — **Experiment.** — Making use of the appa- us of § 433, compare the direction of the current through the wire h that in which the north pole of the needle turns. Cause the cur- t to pass in the reverse direction over the needle; the deflection is

reversed. Now hold the wire below the needle, and the direction of deflection is again reversed.

The direction of the deflection may always be predicted by the following rule: *Stretch out the right hand along the wire, with the palm turned toward the magnetic needle, and with the current flowing in the direction of the extended fingers. The outstretched thumb will then point in the direction of deflection of the north pole of the needle.* By the converse of this rule, the direction of the current may be inferred from the direction in which the needle is deflected.

435. Local Action.—**Experiment.**—Place a strip of commercial zinc in dilute sulphuric acid. Hydrogen is liberated during the chemical action, and after a few minutes the zinc becomes black from particles of carbon exposed to view on dissolving away the surface. If the experiment is repeated with zinc amalgamated with mercury, there will be little or no chemical action.

This experiment shows that the amalgamation of commercial zinc with mercury changes its properties. If in the experiment with the simple voltaic cell, a galvanoscope is inserted in the circuit both before the zinc has been amalgamated and afterward, it will be found that a larger deflection will be obtained in the second case.

The chemical action going on in a voltaic cell which contributes nothing to the current flowing through the circuit is known as *local action*. It is probably due to the presence of carbon, iron, etc., in the zinc; these with the zinc form miniature voltaic cells, the currents flowing round in short circuits from the zinc through the liquid to the foreign particles and back to the zinc again.

This local action is prevented by amalgamating the zinc; that is, by coating it with an alloy of mercury and zinc. The amalgam brings pure zinc to the surface, covers the foreign particles, and above all forms a smooth

surface, so that a film of hydrogen clings to it and protects it from chemical action save when the circuit is closed.

436. Polarization. — **Experiment.** — Connect the poles of the voltaic cell with a galvanoscope and note the deflection. Let the cell remain in circuit with the galvanoscope for some time, the deflection will gradually become less and less. Now stir up the liquid vigorously with a glass rod, inserting the rod between the plates and rushing off the adhering gas bubbles; the deflection will increase to nearly its original amount.

The diminution in the intensity of the current is due to several causes, but the chief one is the film of hydrogen which gathers on the copper plate, causing what is known as the *polarization* of the cell. The hydrogen on the positive plate not only introduces more resistance to the flow of the current, by diminishing the available surface of copper, but it diminishes the electromotive force to which this flow is due. The presence of hydrogen on the copper plate sets up an inverse E.M.F., which either reduces or stops the flow of current.

437. Remedies for Polarization. — **Experiment.** — Place enough pure mercury in a quart jar to cover the bottom, and hang above it a piece of sheet zinc. Fill the jar with a nearly saturated solution of salt water, and place in the mercury the exposed end of a copper wire insulated with gutta-percha, the upper end forming the positive pole of the battery.

If now the circuit is closed through a telegraph sounder (§ 510) of ten or fifteen ohms resistance, the armature will at first be attracted strongly; but in the course of a few minutes it will be released and will be drawn back by the spring. Polarization has then set in to the extent that the current is insufficient to operate the instrument.

Next take a small piece of mercuric chloride (HgCl_2) no larger than the head of a pin, and drop it in on the surface of the mercury.

The armature of the sounder will instantly be drawn down, showing that the current has recovered its normal value. The hydrogen has been removed by the chlorine of the mercuric chloride. In a few minutes the chlorine will be exhausted, and polarization will again set in. A little more of the chloride will again restore the activity of the cell.

This experiment illustrates a chemical method of reducing polarization. Means are adopted to replace the hydrogen ions with others, such as copper or mercury, which do not produce polarization when they are deposited on the positive electrode; or else the positive electrode is surrounded with a chemical which furnishes oxygen or chlorine to unite with the hydrogen before it reaches the electrode. In both cases the electrode is kept nearly free from hydrogen.

438. The Daniell Cell illustrates the first of these chemical methods of preventing polarization.

In its most common form (Fig. 237) it consists of a glass jar containing a saturated solution of copper sulphate (CuSO_4), and in it a cylinder *C* of copper, which is usually cleft down one side. Within the copper cylinder is a porous cup of unglazed earthenware containing dilute sulphuric acid, or preferably a dilute solution of zinc sulphate (ZnSO_4). The porous cup contains also the zinc prism *Z*. The porous

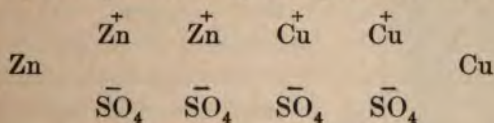


Fig. 237.

cup allows the ions to pass through its pores, but it prevents the rapid admixture of the two sulphates. The copper sulphate must not be allowed to come in contact with the zinc electrode.

th sulphuric acid round the zinc, the hydrogen ions intercepted at the porous cup by the copper sulphate. Positive copper ions then migrate toward the copper electrode and are there deposited as metallic copper. SO_4 ions go to the zinc electrode with their negative charges, as in the case of the simple voltaic cell.

When the zinc is immersed in zinc sulphate, then both sulphates undergo partial ionization or dissociation, and there are free hydrogen ions. Only zinc and copper ions travel toward the copper electrode. Zinc ions never reach the copper, because zinc in copper sulphate invariably replaces copper, forming ZnSO_4 in place of the CuSO_4 . Chemically the operation may be represented as follows:—



It is easy to see that as soon as the circuit is closed, ZnSO_4 will be formed at the zinc electrode, and copper will be precipitated on the copper electrode. At the same time there is a loss of copper sulphate corresponding exactly to the increase in zinc sulphate. Polarization is thereby obviated in the Daniell cell, and it is one of the most convenient elements yet devised.

The Gravity Cell (Fig. 238)
Modified Daniell. The porous cup is omitted, the partial separation of the liquids being secured by difference in density. The copper electrode is placed at the bot-

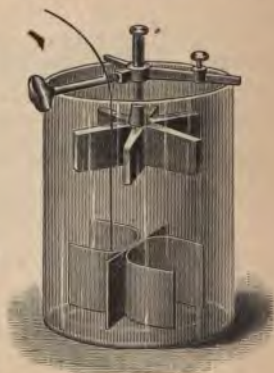


Fig. 238.

tom in saturated copper sulphate, while the zinc is suspended near the top in a weak solution of zinc sulphate, floating on top of the copper sulphate. The zinc should never be placed in the solution of copper sulphate. The saturated copper sulphate is more dense than the dilute zinc salt, and so remains at the bottom, except as it slowly diffuses upwards.

omit
440. The Bunsen Cell (Fig. 239) consists of a glass jar containing dilute sulphuric acid, and a hollow cylinder of zinc immersed in it. Within the zinc cylinder is a porous cup containing strong nitric acid and a prism of compressed carbon.

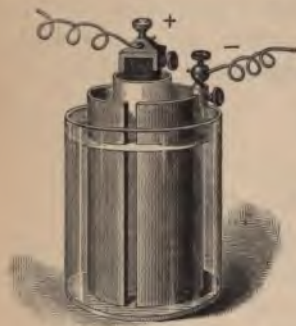


Fig. 239.

The hydrogen ions from the sulphuric acid are intercepted on their way to the positive electrode by oxygen from the nitric acid, and are oxidized with the production of water.

The nitric acid molecule is broken up and yields a brownish red gas, which is very corrosive and poisonous.

The Grove cell differs from the Bunsen in no respect except that the positive electrode is a sheet of platinum instead of a prism of carbon.

omit
441. The Chromic Acid Cell usually consists of a plate of zinc between two carbon plates dipping into a glass vessel containing dilute sulphuric acid, to which is added either chromic acid or the bichromate of potassium or of sodium. The sodium salt is much to be preferred to the potassium

With the bichromates an additional quantity of sulfuric acid is needed to liberate chromic acid.

Fig. 240 illustrates a form of this cell, called the *Grenet* cell, which is very convenient, but is open to the objection, since the carbon plates are left standing in the solution, the liquid soon runs up and attacks the connections at the top, making it difficult to keep the cell in good order. The zinc is attached to a sliding rod, *A*, so that it can be lifted out of the liquid when the cell is not in use.

The hydrogen coming from the ionized zinc is oxidized to water by the chromic acid, and polarization is prevented.

Fig. 241 illustrates a form of chromic acid battery, where the several cells composing it have their carbons and zincs suspended from a frame.

It is known as a *plunge battery*, and is a very convenient form for experimental work.



Fig. 240.



Fig. 241.

Omit
442. The Leclanché Cell (Fig. 242) consists of a glass vessel containing a saturated solution of ammonium chloride (sal ammoniac) in which stands a zinc rod and a porous cup. In this porous cup is a bar of carbon very tightly packed in a mixture of manganese dioxide and graphite, or granulated carbon.



Fig. 242.

The zinc is acted on by the chlorine of the ammonium chloride, liberating ammonia and hydrogen. The ammonia in part dissolves in the liquid, and in part escapes into the air. The hydrogen is slowly oxidized by the manganese dioxide. The cell is not adapted to continuous use, as the hydrogen is liberated at the positive electrode faster than the oxidation goes on, and the cell is polarized. If, however, it is allowed to rest, it recovers from polarization.

XIII. EFFECTS OF ELECTRIC CURRENTS.

a. HEATING EFFECTS.

443. Heating of a Conductor. — **Experiment.** — Close the circuit of a chromic acid battery through a piece of No. 30 platinum wire about 3 cm. long. The wire becomes red hot and possibly may fuse. If copper wire is substituted for the platinum, a smaller change of temperature will be observed.

In a battery, the potential energy of chemical separation is transformed into the energy of an electric current. When the current does no work this energy is all converted into heat in the circuit. The relative amounts of the heat generated in the external circuit and in the battery itself are strictly proportional to the external and internal resistances (§ 460). If in the experiment the

circuit is closed with the fine wire omitted, more heat is generated in the liquid of the battery than with the fine wire in the circuit.

444. Generation of Heat by a Current. — In 1841 Joale demonstrated experimentally that the heat generated by an electric current in any part of a circuit is *proportional to the square of the current, to the resistance of that part of the circuit, and to the time during which the current flows.* *H = I^2 R t*

Let H be the heat in calories, I the current in amperes (§ 464), R the resistance in ohms (§ 461), and t the time in seconds. Then $H = 0.24 I^2 R t$, in which 0.24 is the number of calories in one joule.

The heating effects of an electric current are utilized in firing blasts and cannon, in exploding torpedoes, in cauterizing, in welding, in heating rooms, in cooking, in producing high temperatures for melting refractory substances, and for conducting chemical processes which require extreme heat.

b. CHEMICAL EFFECTS.

445. Electrolysis. — **Experiment.** — Bend a glass tube of about 1.5 cm. diameter and 15 cm. long into a V-form (Fig. 243). Close the ends with corks and thrust through them platinum wires, terminating within the tube in narrow strips of platinum foil. Support the tube in some convenient way after filling it two-thirds full of a solution of sodium sulphate, colored with the extract of purple cabbage. Connect the terminals to the poles of two or three cells joined in series (§ 468). After the circuit has been closed for a few minutes, the liquid around the anode will turn red, showing the presence of an acid, while that around the cathode will turn green, showing the presence of an alkali.



Fig. 243.

The experiment shows that the electric current in its passage through a liquid decomposes it. To this process of decomposing liquids by means of an electric current Faraday gave the name of *electrolysis*; to the substance decomposed, *electrolyte*; to the parts of the separated electrolyte, *ions*. The *anode* is the electrode by which the current enters the electrolytic cell, and the *cathode* the electrode by which it leaves.

446. Electrolysis of Copper Sulphate. — **Experiment.** — Fill the V-tube of the last experiment about two-thirds full of a solution of copper sulphate. After the circuit has been closed a few minutes, the cathode will be covered with a deposit of copper, and bubbles of gas will rise from the anode. These bubbles are oxygen.

When copper sulphate is dissolved in water it suffers dissociation to some extent. If, therefore, electric pressure is applied to the solution through the electrodes, the electropositive ions (Cu^+) are set moving from higher to lower potential, while the electronegative ions (SO_4^-) carry their negative charges in the opposite direction. The Cu ions are therefore driven against the cathode, and, giving up their charges, become metallic copper. The sulphions (SO_4^-) go to the anode; and, giving up their charges, they take hydrogen from the water present, forming sulphuric acid (H_2SO_4) and setting free oxygen, which comes off as bubbles of gas. If the anode were copper, the sulphion would unite with it, and copper would then be removed from the anode as fast as it is deposited on the cathode. The result of the passage of a current would then be the transfer of copper from the anode to the cathode. This actually takes place in the electrolytic refining of copper.

It will be seen that the passage of an electric current

through an electrolyte is accomplished in the same way, whether it is in a voltaic cell or in an electrolytic cell.

447. Electrolysis of Water. — Experiment. — Insert two burette tubes into a cork which has been boiled in paraffin, and which fits a wide-mouthed bottle. The glass stopcocks are at the top, as represented in Fig. 244. The thistle-tube is added for convenience in filling. Solder platinum wires to two copper wires, insulated with gutta-percha, the platinum wires terminating in strips of platinum foil. The copper must be completely insulated, leaving only platinum exposed. Insert the wires and the foil as shown in the figure. Open the stopcocks and fill through the thistle-tube with water, acidulated with sulphuric acid, till the solution rises to the stopcocks, which should then be closed. Connect the apparatus by means of the connectors with two or more chromic acid cells joined in series (§ 468). Bubbles of gas will at once begin to rise from the platinum electrodes. After a few minutes the tube over the cathode will be found to contain about twice as much gas as the other one. The gas given off at the cathode is hydrogen and at the anode oxygen. The apparatus is called a *voltameter* or a *coulometer*.



Fig. 244.

During the passage of the current the hydrogen ions, coming from the dissociated sulphuric acid, move with the current and the sulphions (SO_4) against it. The latter release oxygen from the water by taking away hydrogen, precisely as in the electrolysis of copper sulphate. If a copper or brass anode be used, the sulphion will attack it and no oxygen will appear.

448. Laws of Electrolysis. — The following fundamental laws of electrolysis were established by Faraday : —

I. *The mass of an ion liberated is proportional to the quantity of electricity which passes through the electrolyte.*

The mass of an ion liberated in one second is, therefore, proportional to the strength of current.

II. *The masses of different ions liberated per second by the same current are proportional to their chemical equivalents.*

By "chemical equivalents" are meant the relative quantities of the ions which are chemically equivalent to one another, or take part in equivalent chemical reactions. Thus, 32.5 gm. of zinc or 31.7 gm. of copper take the place of one gm. of hydrogen in sulphuric acid (H_2SO_4) to form zinc sulphate (ZnSO_4) or copper sulphate (CuSO_4) respectively.

The first law of electrolysis affords a valuable means of comparing the strength of two electric currents by determining the relative masses of any ion, such as silver or copper, deposited by the two currents in succession in the same time (§ 464).

449. Electroplating consists in covering bodies with a coating of any metal by means of the electric current. The process may be summarized as follows: Thoroughly clean the surface to remove all fatty matter. Attach the article to the negative electrode of a battery, and suspend it in a solution of some chemical salt of the metal to be deposited. If silver, cyanide of silver dissolved in cyanide of potassium is used; if copper, sulphate of copper. To maintain the strength of the solution a piece of the metal of the kind to be deposited is attached to the positive electrode of the battery. The action is similar to that heretofore given. Articles of iron, steel, zinc, tin, and lead cannot be silvered or gilded unless first covered with a thin coating of copper.

450. Electrotyping consists in copying medals, woodcuts, type, and the like in metal, usually copper, by means of the electric current. A mould of the object is taken in wax or plaster of Paris. This is evenly covered with powdered graphite to make the surface a conductor, and treated very much as an object to be plated. When the deposit has become sufficiently thick it is removed from the mould and backed or filled with type-metal.

451. The Secondary or Storage Battery. — **Experiment.** — Connect the apparatus of § 447 to a suitable battery. After passing the current for a short time, causing an evolution of gas, disconnect the battery and put a galvanoscope in its place. The needle will be deflected, showing that a current is now passing through the apparatus in a direction opposite to the battery current.

Experiment. — Support two lead plates with attached copper wires by a strip of wood (Fig. 245), and immerse the plates in dilute sulphuric acid, one part of acid by measure to five of water. Pass a current from a suitable battery through this electrolytic cell for a few minutes, and then disconnect the battery and connect the cell with an electric bell (§ 515). It will furnish for some time a current sufficient to ring the bell. As soon as it is discharged connect again to the battery as before and repeat the operation.

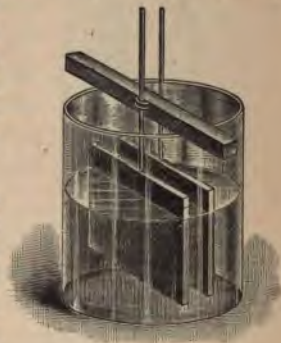


Fig. 245.

The last experiment illustrates the *lead storage battery*. The electrolysis of water liberates oxygen at the anode, which combines with the lead electrode to form a chocolate-colored deposit of lead peroxide (PbO_2). Hydrogen accumulates on the cathode. When the battery is disconnected and the lead plates are joined by a conductor, a

current flows in the external circuit from the oxidized plate, which is called the positive electrode, to the other one, called the negative, the lead peroxide is reduced to spongy lead on the positive plate, while some lead sulphate is formed on the negative. During subsequent charging this lead sulphate is reduced by the hydrogen to spongy lead. Note that the charging current passes through the storage cell in the opposite direction to the discharge current furnished by the cell itself.



Fig. 246.

The storage battery stores energy and not electricity. The energy of the charging current is converted into the potential energy of chemical separation in the storage cell. When the circuit of the charged secondary cell is closed, the potential chemical energy is reconverted into the energy of an electric current in precisely the same way as in a primary cell.

Fig. 246 shows a complete storage cell containing one positive and two negative plates.

C. MAGNETIC EFFECTS.

452. Magnetic Character of the Current. — Experiment. — Connect two or three chromic acid cells in parallel (§ 469). Close the circuit through a heavy wire, and then dip a portion of it into fine iron filings. A thick cluster of them will adhere to the wire (Fig. 247).



Fig. 247.

This experiment illustrates the fact that a conductor through which a current is passing possesses magnetic properties. The iron filings are magnetized by the cur-

rent and set themselves at right angles to the wire. When the circuit is broken, they lose their magnetism and drop off.

453. Magnetic Field about Conductor. — **Experiment.** — Bend a wire *AB* and support it in the shallow vessel nearly full of water, as shown in Fig. 248. Magnetize a sewing-needle and oil it so that it will float on the water. It will turn with its north pole pointing north; but when a current is passed through the wire, it will set itself perpendicular to a line joining the needle and the wire. Reverse the current, and the needle will also reverse its direction.

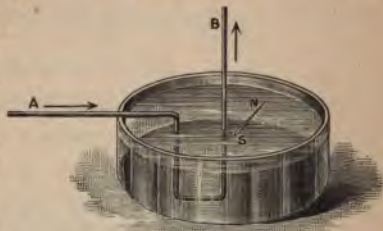


Fig. 248.

Experiment. — Support horizontally a sheet of stiff paper. Pass

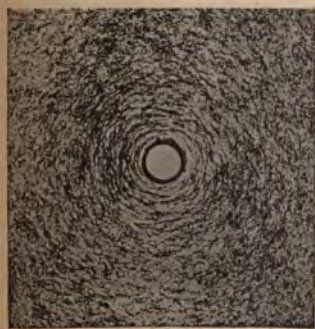


Fig. 249.

vertically through it a wire which connects the poles of a battery of two or more chromic acid cells. Close the circuit and sift a few very fine iron filings on the paper, jarring it slightly with a pencil as they fall. They arrange themselves in circles with the wire at the centre. Place a small mounted magnetic needle on the paper near the wire. The needle sets itself tangent to these circles, and points in the opposite direction to that traversed by the hands of a watch, when the current comes up through the paper from below. (What is the direction of the lines of force?)

These experiments show that the lines of magnetic force about a wire through which an electric current is flowing, are concentric circles. Fig. 249 was made from a photograph of these circular lines of force as shown by

iron filings on a plate of glass. Their direction relative to the current is given by the following rule:—

Grasp the wire by the right hand so that the extended thumb points in the direction of the current; then the fingers indicate the direction of the lines of force round the wire.

Fig. 250 is a sketch intended to show the direction of these circular lines of magnetic force (or magnetic whirl) which everywhere surround a wire conveying a current.



Fig. 250.

454. Properties of a Circular Conductor. — Experiment. —

Bend a piece of No. 16 copper wire into the form shown in Fig. 251, the diameter of the circle being about 20 cm. Suspend it by a long thread, so that the ends dip into the mercury cups shown in section in the lower part of the figure. Send a current through the suspended wire by connecting a battery to the binding posts. A magnet brought near the face of the circular conductor will cause the latter to turn about a vertical axis and take up a position at right angles to the axis of the magnet.



Fig. 251.

This experiment, due to Arago, shows that a circular current acts like a disk magnet, whose poles are its faces. The lines of force surrounding the conductor in this form pass through the circle and come

and from one face to the other through the air outside the loop. The north-seeking side is the one from which the lines issue; and to an observer looking toward this side, the current flows round the loop counter-clockwise (Fig. 252). If instead of a single turn we take a long insulated wire and coil it into a number of parallel circles close together, the magnetic effect will be increased. Such a coil is called a *helix* or *solenoid*; and the passage of an electric current through it gives to it all the properties of a cylindrical bar magnet.



Fig. 252.

455. The Electromagnet. — Experiment. — Wind neatly on a paper tube, about 2 cm. in diameter and 15 cm. long, three layers of No. 18 insulated copper wire. Pass an electric current through it and observe its magnetic properties by bringing it near a mounted magnetic needle. Now fill the tube with straight pieces of soft iron wire and again bring it near the needle. Its magnetic effect will be greatly increased.¹

This device, consisting of a helix encircling an iron core, is called an *electromagnet*. The presence of the iron

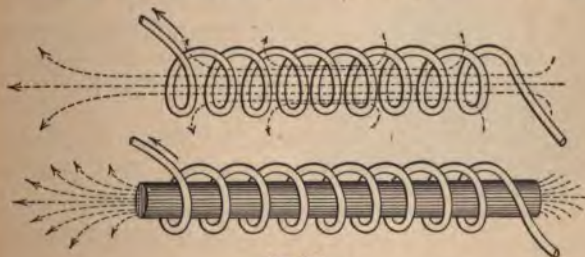


Fig. 253.

¹ The magnetic field in and around a helix may be shown by means of iron filings.

core greatly increases the number of lines of force running through the helix from end to end, by reason of its greater permeability (§ 377) as compared with air (Fig. 253).

When the iron core is not used, many of the lines leak out at the sides of the helix, and but few extend through from end to end. The core not only diminishes the leakage of the lines of force, but also adds many more to those previously running through the solenoid. Hence the magnetic strength of a helix is greatly increased by the iron core.

456. Mutual Relation of the Current and Lines of Force.—

Experiment.—Support a long magnetized steel rod in a wooden clamp



Fig. 254.

(Fig. 254); make a flexible conductor, by twisting together two or three strands of conducting tinsel (which can be bought in skeins), and connect as shown in the figure. When the current from two chromic acid cells, or from a single storage cell, is turned on, the flexible conductor will wind itself round the magnet in a helix. Use some convenient device to reverse the current, and the conductor will unwind and rewind itself in the opposite direction.

Here the conductor tends to wind itself in circles round the lines of force through the long magnet.

If the magnet be grasped by the right hand, with the thumb pointing toward its north pole, the current will be found to flow round in the direction of the fingers (§ 453).

457. The Horseshoe Magnet.—The form given to an electromagnet depends on the use to which it is to be put. The *horseshoe* or *U-shape* (Fig. 255) is the most common. The advantage of this form becomes apparent when attention is directed to the fact that *all magnetic lines are closed curves*; that is, the lines of magnetic force or of magnetic induction are continuous, passing through the core from the south to the north pole, and completing the circuit through the air from the north

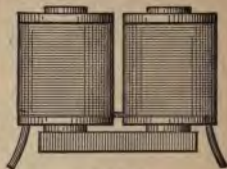


Fig. 255.



Fig. 256.

pole back to the south. Now, the shorter the air path of the magnetic lines, the larger their number and the stronger the magnet. The approach of the two poles in the U-shaped magnet shortens the air gap, and hence increases the number of lines of force.

If then a bar of soft iron, called an *armature*, be placed across the poles, the air gap is fur-

ther diminished when the armature is made to approach the poles (Fig. 256). When the armature is in contact with the poles, the magnetic circuit is all iron, and is said to be a closed circuit. The maximum number of magnetic lines then traverse it (Fig. 257).



Fig. 257.

458. The Polarity of a Solenoid may be determined by the following rule:—

Grasp the coil with the right hand so that the fingers point in the direction of the current; the north pole will then be in the direction of the extended thumb.

In Fig. 258, if the fingers of the right hand are parallel to the arrows and point in the same direction, the extended thumb will point toward *N*, as the north pole of the coil. The converse of the rule enables one to ascertain the direction of the current when the polarity of the solenoid is known.

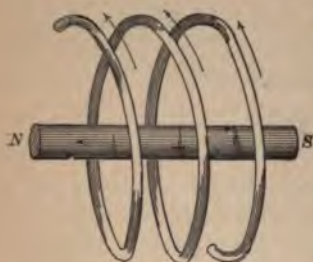


Fig. 258.

459. Mutual Action of Two

Currents. — Experiment. — Construct a rectangular frame, about 25 cm. square, out of insulated copper wire No. 20, by winding four or five layers round the edge of a square board. Slip the wire off the board and tie the parts together in a number of places with thread. Bend the ends at right angles to the frame, remove the insulation, and give them the shape shown in Fig. 259. Suspend the wire frame by a long thread so that the ends dip into the mercury cups.

Wind several layers of the same size of insulated wire round a form about 10 cm. by 20 cm. Connect this coil in the same circuit with the rectangular coil and a battery of two or three cells joined in series.

First. Hold the coil *HK* with its plane perpendicular to the plane of the coil *EF*, with its edge *H* parallel to *F*, and with the currents

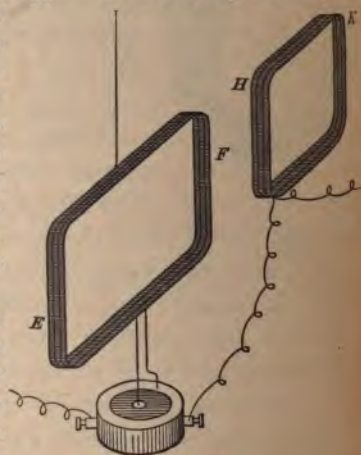


Fig. 259.

two adjacent portions flowing in the same direction. The coil will turn upon its axis, the edge *F* approaching *H*, attraction.

2. Turn *HK* over so that the currents in the adjacent portions flow in opposite directions. The edge *F* of the suspended coil will be repelled by *H*.

Give *HK* a quarter turn around a vertical axis from the position shown in the figure. If it is near *F*, the suspended coil will turn to make the two parallel currents flow in the same direction.

3. Hold the coil *HK* within the rectangular coil *EF*, and edge *H* making an angle with the lower edge of *EF*. The coil will turn till the currents in its lower edge are parallel with *H*, and flowing in the same direction.

The results may be expressed by the following laws between currents :—

Parallel currents flowing in the same direction attract.

Parallel currents flowing in opposite directions repel.

Currents making an angle with each other tend to become parallel and to flow in the same direction.

Ampère included the entire phenomenon under one law, *the two circuits tend to move so that the number of lines common to them shall be a maximum.*

Fig. 260 was obtained from a photograph of the magnetic field around two parallel wires passing through the holes of a glass plate carrying parallel currents in the

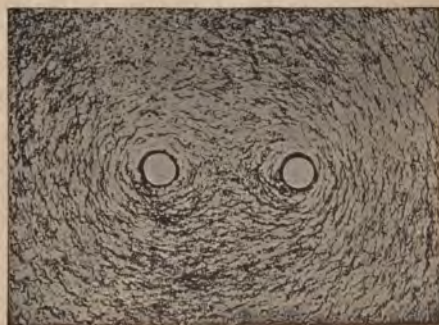


Fig. 260.

same direction. Many of the circular lines of force are extended so as to include both wires, and the tension along them tends to draw the two conductors together.

Figure 261 was made in the same manner from the magnetic field around the two wires when the parallel cur-

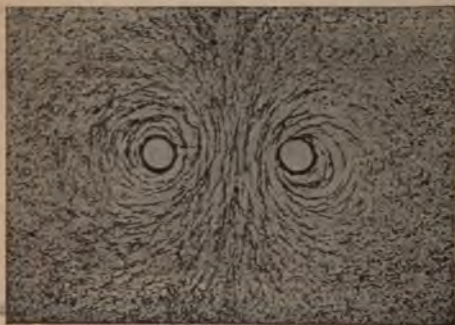


Fig. 261.

rents were in opposite directions. The circular lines of force are crowded together between the wires, and their reaction to recover their normal position forces the wires apart.

XIV. ELECTRICAL QUANTITIES.

a. OHM'S LAW.

460. Resistance. — Frequent mention has already been made of *electrical resistance*, *electromotive force*, and *strength of current*. We shall now consider these quantities more closely, and shall study the relation between them which is known as *Ohm's Law*. This law forms the basis of most electrical measurements of steady currents.

No conducting body possesses perfect conductivity, but every conductor presents some obstruction to the passage of electricity. This obstruction is called its *electrical resistance*. It is the reciprocal of *conductance*. The greater the conductance of a conductor the less its resistance, the one decreasing in the same ratio as the other increases.

461. Unit of Resistance. — The practical unit of resistance is the *ohm*. It was defined by the International Congress of Electricians in Chicago in 1893 as follows: "The international ohm is represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 gm. in mass, of constant cross-sectional area, and of the length 106.3 cm." The cross-sectional area of this thread of mercury is very nearly one square millimetre.

462. Laws of Resistance. — I. *The resistance of a conductor is proportional to its length.* For example, if 39 ft. No. 24 copper wire (B. & S. gauge) have a resistance of 1 ohm, then 78 ft. of the same wire will have a resistance of 2 ohms.

II. *The resistance of a conductor is inversely proportional to its cross-sectional area, and in the case of round wire is inversely proportional to the square of its diameter.* For example, No. 24 wire has a diameter of .0201 inch, and No. 30 has a diameter of .01 inch, or nearly one-half of No. 24. 39 ft. of No. 24 has a resistance of 1 ohm, and 7 ft. of No. 30, which is nearly $\frac{39}{2^2}$, has a resistance also of 1 ohm, both at 22° C.

III. *The resistance of a conductor of a given length and cross-sectional area depends upon the material of which it is made, and is affected by any cause which modifies its molecular condition.* For example, the resistance of 2.2 ft. of No. 24 German-silver wire is 1 ohm, whereas it takes 39 ft. of copper wire of the same diameter to give the same resistance. Moreover, this is true only at a definite temperature; if these substances are heated, the percentage increase of resistance of the copper is nearly ten times as much per degree as that of German-silver, and twenty

times as much as that of an alloy called *platinoid*. All metals have their resistance increased by increase of temperature. Carbon and most electrolytic conductors decrease in resistance as the temperature rises.

Experiment. — Connect the poles of a fresh chromic acid cell with a piece of fine platinum wire of such a length that the current heats it to a dull red color. Now apply a piece of ice to one portion and notice the rise in temperature of the remaining portion. Explain.

463. Formula for Resistance. — The above laws are conveniently expressed in the following formula for the resistance of a wire: —

$$r = k \frac{l}{C.M.},$$

in which k is a constant depending on the material, l the length of the wire in feet, and $C.M.$ denotes "circular mils." A "mil" is a thousandth of an inch, and circular mils are the square of the mils, that is, the square of the diameter of the wire in thousandths of an inch. The constant k becomes then the resistance in ohms of one mil-foot, that is, of a wire one foot long and one mil in diameter. The following are the values of k in ohms for several metals, at 20° C.: —

Silver	9.53	Iron	61.3	German-silver	181.3
Copper	10.19	Platinum	70.5	Mercury	574

464. The Strength of a Current is measured by the magnitude of the effects produced by it. Either the chemical, the electromagnetic, or the heating effects may be made the basis of a system of measurement. The quantity of an ion deposited in a second furnishes a convenient magnitude for the determination of unit strength of current. The unit of current strength is the *ampere*. It is that current which will deposit by electrolysis, under suitable

conditions, 0.001118 gm. of silver, or 0.0003287 gm. of copper in one second. The ampere deposits 4.025 gm. of silver in one hour. A milliampere is a thousandth of an ampere. It is to be noted that the electrolytic method measures only the quantity of electricity passing through the decomposing cell, called a *voltameter*, in the given time.

465. Electromotive Force is the name given to the cause of an electric flow. It is often called *electric pressure* from its superficial analogy to water pressure. The unit of electromotive force (E.M.F.) is the *volt*. It is the E.M.F. which will cause a current of one ampere to flow through a resistance of 1 ohm. The E.M.F. of a voltaic cell depends upon the materials employed, and is entirely independent of the size and shape of the plates. The E.M.F. of a Daniell cell is about 1.1 volts; of a fresh chromic acid cell, 2 volts; and of a Leclanché cell, 1.5 volts. The E.M.F. of a Carhart-Clark standard cell is 1.44 volts at 15° C.

466. Ohm's Law. — The definite relation existing between strength of current, resistance, and E.M.F. is known as *Ohm's law*. It may be expressed as follows:—

The strength of a current equals the electromotive force divided by the resistance; or in symbols:—

$$C = \frac{E}{R}$$

$$I = \frac{E}{R}, \quad (30)$$

where I is the current in amperes, E the E.M.F. in volts, and R the resistance in ohms. Applied to a battery, if R is the resistance external to the cell, and r the internal resistance of the cell itself, then

$$C = \frac{E}{R + r}$$

$$I = \frac{E}{r + R}$$

$$H = \frac{C^2 R}{4 \pi^2}$$

Thus, if a chromic acid cell of 2 volts E.M.F. and half an ohm internal resistance is closed with a wire having a resistance of one and a half ohms, the current will be

$$\frac{2}{1.5 + 0.5} = 1 \text{ ampere.}$$

From the equation $I = \frac{E}{R}$, we derive $E = IR$; that is, the effective E.M.F. equals the product of the current and resistance.

Again, $\frac{E}{I} = R$, or the resistance of a conductor is the constant ratio between the E.M.F. and the current which it produces through the conductor.

467. Methods of Varying Strength of Current. — It is evident from Ohm's law that the strength of the current furnished by an electric generator may be increased in two ways: 1. By increasing the E.M.F. 2. By reducing the internal resistance.

The E.M.F. may be increased by joining several cells in series, and the internal resistance may be diminished by connecting them in parallel. Enlarging the plates of a battery or bringing them closer together diminishes the internal resistance.

468. Connecting in Series. — If several cells are connected so that the positive pole of one is joined to the negative pole of the next and so on, then the total E.M.F. is the sum of the E.M.F.'s of the several cells. The cells are then said to be joined in *series*. Figure 262 is the conventional sign for a single cell. The short, thick line represents the

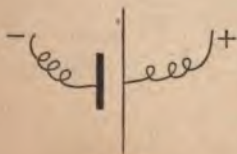


Fig. 262.

the electrode, and the long, thin line the positive terminal. Figure 263 shows six cells joined in series. If each cell has an E.M.F. of 2 volts, then the total E.M.F. will be 6×2 or 12 volts. In connecting cells in this manner the internal resistance is also increased about six times, since the liquid conductor is six times as long as in one cell. The current for any external resistance

$$R \text{ is then } I = \frac{12}{6r + R}$$

469. Connecting in Parallel. —

When all the positive terminals are connected together on one side and the negative on the other, the cells are grouped in *parallel*. With n similar cells the effect of such a grouping (Fig. 264) is to reduce the inter-

64.

distance to $\frac{1}{n}$ th that of a single cell. It is equivalent to increasing the area of the plates n times. All the cells by side contribute equal shares to the output of the battery.

is small || when R is large series gives more current

Relative Advantages of the Series and Parallel
g. — It is evident from Ohm's law that when the internal resistance is small, there is nothing gained by increasing E and at the same time increasing r , since E remains practically unchanged. But when R is large the increase in r due to joining cells in series is more than counterbalanced by the increase in E . Con-

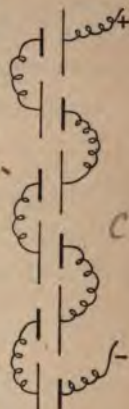


Fig. 263.

sequently I is greater the larger the number of cells joined in series. For example: A battery of 6 cells, each having an E.M.F. of 2 volts and an internal resistance of 0.5 ohm, acts, first, through an external resistance of 0.1 ohm; and, secondly, through one of 500 ohms. If joined in parallel circuit, then when $R = 0.1$, $I = \frac{2}{0.1 + \frac{0.5}{6}} = 10.9$.

amperes; and when $R = 500$, $I = \frac{2}{500 + \frac{0.5}{6}} = 0.004$ ampere.

If joined in series, then when $R = 0.1$, $I = \frac{6 \times 2}{0.1 + 6 \times 0.5} = 3.87$ amperes; and when $R = 500$, $I = \frac{6 \times 2}{500 + 6 \times 0.5} = 0.024$ ampere. A comparison of these results shows that when the external resistance is small, the greater current is obtained by grouping in parallel; but when the external resistance is large, the series arrangement gives the greater current.

The largest current is obtained in any case by so grouping the cells that the external and internal resistances are equal to each other.

b. INSTRUMENTS FOR MEASUREMENT.

471. The Galvanometer. — If a comparison of currents is made by means of their magnetic effects, the instrument used for the purpose is called a *galvanometer*. If the galvanometer is calibrated, so as to read directly in amperes, it is called an *ammeter*. A galvanoscope becomes a galvanometer by providing it with a scale so that the deflections may be measured. In very sensitive instruments a small mirror is attached to the movable part of the instrument; it is then called a *mirror galvanometer*. Sometimes

am of light from a lamp is reflected from this small or back to a scale, and sometimes the light from the scale is reflected back to a small telescope, by means of which the deflections are read. In either case the path of light then becomes a long pointer without doubt.

2. **The Tangent Galvanometer** consists of a vertical coil of wire (Fig. 265) from twenty-five to thirty centimetres in diameter, at the centre of which is supported a magnet-needle about two centimetres long, furnished with a light pointer moving over a graduated scale. Owing to the size of the coil the magnetic field at the centre is nearly uniform; that is, the lines of force there are nearly parallel straight lines as in Fig. 261; and the movement of the short coil round a vertical axis does not carry its poles into a magnetic field of different strength. Under these conditions the strength of current is proportional to the tangent of the angle of deflection.



Fig. 265.

For example, if two different batteries, used successively in circuit with a tangent galvanometer, give deflections of 55° and $35\frac{1}{2}^\circ$ respectively, then the strengths of these currents are as $\frac{\tan 55^\circ}{\tan 35\frac{1}{2}^\circ} = \frac{1.428}{0.714} = \frac{2}{1}$; that is, the strength of one current is double that of the other. (See Appendix.)

*- tangent to
tangent to*

The meaning of the "tangent of an angle" may be made clear by consulting Fig. 266. The line AH is

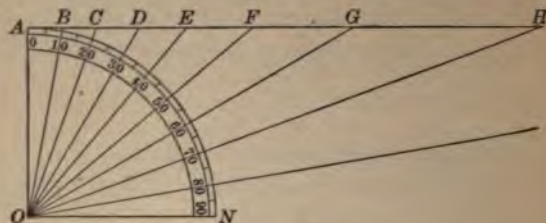


Fig. 266.

drawn perpendicular to OA . The tangents of 10° , 20° , 30° , etc., are equal to the quotients of AB , AC , etc., divided by OA ; that is, they are proportional to the lengths of the lines AB , AC , AD , etc.

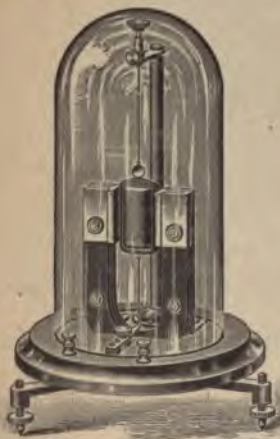


Fig. 267.

473. The d'Arsonval Galvanometer. — One of the most useful forms of galvanometer is the d'Arsonval, Fig. 267. Between the poles of a strong permanent magnet swings a coil suspended by a fine wire in such a way that the current is led in by the suspending wire and out by the wire connecting the coil to a spring at

the bottom. A small mirror for reflecting a beam of light is attached to the coil. Inside the coil is a soft iron tube supported from the back. In some of the most recent forms of this instrument the coil is made narrower and

tube is omitted. Figure 268 illustrates a mirror meter with horizontal magnets. The glass in front of the movable mirror is silvered half way up; and the observer, looking through a slit in the upright strip, sees the reflection of the scale on the fixed and a movable part of the scale.

In the d'Arsonval galvanometer the coil is movable and the magnet is fixed. Its advantages are simplicity of construction, almost complete independence of the position and strength of



Fig. 268.



Fig. 269.

the magnetic field at the place where it is used, and the ease with which it comes to rest after the coil has been deflected by a current through it.

474. Voltmeters. —

An instrument designed to measure the difference of potential in volts is called a *voltmeter*. For direct currents the most convenient portable voltmeter is made on the

principle of the d'Arsonval galvanometer. The appearance of one of the best-known instruments of this class is shown in Fig. 269. The interior is represented by Fig. 268. A portion of the instrument is cut away to show

the coil and the springs. The current is led in by one spiral spring and out by the other. Attached to the coil is a very light aluminum pointer, which moves over the scale seen

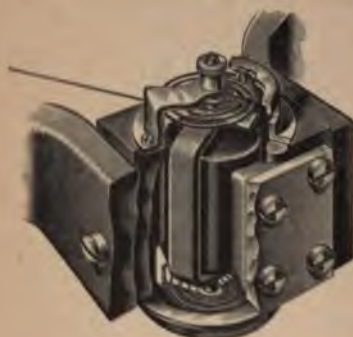


Fig. 270.

in Fig. 269, where it stands at zero. Soft iron pole-pieces are screwed fast to the poles of the permanent magnet, and they are so shaped that the divisions of the scale in volts are equal. Such an instrument needs to be frequently recalibrated because of slight changes in the strength of the magnet.

A similar instrument, called an *ammeter*, is designed to measure currents in amperes. The resistance of a voltmeter should be high, so that it will take the smallest practicable current; the resistance of an ammeter should be as small as possible, so that it will not increase the resistance of the circuit in which it is placed.

475. Resistance Coils.—Coils of wire of known resistance, for use with a galvanometer in measuring resistance, can be purchased of makers of electrical instruments. They should be mounted as shown in Fig. 271. Each coil is wound on its spool double, the inner ends being connected so that the current passes an equal number of times in both directions round the spool; the coil does not then produce a



Fig. 271.

magnetic field. This method of winding diminishes what is known as self-induction (§ 483), which is an E.M.F. affecting the current on opening and closing the circuit. The ends of the coils are connected to the heavy brass blocks, C^1 , C^2 , C^3 , on top of the box. When a brass plug, as P , is inserted between two blocks, as C^2 and C^3 , the current goes through the plug; but when the plug is withdrawn, as between C^1 and C^2 , the current must go through the corresponding coil.

The resistance of these coils is often arranged in ohms of the following numbers :—

1, 2, 2, 5, 10, 10, 20, 50, 100, 100, 200, 500, etc. In this way any number of ohms from one up to the full capacity of the resistance box can be thrown into circuit by withdrawing the proper plugs.

476. Divided Circuits.—When the wire leading from any electric generator is divided into two branches, as at B (Fig. 272), the current also divides, part flowing by one path and a part by the other.

The sum of these two currents is always equal to the current in the undivided part of the circuit, since there is no accumulation of electricity

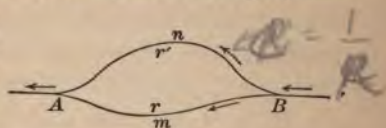


Fig. 272.

at any point. Either of the branches between B and A is called a *shunt* to the other, and *the currents through them are inversely proportional to their resistances*.

477. Resistance of a Divided Circuit.—Let the total resistance between the points A and B be represented by R , that of the branch BmA by r , and of BnA by r' . The conductivity of BA equals the sum of the conductivities

of the two branches; and, as conductivity is the reciprocal of resistance, the conductivities of BA , BmA , and BnA are $\frac{1}{R}$, $\frac{1}{r}$, and $\frac{1}{r'}$ respectively, and $\frac{1}{R} = \frac{1}{r} + \frac{1}{r'}$.

From this we easily derive $R = \frac{rr'}{r+r'}$. To illustrate, let a galvanometer whose resistance is 100 ohms have its binding posts connected by a shunt of 50 ohms resistance; then the total resistance of this divided circuit is $\frac{100 \times 50}{100 + 50} = 33\frac{1}{3}$ ohms. The introduction of a shunt always lessens the resistance between the points connected, as A and B .

478. Fall of Potential along a Conductor. — When a current flows through a conductor a difference of potential exists, in general, between different points on that conductor. Let A , B , C be three points on a conductor conveying a current, and let there be *no source of E.M.F. between these points*. Then if the current flows from A to B , the potential at A is higher than at B , and the potential at B is higher than at C . If the potential difference between A and B and that between B and C be measured, the ratio of the two will be the same as the ratio of the resistances between the same points. This is only another statement of Ohm's law. For since $I = \frac{E}{R}$, and the current is the same through the two adjacent sections of the conductor, the ratio of the potential differences to the resistances of the two sections is the same. This important principle, of which great use is made in electrical measurements, may be expressed by saying that, when the current is constant, *the loss of potential along a conductor is proportional to the resistance passed over.*

479. Wheatstone's Bridge consists of four resistances connected as shown in Fig. 273. The four conductors R , R' , R'' , X constitute the arms, and the conductor CD is the bridge. When the circuit is closed the current divides at B , the two parts reuniting at A . The loss of potential along BCA is the same as that along BDA . Now if no current flows through the galvanometer G , then the potentials of C and D must be the same. Under

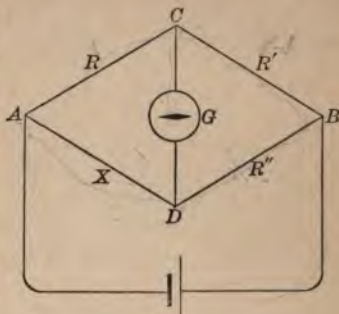


Fig. 273.

these conditions the fall of potential from B to C is the same as from B to D . We may get an expression for these two potential differences, and place them equal to each other. Let I' be the current through R' ; it will also be the current through R , because none flows across through the galvanometer. Also let I'' be the current through the branch BDA . Then the difference of potential between B and C is the same as between B and D , and by Ohm's law (§ 466)

$$R'I' = R''I'' \quad \dots \quad (a)$$

In the same way, the equal potential differences between C and A and D and A give

$$RI' = XI'' \quad \dots \quad (b)$$

Dividing (a) by (b), we have

$$\frac{R'}{R} = \frac{R''}{X} \quad (31)$$

When no current passes through the galvanometer, the four resistances in the arms of the bridge form a proportion. If three of them are known, the proportion gives the formula, $X = \frac{RR''}{R'}$.

Problems.

1. No. 30 wire has a diameter of 0.01 in. Calculate the resistance of 50 ft. of copper wire ($k = 10.19$) of this number.

2. No. 36 wire has a diameter of 0.005 in. How many feet of German silver ($k = 181.3$) wire of this number will there be in a 5000-ohm coil?

3. Iron ($k = 61.3$) wire No. 10 is to be used in making a resistance coil of 20 ohms. The diameter of this wire is 0.1014 in. Calculate the number of feet required.

✓ 4. What strength of current will be necessary to deposit silver by electrolysis at the rate of 1 gm. per minute?

5. If in 50 min. an electric current deposits by electrolysis 19.722 gm. of copper, what is the strength of the current?

6. How long will it require a current of 3 amperes to deposit by electrolysis 6 gm. of silver?

✓ 7. Six Daniell cells, each having an E.M.F. of 1.1 volts and an internal resistance of 3 ohms, are joined in parallel through an external resistance of 1 ohm. Calculate the current. Calculate the current if the cells are joined in series.

8. A chromic acid cell has an E.M.F. of 2 volts and an internal resistance of 0.4 ohm. What external resistance must be placed in circuit in order that the current may be 2 amperes? 5.6 ohm

✓ 9. A current of 10 amperes flows through an electric lamp; the difference of potential between the terminals of the lamp is 75 volts. What is the apparent resistance of the lamp? 7.5 ohm

10. What must be the resistance of a set of resistance coils carrying a current of 10 amperes and showing a fall of potential between its terminals of 450 volts? 45 ohm

11. What is the resistance of an incandescent lamp taking 0.25 ampere at a pressure of 220 volts? 880 ohm

12. A galvanometer whose resistance is 10,000 ohms has its binding posts joined by a shunt wire of 100 ohms resistance. What is the combined resistance between the binding posts?

13. A galvanometer of 100 ohms resistance is to be provided with a shunt such that one-fifth of the whole current shall pass through the galvanometer. Compute the resistance of the shunt. ✓

[The current divides inversely as the resistance of the branches.]

14. A Leclanche cell whose E.M.F. is 1.5 volts and internal resistance is 0.5 ohm is connected to a galvanometer whose resistance is 100 ohms. Calculate the change in the current through the galvanometer when it is shunted by a wire whose resistance is 1 ohm.

[Calculate the current before the shunt is introduced, then calculate the resistance after introducing the shunt, and from that calculate the current.]

15. Three cells, each having an E.M.F. of 1 volt and an internal resistance of 2 ohms, are to be joined so as to send the largest current through an external resistance of 1 ohm. Determine the method of connection and the current given. *0.6 amperes*

16. A uniform copper wire, 500 ft. long, connects the two poles of an electric generator whose E.M.F. is 50 volts. If the positive pole of the generator is electrically connected through a voltmeter to a point 100 ft. along the wire, what will be its reading? [Consult § 478.] ✓

17. A cell whose E.M.F. is 2 volts gave a current of $\frac{1}{2}$ ampere through an external resistance of 3 ohms. What was the internal resistance of the battery? *1 ohm*

18. What external resistance must be used in order that a battery of 5 volts E.M.F. and 2 ohms internal resistance may give a current of 2 amperes? *0.5 ohms*

19. Four similar cells, each with an E.M.F. of 1.5 volts, are joined in series through a resistance and found to give a current of 1 ampere, and when joined in parallel through the same resistance the current is a third less. What is the resistance of each cell?

20. A current of 10 amperes is required through a resistance of 1 ohms from a number of cells joined in series, each cell having an E.M.F. of 2 volts and a resistance of 0.1 ohm. Find the number of cells. *40 cells* ✓

XV. ELECTROMAGNETIC INDUCTION.

✓ 480. **Electromotive Forces induced by Magnets.** — Experiment. — Connect a coil of insulated wire, consisting of a large number



Fig. 274.

of turns, in circuit with a sensitive galvanometer (Fig. 274). Thrust quickly into the coil the north pole of a bar magnet. The needle of the galvanometer will be deflected, and the direction of the deflection will show that a momentary current has been set flowing round the coil counter-clockwise to one looking down upon it. The current ceases as soon as the magnet stops moving. When the magnet is removed, a current is produced in the opposite direction to the first one. If the south pole be thrust into the coil, and then withdrawn, the currents in both cases are the reverse

of those produced by the north pole. If we substitute a helix of a smaller number of turns, or a weaker bar magnet, the deflection will be less, showing that a weaker current is set flowing, or a smaller E.M.F. has been generated.

The momentary electromotive forces generated in the helix are known as *induced electromotive forces*, and the currents as *induced currents*. The magnet carries with it into the coil its lines of force; and when the relative positions of a magnet and a closed conductor are so altered that a variation is produced in the number of the lines of force linked with the coil, an induced electromotive force is generated in the conductor in accordance with the following laws:—

I. *An increase in the number of lines of force linked with a coil produces an indirect electromotive force, while*

a decrease in the number of lines produces a direct electromotive force. By direct electromotive force is meant one in the direction of the motion of watch-hands, and by indirect electromotive force, one in the opposite direction. The observer must always be looking along the lines of force. Thus, if in Fig. 275 the magnet be thrust into the coil in the direction of the arrow, the current will flow counter-clockwise, as shown by the arrows on the coil. The motion of the magnet into the coil carries its lines of force with it, and thus increases the number passing through the coil.

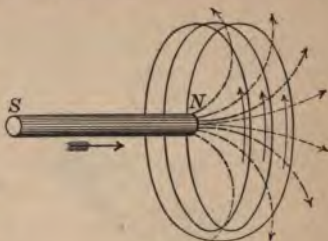


Fig. 275.

II. *The electromotive force produced is equal to the rate of increase or decrease in the number of lines of force linked with the coil.*

Experiment.—Wind a number of turns of fine insulated wire around the armature of a horseshoe magnet, leaving the ends of the iron free to come in contact with the poles of the permanent magnet. Connect the ends of the coil to a sensitive galvanometer, the armature being in contact with the magnetic poles, as shown in Fig. 276. Keeping the magnet



Fig. 276.

fixed, suddenly pull off the armature. The galvanometer will show a momentary current. Suddenly bring the armature up to the poles of the magnet; another momentary current in the reverse direction will flow through the circuit.

When the armature is in contact with the poles of the magnet, the number of lines of force passing through the coil is a maximum. As the armature is pulled away, the num-

ber of magnetic lines threading through the coil rapidly diminishes. This experiment illustrates Faraday's method of obtaining electric currents by the agency of magnetism.

481. Currents induced by Other Currents. — Experiment. —

Insert within a coil connected to a sensitive galvanometer a second coil in circuit with a battery (Fig. 277). The effects are exactly like those obtained by the use of the magnet, even to the direction of the galvanometer deflection, when the polarity of the coil (§ 458) corresponds with that of the magnet. Furthermore, the effects are the same if the coil is inserted before closing the circuit.



Fig. 277.

This experiment shows that a coil through which a current is passing has the same effect in producing an induced electromotive force as a magnet. This was to be expected, since we have seen that a coil carrying a current has properties identical with a magnet. The coil connected with the battery is

called the *primary coil*, and the other the *secondary coil*.

482. Lenz's Law. — *The direction of an induced current is always such that it produces a magnetic field opposing the motion or change which induces the current.* This is known as *Lenz's law*. It is only the law of conservation of energy applied to electricity. To illustrate the law, consider Fig. 275. When the north pole of the magnet is thrust into the coil, the induced current flowing in the

direction of the arrows produces lines of force running in the opposite direction to those from the magnet (§ 480). These lines of force tend to oppose the change in the magnetic field within the coil, or the magnetic field set up by the coil opposes the motion of the magnet.

Again, when the primary coil of Fig. 277 is inserted into the secondary, the induced current in the latter is opposite in direction to the primary current, and parallel currents in opposite directions repel each other. In every case of electromagnetic induction the change in the magnetic field which produces the induced current is always opposed by the magnetic field due to the induced current itself.

Hand

483. Self-induction. — Experiment. — Make a helix of thin brass wire with the turns close together and about 2 cm. in diameter. Solder to one end a small weight and to the weight a short piece of platinum wire. Support the other end of the helix in some convenient way so that its height can be readily adjusted, and let the platinum point just touch the surface of some mercury in a small cup. Connect one electrode of a battery to the upper end of the helix and the other electrode to the mercury. The current through the parallel turns of the helix will cause them to attract one another (§ 459), thus shortening the helix and lifting the platinum wire out of the mercury with a brilliant spark. The attraction then immediately ceases, and the platinum point again drops into the mercury. The operation is thus repeated, and the helix is set vibrating with a succession of bright sparks at the break. (If the helix is long enough and properly weighted, it will divide into vibrating segments separated by nodes, like a stretched cord (§ 214)).

Experiment. — Attach to one pole of a chromic acid battery a flat file and to the other a piece of iron wire. Draw the iron wire quickly across the ridges of the file. Only slight sparking will be visible. Put an electromagnet (§ 457) in the circuit and repeat the experiment. A series of bright sparks will now be seen as the circuit is broken at the ribs of the file.

The sparks in both cases are due to the relatively high E. M. F. induced by the mutual inductive action of adjacent turns of the helix. When there is only a single circuit, as in these cases, the action is called *self-induction*. When the circuit is opened, the self-induced E. M. F. tends to prolong the current, and a spark breaks over the opening. The current flowing across under the impulse of self-induction is often called the *extra current*. There is a similar self-induced E. M. F. when the circuit is closed, and this prevents the instantaneous rise of the current to its maximum value given by Ohm's law.

Let a coil be wound round a wooden cylinder (Fig. 278). Then some of the lines of magnetic force around one turn



Fig. 278.

will thread through adjacent turns. If the cylinder *PR* is iron instead of wood, the magnetic flux through all the parallel turns of wire will be greatly increased (§ 455). At the instant when the circuit is closed, the increase of magnetic induction through the parallel turns of wire will give rise to a counter

E. M. F. of self-induction; and when the circuit is opened, the decrease of magnetic induction through the coil causes a direct E. M. F.; that is, one in the same direction as the current itself (§ 481).

484. Self-induction with a Wheatstone's Bridge. — Experiment. — Connect an electromagnet in one of the arms of a Wheatstone's bridge (§ 479) as a resistance to be measured. Adjust for a balance so that no current flows through the galvanometer when the battery circuit is closed first, and the galvanometer circuit a few seconds later. Then, keeping the galvanometer circuit closed, close and open the battery circuit. The galvanometer will show a sharp

deflection in one direction when the main circuit is closed, and in the other direction when it is opened.

The E. M. F. generated by self-induction in the electro-magnet destroys the balance. The deflections of the galvanometer demonstrate that the induced E. M. F. is in one direction when the circuit through the electro-magnet is closed, and in the other direction when it is opened.

✓ **485. The Induction Coil.** — An apparatus for producing a high E. M. F. by means of induction is called an *induction coil* or a *transformer*.

It consists of a coil of coarse insulated wire surrounding an iron core, and a second coil of a very large number of turns of fine wire surrounding the first (Fig. 279).

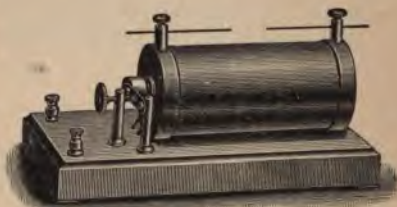


Fig 279

The inner or primary coil is connected to a battery through a current interrupter; also through a device called a *commutator* for changing the direction of the current. At the “make” and “break” of the circuit currents are induced in the secondary coil in accordance with the laws of current induction (§ 480). In coils designed to give sparks between the terminals of the secondary coil a *condenser* is added. It is placed in the supporting base of the coil, and consists of two sets of interlaid layers of tin-foil, separated by sheets of paper saturated with oil or paraffin. The two sets are connected with two points of the primary circuit on opposite sides of the current interrupter.

486. Action of the Coil. — Figure 280 shows the arrangement of the various parts of an induction coil. The current first passes through the heavy primary wire *PP*, thence through the spring *k*, which carries the soft iron block *F*, then across to the screw *b*, and so back to the negative pole of the battery. This current magnetizes the iron core of the coil, and the core attracts the soft iron block *F*, thus breaking the circuit at the point of the

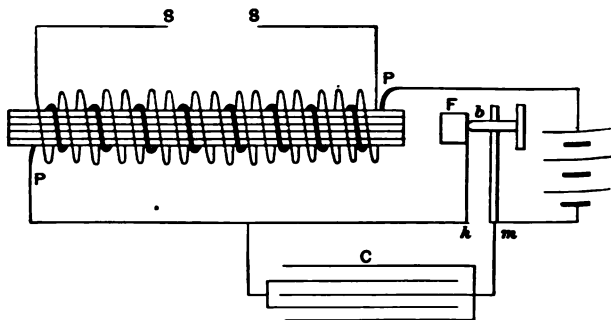


Fig. 280.

screw *b*. The core is then demagnetized, and the release of *F* again closes the circuit. Electromotive forces are thus induced in the secondary coil *SS*, both at the make and the break of the primary. Hence, if the secondary is closed, alternating currents are induced in it by the intermittent current in the primary. The high E. M. F. of the secondary is due to the large number of turns of wire in it and to the influence of the iron core in increasing the number of lines of force which pass through the entire coil.

The self-induction of the primary has a very important bearing on the action of the coil. At the instant the circuit is closed, the counter E. M. F. opposes the battery current, and prolongs the time of reaching its greatest

length. Consequently the E. M. F. of the secondary will be diminished by self-induction in the primary. The E. M. F. of self-induction at the "break" of the primary is direct, and this added to the E. M. F. of the battery produces a spark at the break points.

487. The Condenser. — The addition of a condenser increases the E. M. F. of the secondary coil in two ways:

It gives such an increase of capacity to the primary coil that at the moment of breaking the circuit the potential difference between the contact points does not rise high enough to cause a spark discharge across the air gap. The interruption of the primary is therefore more abrupt, and the E. M. F. of the secondary is increased. 2. After the break, the condenser *C*, which has been charged by the E. M. F. of self-induction, discharges back through the primary coil and the battery. The condenser causes an electric recoil in the current, and returns the stored charge as a current in the reverse direction through the primary, thus demagnetizing the core, increasing the rate of change of magnetic flux, and increasing the induced E. M. F. in the secondary. The condenser momentarily stores the energy presented by the spark when no condenser is used, and then returns it to the primary and by mutual induction to the secondary, as indicated by the longer spark or the greater current. When the secondary terminals are separated, the discharge is all in one direction and occurs when the primary current is broken.

488. Experiments with the Induction Coil. — 1. *Physiological Effects.* — Hold in the hands the electrodes of a very small induction coil, of the style used by physicians. When the coil is working, a peculiar muscular contraction is produced.

The "shock" from large coils is dangerous on account of the high E.M.F. The danger decreases with the increase in the rapidity of the impulses or alternations. Experiments with induction coils, worked by alternating currents of very high frequency, have demonstrated that the discharge of the secondary may be taken through the body without injury.

2. *Mechanical Effects.*—Hold a piece of cardboard between the electrodes of an induction coil giving a spark 3 cm. long. The card will be perforated, leaving a burr on each side. Thin plates of any non-conductor can be perforated in the same manner.

3. *Heating Effects.*—Make a torpedo (§ 419) and place it in circuit with an induction coil. Close the battery circuit and the torpedo will explode.

4. *Chemical Effects.*—Place on a plate of glass a strip of white blotting-paper moistened with a solution of potassium iodide and starch paste. Attach one of the electrodes of a small induction coil to the margin of the paper. Handle a wire leading to the other electrode with an insulator, and trace characters with the wire on the paper when the coil is in action. The discharge decomposes the chemical salt, as shown by the blue mark. This blue mark is due to the reaction between the iodine and the starch.

489. Discharges in Partial Vacua.—**Experiment.**—Cover the inside of a vase or glass goblet with tin-foil, a little over halfway up, and place the goblet on the table of the air-pump, under a bell-jar provided with a brass sliding rod passing air-tight through the cap at the top (Fig. 281). Surround the part of the rod inside the jar with a glass tube and push the rod down till it touches the tin-foil. Connect the rod and the air-pump table to the terminals of the induction coil. When the air is exhausted a beautiful play of light will fill the bell-jar. If the vase is made of uranium glass, or if it stands on a block of that material, the effect is still more beautiful. This experiment is known as *Gassiot's cascade*.



Fig. 281.

The best effects are obtained with discharges from the secondary of an induction coil in glass tubes when the exhaustion is carried to a pressure of about 2 mm. of mercury, and the tubes are permanently sealed. Platinum electrodes are melted into the glass at the two ends. Such tubes are known as *Geissler tubes*.

They are made in a great variety of forms (Fig. 282), and the luminous effects



Fig. 282.

are more intense in the narrow connecting tubes than in the large bulbs at the ends. The colors are determined by the nature of the residual gas. Hydrogen glows with a brilliant crimson; the vapor of water gives the same color, indicating that the vapor is dissociated by the discharge. An examination of this glow by the spectroscope gives the characteristic lines of the gas in the tube.

Geissler tubes often exhibit *stratifications*, which consist of portions of greater brightness separated by darker intervals. Stratifications have been produced throughout a tube 50 feet long. These striæ present an unstable flickering motion, resembling that sometimes observed during auroral displays.

490. Discharges in High Vacua. — When the exhaustion of a tube is carried to about a millionth of an atmosphere, the effects of an electric discharge through it are entirely changed in character. The light emanating from the residual gas nearly disappears. The dark space, which is present about the negative or cathode of a Geissler tube, broadens till it reaches the opposite wall of the bulb. Tubes of this

kind were first investigated by Sir William Crookes; they are therefore known as *Crookes tubes*. A stream of the electrified particles, or of much more minute parts called *corpuscles*, is then projected from the cathode across to the opposite wall of the bulb. The impact excites remarkable luminous effects in the glass.

Other substances, such as diamond, ruby, and various sulphides, under the impact of the cathode rays, as they are called, exhibit beautiful fluorescent colors (Fig. 283).

These cathode rays are straight and at right angles to the cathode surface, except when they are deflected by a magnet or by mutual repulsion. The cathode stream, when once deflected by a magnet, does not recover its former direc-



Fig. 283.



Fig. 284.

tion after passing the magnet (Fig. 284). A screen, *bd*, placed in the path of cathode rays stops them more or less completely, and appears to cast a shadow by protecting the wall of the tube from their impact. At the same time the obstruction is acted on mechanically, as if by a bom-

rdment or a wind. If the cathode is concave, the rays pass at a focus, and at this focus platinum foil can be raised to a white heat (Fig. 285).

491. Roentgen Rays. — The rays of radiant matter, as Crookes called it, emanating from the cathode, give rise to another kind of rays when they strike the walls of the tube, or a piece of platinum placed in their path. These last rays, which Roentgen, their discoverer, gave the name of "*X-rays*," can pass through glass, and so get out of the tube. They also pass through wood, paper, flesh, and many other substances opaque to light. They are stopped by bones,

metals (except in very thin sheets), and by some other substances. Roentgen discovered that they affect a photographic plate like light. Hence, photographs can be taken of objects which are entirely invisible to the eye, such as the bones in a living body, or bullets embedded in the flesh.



Fig. 286.



Fig. 285.

A Crookes tube adapted to the production of Roentgen rays (Fig. 286) has a concave cathode *K*, and at its focus an inclined piece of platinum

A, which serves as the anode. The X-rays originate at A and issue from the side of the tube. Figure 287 is a photograph taken by the aid of one of these tubes. The sensitized plate was enclosed in an ordinary plate-holder, the hand was laid on the holder next to the sensitized side



Fig. 287.

of the plate, and the X-ray tube was held about a foot above the hand.

The tube of Fig. 286 is called a *focus tube*, because the cathode is so shaped that all the cathode rays reach the inclined platinum anode at nearly the same point. The X-rays themselves cannot be focussed; all photographs taken by them are only shadow pictures; but, as in the case of light, the shadow will be sharper when the source of light is of small area than when it is larger (§ 232). When the tube is working well, the anode A becomes red-hot near its centre, and is then a source of copious X-rays.

492. The Fluoroscope. — Soon after the discovery of X-rays it was found that certain fluorescent substances, like platino-barium-cyanide, and calcium tungstate, become luminous under the action of X-rays. This fact has been turned to account in the construction of a *fluoroscope* (Fig. 288), by means of which shadow pictures of con-

al objects become visible. An opaque screen is covered on one side with the fluorescent substance; this screen is placed into the larger end of a cone which is blackened inside, and looking through the opening at the other end an image of the object being examined is seen. By closing the opening adapted to fit closely around the eyes, so as to exclude all outside light, when an object, such as a hand, is held against the fluorescent screen and the fluoroscope is turned toward the Roentgen tube, the bones are plainly visible as darker objects than the flesh because they are more opaque to X-rays. The beating heart may be made visible in a similar manner.



Fig. 288.

XVI. DYNAMO-ELECTRIC MACHINES.

493. A Dynamo-electric Machine is a device designed to produce electric currents by the expenditure of mechanical work. The so-called generation of electric currents consists always in the generation of electromotive forces or electric pressures. Every dynamo-electric machine has three essential parts: 1. The *inductor* or *magnet* by which the magnetic field is produced. 2. The *armature*, or system of conductors in which electromotive forces are generated by a change in the magnetic field or flux through it. 3. The *brushes* or *collecting rings*. When the field is produced by a permanent magnet, the machine is called a *magneto*; when the electromagnet is used, the machine is a *dynamo*.

494. The Ideal Simple Dynamo. — Suppose a single loop of wire to revolve between the poles of a magnet *NS* (Fig. 289) in the direction of the arrow and round a horizontal

axis. The magnetic flux runs across from *N* to *S*, as indicated by the light lines. The loop encloses in the position shown the largest possible magnetic flux. When it has rotated through a quarter of a turn, the magnetic flux will be parallel to its plane, and none will then pass through it. During this quarter turn, the decrease in the magnetic flux through the loop generates a direct E.M.F.

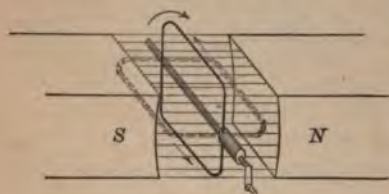


Fig. 289.

looking in the direction *N* to *S*, as indicated by the arrows on the loop. During the next quarter turn the magnetic flux through the rotating loop will increase again, but it will run through the

loop in the opposite direction. This is equivalent to a continuous decrease in the original direction, and therefore the direction of the induced E.M.F. round the loop remains the same for the entire half turn. If the loop is part of a closed circuit, a current will flow in the direction of the E.M.F. induced. During the next half revolution, the current will flow in the opposite direction round the loop. Hence the E.M.F. and the current through the loop reverse twice during every revolution.

495. The Shuttle Armature. — If the loop is composed of several turns of wire instead of one, the E.M.F. generated by the rotation of the coil is increased in the same ratio as the number of turns of wire. The E.M.F. can also be increased by winding the coil on iron, for the iron increases the magnetic flux through the coil (§ 377). Fig. 290 is a cross-section of a



Fig. 290.

oil with a large number of turns of wire wound in grooves roughed in an iron cylinder. It is called the *shuttle armature*. Such an armature is very inefficient and not suitable for large currents. Currents are produced in the solid iron core, as well as in the insulated wire, and these absorb energy and heat the iron. It is used extensively in "magnetos" for ringing bells on telephone lines.

496. The Commutator. — When it is desired to convert the alternating currents flowing in the armature into a current in one direction through the external circuit, a special device called a *commutator* is employed. For a single coil in the armature, the commutator consists of two parts only. It is a split tube with the two halves insulated from each other and from the shaft on which they are mounted (Fig. 291). The two ends of the coil are connected with the two halves of the tube. Two brushes, with which the external circuit is connected, bear on the commutator, and they are so placed that they exchange contact with the two commutator segments at the same time that the current reverses in the coil. In this way one of the brushes is always positive and the other negative, and the current flows in the external circuit from the positive brush back to the negative, and thence through the armature to the positive again.



Fig. 291.

497. The Gramme Ring. — The use of a commutator with more than two parts is conveniently illustrated in connection with the *Gramme ring*. This armature has a core made either of iron wire, or of thin disks at right angles to the axis of rotation. The iron is divided for the pur-

pose of preventing induction currents in it, which waste energy. The relation of the several parts of the machine is illustrated by Fig. 292. A number of coils are wound in one direction and are all joined in series. Each junction between coils is connected with a commutator bar. When



Fig. 292.

a coil is in the highest position in the figure, the maximum flux passes through it; as the ring rotates, the flux through the coil decreases, and after a quarter of a revolution there is no flux through it. The current

through each coil reverses twice during each revolution, exactly as in the case of the single loop. No current flows entirely round the armature, because the E.M.F. generated in one coil at any instant is exactly counterbalanced by the E.M.F. generated by the coil opposite. But when the external circuit connecting the brushes is closed, a current flows up on both sides of the armature. The current has then two paths through the armature, and one brush is constantly positive and the other negative. By increasing the number of coils, the potential difference between the brushes never drops to zero, as it does with a single coil twice during each revolution, but it is kept nearly constant and at the highest value given by half the coils in series. The brushes must bear on the commutator near the part of the field where the E.M.F. in any coil passes through zero and reverses.

498. The Field-magnet.— In dynamos the magnetic field is produced by a large electromagnet excited by the current flowing from the armature, which is led either wholly

or in part round the field-magnet cores. When the entire current is carried round the coils of the field-magnet, the dynamo is said to be *series wound*. When the field-magnet is excited by coils of many turns of fine wire connected as a shunt to the external circuit, the dynamo is said to be *shunt wound*. The connections of the field and external circuit of the two kinds of winding are shown in Fig. 293. A combination of these two methods of exciting the field-magnet is called *compound*

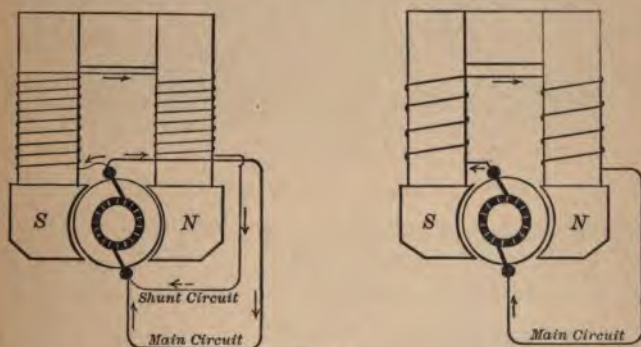


Fig. 293.

winding. The residual magnetism remaining in the cores is sufficient to start the machine. The current thus produced increases the magnetic flux through the armature and so increases the E.M.F.

499. The Drum Armature.—The modern *drum armature* for direct currents is an evolution from the one-coil shuttle armature. It is shown at A, Fig. 294, which represents the parts of a four-pole machine. Fig. 295 is the same machine ready to run.

In the drum armature the iron core, which is made up

of laminated disks, contains a series of grooves, parallel to the shaft, and coils are wound in them at equal angular



Fig. 294.

distances round the circumference.



Fig. 295.

These sections of the armature may all be joined in series, and the junctions between them are then connected with the commutator bars *C*, as in the Gramme ring. If there are only two poles to the field-magnet, there are only two brushes and two circuits through the armature. The ma-

ne of Fig. 294 has four poles P , four armature circuits, and four brushes. A four-pole machine may be wound so as to require only two brushes. When there are four sets of brushes, two of them are positive and two are negative; the two positives are connected in parallel as the positive terminal, and the two negatives as the negative terminal. With such a winding, divided into numerous sections with corresponding commutator bars, the current in the external circuit is almost perfectly continuous and free from all fluctuations.

200. The Electric Motor.—An *electric motor* for direct currents is exactly like a generator. In fact, any direct current generator may be used as a motor. A study of the magnetic field across which a current is flowing aids greatly in understanding both a generator and a motor.

Fig. 296 is the magnetic field (shown by iron filings) distorted by the magnetic influence of a current in a wire looped through the two holes between the two like magnetic poles. These lines of force are under tension, and the loop has therefore a magnetic stress acting

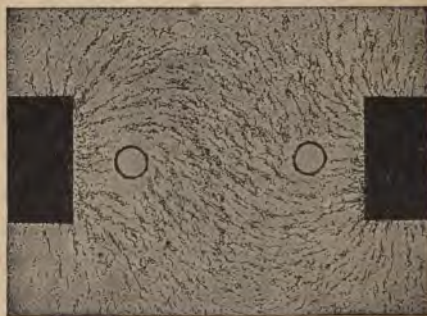


Fig. 296.

on it and tending to turn it counter-clockwise. If the loop is rotated clockwise by mechanical means, it turns against the magnetic torque on it, and work must be done against the resistance of this magnetic drag. When, therefore, the armature of a dynamo is rotated by mechanical

means against the internal magnetic action between the field and the armature, mechanical power is converted into electrical energy, and the machine is a generator. If, on the other hand, the rotation is in the direction of the magnetic effort between the field and the armature, work is done by the machine as a motor. With the current through the field and the armature in the same direction in both cases, the armature rotates in one direction as a generator and in the other direction as a motor; but if the field be reversed when the machine is used as a motor, the armature will turn in the same direction as when acting as a generator without reversing the field. A series-wound machine turns one way as a generator and the other as a motor; a shunt-wound machine turns in the same direction whether operating as a generator or as a motor.

501. Alternators.—If the brushes of a dynamo bear on two continuous rings on the shaft instead of on a commutator, the current in the external circuit will alternate or reverse every time the armature turns through the angular distance from one field pole to the next. A complete series of changes in the current and E.M.F. in both directions takes place while the armature is turning the distance from one pole to the next one of the same name. This is called a *cycle*. The *frequency* is the product of the number of *pairs of poles* and the number of rotations per second. In two-pole machines the frequency is the same as the number of rotations per second. Frequencies are now restricted between the limits of about twenty-five and sixty cycles per second. Multipolar machines are used to avoid excessive speed of rotation.

Figure 297 is a diagram of an alternator with a stationary

ld and a rotating armature. In large machines the nature is generally the stationary member and the field revolves. The field is excited by a direct current machine. The armature coils are reversed in windings from pole to pole; they are then all joined in series, and the terminals are brought out to the slip rings at *B*. The brushes bearing on these slip rings lead to the external circuit.

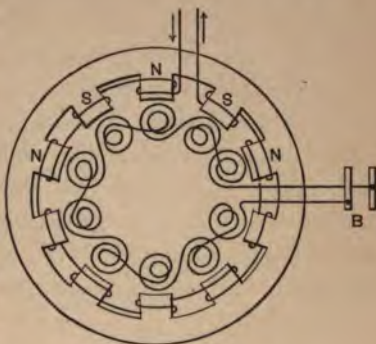


Fig. 297.

502. Transformers.—A *transformer* is an induction coil with a primary of many turns of wire and a secondary of

a smaller number, both wound round a divided iron core forming a closed magnetic circuit; that is, the magnetic flux is entirely through iron (Fig. 298). A transformer is employed with alternating currents either to step



Fig. 298.

down from a high E.M.F. to a low one, or the reverse. The two electromotive forces are directly proportional to the number of turns of wire in the two coils. While the primary and the secondary are wound round the same iron core, they are as perfectly insulated from each other as possible. The iron serves as a path for the flux of mag-

netic induction, and all the lines of force produced by the primary also pass through the secondary, and *vice versa*. When the secondary is open, the transformer acts simply as a "choke coil"; the current in the primary is then only the very small one required to magnetize the iron and to furnish the small amount of energy lost in it, for the counter E.M.F. of self-induction is then nearly equal to the impressed E.M.F. When the secondary is closed, the energy of the current supplied by it lacks only a few per cent of the energy absorbed by the primary; for, while the secondary E.M.F. is less than the primary, the secondary current is greater in the same ratio.

XVII. THE ELECTRIC LIGHT.

✓ **503. Electric Lights** are of two classes, *arc* and *incandescent*. The former is produced by a current of a few



Fig. 299.

amperes passing across from one carbon rod to another, the highly heated vapor between the two rods acting as a conductor. The carbon rods are placed in electrical connection with the two poles of a dynamo, and the current is established by bringing the carbon points together. Upon separation they are heated to an exceedingly high temperature and the current continues to pass across through the heated vapor. The ends of the carbon rods in the open air are disintegrated, a depression or crater forming in the positive (Fig. 299) and a cone on the

gative carbon. Most of the light of the open electric arc comes from the bottom of the crater, which is the hottest part of the carbon. Sir Humphry Davy, who first produced the electric arc light, obtained an arc four inches in length between two charcoal points. For this purpose he used a great battery of 2000 cells. The pores of the charcoal blocks had been previously filled with mercury. Despretz, with 600 Bunsen cells, obtained an arc 7.8 inches long. The arc light may be produced in a vacuum. The intense heat is not due, therefore, to combustion, but to the conversion of the energy of the current into heat in the arc. The electric arc light may be produced even in water, but its brilliancy is much reduced and the water is rapidly decomposed.

504. The Open and the Enclosed Arc. — When the electric arc is maintained between rods of hard retort carbon in the open air, the carbon burns away rather rapidly, the positive about twice as fast as the negative. The E.M.F. required is then from 45 to 55 volts for a 10-ampere current. Modern arc lamps are mostly "enclosed arcs"; that is, the lower carbon and a part of the upper carbon are enclosed in a small glass globe, which is airtight at the bottom, but allows the upper carbon to slip through a check-valve at the top. Soon after the arc begins to burn, the oxygen is absorbed and the arc is then enclosed in an atmosphere of nitrogen and carbon monoxide. The enclosed arc is longer than the open arc, and the E.M.F. required is about 80 volts, while the current for the same consumption of energy is smaller than the open arc requires. The carbons for the enclosed arc last about ten times as long as in the open air.

505. The Arc Lamp. — The wasting away of the carbons necessitates some automatic devices to make them ap-

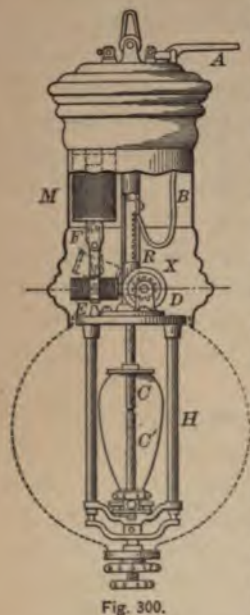


Fig. 300.

proach each other. Most arc lamps contain one electromagnet for separating the carbons when the current is turned on, and a second one to make the upper carbon feed down when the arc requires it. Fig. 300 represents one form of enclosed arc lamp with automatic feed of the upper carbon. *M* is a lifting magnet to start the arc by separating *C* from *C'* in the small inner globe. The magnet *E* has a circular or disk armature *D*, to which is attached a pinion engaging the rack *R*. The upper electromagnet is a solenoid containing an iron core, which is drawn into it by the magnetism induced in the iron. The lower magnet has both a series and a shunt winding, and the two are so connected

that they oppose each other magnetically; that is, they act differentially. The ends of the fine wire winding composing the shunt are attached to the two carbon holders. When the arc increases in length, the potential difference between the carbons increases and a larger current flows through the shunt winding. The magnetism due to the series winding, which is connected in series with the carbons themselves, is then neutralized by the effect of the shunt coil. This differential magnet, which is quick acting, then releases its armature, and the upper carbon drops slightly by its own weight. It does

not reach the lower carbon, because the differential magnet again grips its armature and stops the descent before the arc is extinguished. The upper carbon is thus fed down as needed till the carbon holder touches the check-valve.

506. Incandescent Lamps.—The smaller subdivision of the electric light is made by means of carbon filaments enclosed in exhausted glass bulbs (Fig. 301). These small lamps are usually placed in parallel across from one main conductor to the other (Fig. 302). The carbon filaments have a resistance of from 25 to 800 or 900 ohms when hot, according to the voltage used and the candle power of the lamp. On account of this resistance, the current heats the filament to incandescence. No combustion takes place, but the carbon gradually deteriorates, is thrown off, and blackens the bulb. The lamp then gives less light, and if burned long enough, the carbon filament will break and interrupt the current.



Fig. 301.

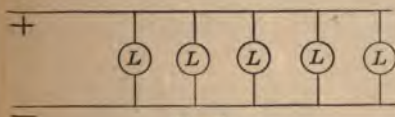


Fig. 302.

The Nernst incandescent lamp employs a refractory earth as the luminescent body. This material is not conducting till it is preheated by a special device for the purpose. It is then traversed by a current which heats it to brilliant incandescence. The Nernst lamp gives light of an intensity intermediate between a carbon filament and an electric arc. Commercial glow lamps take from $2\frac{1}{2}$ to 4 watts for each candle power.

XVIII. THE ELECTRIC TELEGRAPH.

507. The **Electric Telegraph** is a system of transmitting messages by means of simple signals through the agency of an electric current. Its essential parts are the *line*, the *transmitter* or *key*, the *receiver* or *sounder*, and the *battery*.

508. The **Line** is an iron or copper wire, insulated from the earth except at its ends, and serving to connect the signaling apparatus. The ends of this conductor are connected with large metallic plates, or with gas or water pipes, buried in the earth. By this means the earth becomes a part of the electric circuit containing the signaling apparatus.



Fig. 303.

509. The **Transmitter** or **Key** (Fig. 303) is merely a current interrupter, and usually consists of a brass lever *A*, turning about pivots at *B*. It is connected with the line by the screws *C*

and *D*. When the lever is pressed down, a platinum point projecting under the lever is brought in contact with another platinum point *E*, thus closing the circuit. When not in use, the circuit is left closed, the switch *F* being used for that purpose.



Fig. 304.

510. The **Receiver** or **Sounder** (Fig. 304) consists of an electromagnet *A* with pivoted armature *B*. When the circuit is closed through

the terminals *D* and *E*, the armature is attracted to the magnet, producing a sharp click. When the circuit is broken, a spring *C* causes the lever to rise and strike the back stop with a lighter click.

511. The Relay. — When the resistance of the line is large, the current is not likely to be strong enough to operate the sounder with sufficient energy to render the signals distinctly audible. To remedy this defect, an electromagnet,



Fig. 305.

called a *relay* (Fig. 305), whose helix *A* is composed of many turns of fine wire, is placed in the circuit by means of its terminals *C* and *D*. As its armature *H* moves to and fro between the points at *K*, it opens and closes a second and shorter circuit through *E* and *F*, in which the sounder is placed. Thus the weak current, through the agency of the relay, brings into action a current strong enough to do the necessary work.

512. The Battery consists of a large number of cells, usually of the gravity type, connected in series. (Why?) It is generally divided into two sections, one placed at each terminal station, these sections being connected in series through the line. The principal circuits of the great telegraph companies are now worked by means of currents from dynamo machines.

513. The Signals are a series of sharp and light clicks separated by intervals of silence of greater or less duration,

a short interval between the clicks being known as a "dot," and a long one as a "dash." By a combination of "dots" and "dashes" letters are represented and words are spelled out.

514. The Telegraph System described in the preceding sections is known as Morse's, from its inventor. Fig.

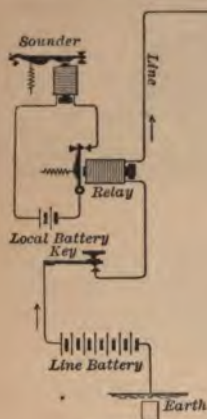


Fig. 306.

306 illustrates diagrammatically the instruments necessary for one terminal station, together with the mode of connection. The arrangement at the other end of the line is an exact duplicate of this one, the two sections of the battery being placed in the line, so that the negative pole of one and the positive of the other are connected with the earth. At intermediate stations the relay and the local circuit are connected with the line in the same manner as at a terminal station.

515. The Electric Bell (Fig. 307) is used for sending signals as distinguished from messages. Besides the gong, it contains an electromagnet, having one terminal connected directly with a binding-post, and the other, through a light spring attached to the armature (shown on the left of the figure) and a contact screw, with another binding post. One end of the armature is supported by a



Fig. 307.

out spring, or on pivots, and the other carries the bent arm and hammer to strike the bell. Included in the circuit are a battery and a push-button *B*, shown in section Fig. 308.

When the springs *S* are brought into contact by pushing *B*, the circuit is closed, the electro-magnet attracts the armature, and the hammer strikes the gong. The movement of the armature opens the circuit by breaking contact between the spring and the point of the screw; the armature is then released, the retractile spring at the bottom carries it back, and contact is again established between the spring and the screw. The whole operation is repeated automatically as long as the circuit is kept closed by the push-button.

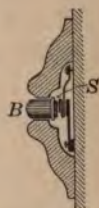


Fig. 308.

XIX. WIRELESS TELEGRAPHY.

516. Electric Waves.—The fundamental discovery which led to telegraphy through space without the use of wires was made by Hertz in 1887–88. Joseph Henry discovered long ago that the discharge of a Leyden jar is oscillatory (§ 426). Later it was found that the spark discharge of an induction coil is likewise oscillatory. Hertz discovered that this oscillatory spark gives rise to electromagnetic waves in the ether, which appear to be the same as waves of light, except that they are very much longer, or of lower frequency. They travel in straight lines, and are reflected and refracted in accordance with the same laws as light.

Evidence of these waves may be readily obtained by setting up an induction coil, with two sheets of tin-foil

on glass, Q and Q' , connected with the secondary terminals, and with two discharge balls, F and F' , as shown in Fig. 309. The receiving apparatus should be tuned so as to have the same period of oscillation as this transmitter; but even without this, so simple a device as a large picture-frame with a conducting gilt border, may be used to detect electric waves in the neighborhood of the induction coil. If the frame has shrunk so as to leave very narrow gaps in the mitre at the corners, then minute sparks may be seen, in a dark room, breaking across these



Fig. 309.

gaps when the induction coil produces vivid sparks between the polished balls, F and F' . The plane of the frame should be held parallel with the sheets of tin-foil. The passage of electromagnetic waves through a conducting circuit produces electric oscillations in it, and these oscillations cause electric surges across a minute air gap. An apparatus tuned to the same frequency as the transmitter or oscillator is called a receiver or resonator.

517. The Coherer.—A very sensitive device for the detection of electromagnetic waves was discovered by Branly. He found that when metallic filings are placed loosely

between solid electrodes in a glass tube they offer a high resistance to the passage of an electric current; but when electric oscillations are produced in the neighborhood of the tube, the resistance of the filings falls to so small a value that a single voltaic cell sends through them a current strong enough to work a relay (§ 511). The tube containing the metallic filings is called a *coherer*. It is shown at *C* in Fig. 310. If the tube is slightly jarred, the filings resume their state of high resistance. A slight discharge from the cover of an electrophorus (§ 413) through

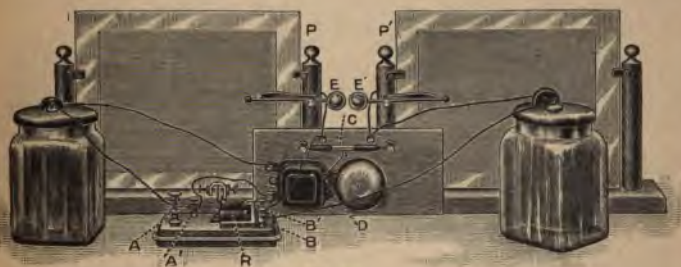


Fig. 310.

the filings lowers the resistance just as electric oscillations do. It is thought that minute sparks between the filings partially weld them together and make them conducting.

518. Receiver for Electromagnetic Waves.—The transmitter of electric oscillations for laboratory experiments is represented by the apparatus of Fig. 309. The receiver is somewhat more complicated, but the different parts and their connection may be easily made out with the help of Fig. 310. The metallic plates, *P* and *P'*, are duplicates of those belonging with the transmitter. The two balls, *E* and *E'*, are connected both with the plates and with the ends of the coherer *C*. A circuit is formed including the

voltaic cell (shown on the right), the coherer, and the fine wire coil R of the relay through the binding posts, B and B' . Another circuit is formed through the contact points of the relay, the voltaic cell at the left, and the vibrating electric bell. The bell is so placed that the ball or hammer strikes the coherer when the bell rings. The transmitter and receiver may then be placed at some distance apart, with the metallic plates parallel to each other. When the key K of the transmitter is firmly pressed for a moment, a spark passes across the gap between the balls; at the same instant the coherer allows a current to pass through, and this current operates the relay and closes the circuit through the electric bell. The blow of the bell hammer on the coherer restores it to its sensitive condition of high resistance, ready for the reception of another short succession of electric waves.

✓ **519. Marconi's System.** — Marconi, who has succeeded in sending messages without wires over very long distances, employs a powerful induction coil in connection with his transmitter, and a coherer as the sensitive detector of electric waves. One terminal of the induction coil is connected with the earth, and the other with a wire running up a tall mast and ending in a ball or a plate of metal. The receiver differs from Fig. 310 in the same way as the transmitter differs from Fig. 309. Another tall mast supports a wire ending in a ball or a plate. The lower end of the wire is connected with one end of the coherer; the other end of the coherer goes to earth. Instead of an electric bell is a sounder (§ 510). Auxiliary devices are used which cannot be described here. Ships fitted with Marconi's system communicate successfully when one hundred fifty miles or more apart at sea.

XX. THE TELEPHONE AND THE MICROPHONE.

20. The Telephone (Fig. 311) consists of a permanent magnet *O*, one end of which is surrounded by a coil of many turns of fine copper wire *b*, whose ends are connected with the binding-posts *t*. At right angles to the magnet, and not quite touching the pole within the coil, is an elastic diaphragm or disk *a* of sheet-iron, kept in place by the conical support-piece *d*. If the instrument is placed in an electric circuit when the current is steady, or alternating in direction, the magnetic field due to the helix, when combined with that due to the magnet, alters intermittently the number of lines of force which branch out from the pole, thus varying the attraction of the magnet for the disk. The result is that the disk vibrates in exact keeping with the changes in the current.

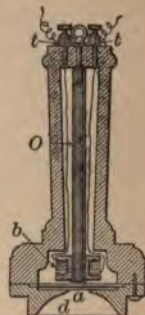


Fig. 311.

21. The Microphone is a device for varying an electric current by means of a variable resistance in the circuit.



Fig. 312.

One of its simplest forms is shown in Fig. 312. It consists of a rod of gas-carbon *A*, whose tapering ends rest loosely in conical depressions made in blocks *C*, *C*, of the same material attached to a sounding-board. These blocks are placed in circuit with a battery and a telephone,

means of the wires *X* and *Y*. While the current is

passing, the least motion of the sounding-board, caused either by sound waves or by any other means, moves the loose carbon pencil and varies the pressure between its ends and the supporting bars. A slight increase of pressure between two conductors resting loosely one on the other lessens the resistance of the contact, and conversely. Hence the vibrations of the sounding-board cause variations in the pressure at the points of contact of the carbons, and consequently make corresponding fluctuations in the current and vibrations of the telephone disk.

522. The Solid Back Transmitter. — The varying resistance of carbon under varying pressure makes it a

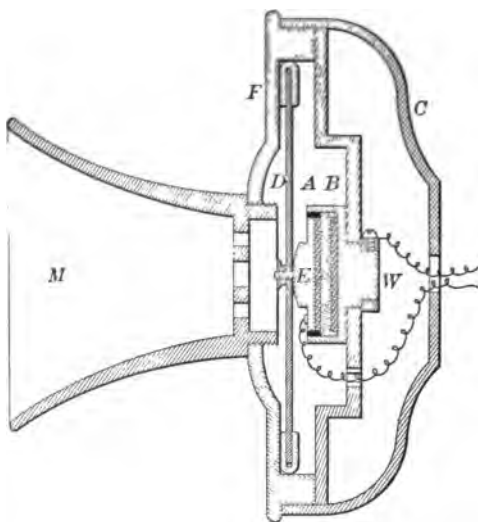


Fig. 313.

valuable material for use in telephone transmitters. Instead of the loose contact of the microphone, carbon in granules between carbon plates is now commonly employed.

The form of transmitter is extensively used for long

distance work is the "solid back" transmitter (Fig. 313). The figure shows only the essential parts in section, minor details being omitted. *M* is the mouthpiece, and *F* and

the front and back parts of the metal case. The aluminum diaphragm *D* is held around its edge by a soft rubber ring. The metal block *W* has a recess in front to receive the carbon electrodes *A* and *B*. Between them are the carbon granules. The block *E* is attached to the diaphragm and is insulated from *W* except through the carbon granules. The transmitter is placed in circuit by the wires connected to *W* and *E*.

Provision is made for an elastic motion of the diaphragm and the block *E*. Sound waves striking the diaphragm cause a varying pressure between the plates and the carbon granules. This varying pressure varies the resistance offered by the granules and so varies the current. The transmitter is in circuit in the line with the primary of a small induction coil, the secondary being in a local circuit with the telephone receiver. The induced currents in the secondary have all the peculiarities of the primary current; and when they pass through a receiver, it responds and reproduces sound waves similar to those which disturb the disk of the transmitter.

523. Common Battery Multiple Switch-Board. — Figure 314 shows the essentials of a common battery or central energy switch-board, together with two subscribers' stations. Assume station 1 at the left as that of the calling subscriber. When not in use the receiver *R* hangs on the hook, which serves as a switch, and the circuit from the central office is open on account of the condenser *C*; but when the receiver is lifted from the hook, the circuit of the line is closed through the talking apparatus *T*. In practice the receiver is in the secondary circuit of a small induction coil (§ 522), and this circuit is connected as a shunt to the condenser *C*.

At the central office the two sides of the subscribers' line pass to the tip and ring contact springs of each "multiple jack," only two of which are shown at J, J' . The talking circuit is drawn in heavy lines.

Suppose now the calling subscriber at station 1 removes his receiver from the hook. The hook rises and closes

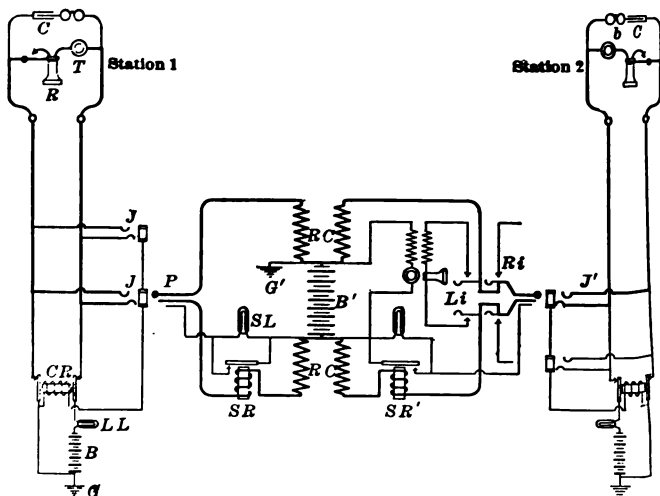


Fig. 314.

the line circuit through the armatures of the cut-off relay CR , the battery B and the line signal lamp LL . The lighting of this miniature lamp notifies the operator at the central office that the subscriber at station 1 is signaling. The operator accordingly inserts the plug P in the jack J , thus completing the talking circuit of the calling subscriber through the repeating coil RC and the battery B' at the central office. At the same time the ring on the jack at J completes another circuit (shown in light lines) from the ground G' through the battery B

relay *CR*, and thence to the ground *G*. When the key draws up its armatures, the line circuit through the calling lamp *LL* is opened and the lamp is extinguished. The central operator then closes the listening key at *Li*. Current flows through the repeating coils *RC* for the number required. Having obtained it, she inserts the plug in the jack *J'* of the called subscriber and makes contact with the ringing circuit *Ri*. The alternating current used rings the bell *b* at station 2 through the condenser *C*. The two talking circuits are then complete as shown by the heavy lines, the plugs with the three-wire cords being the corresponding jacks.

As soon as the calling subscriber hangs up the receiver, he opens the line. Current ceases to flow through the key *SR*. The object of this relay is to close a shunt around the supervisory lamp *SL* to prevent its lighting. As soon as the subscriber opens the line, relay *SR* raises its armature, the shunt around the lamp is opened, and the lamp *SL* lights. This is the operator's signal to withdraw the plug from the jack. The withdrawal of the plug opens the circuit through the supervisory lamp. The operation on the side of the called subscriber is the same.

APPENDIX.

I. GEOMETRICAL CONSTRUCTIONS.

The principal instruments required for the accurate construction of diagrams on paper are the *compasses* and the *ruler*. For the construction of angles of any definite size the *protractor* (Fig. 315) can be used.

There are, however, a number of angles, as 90° , 60° , and those which can be obtained from these by bisecting them and combining their

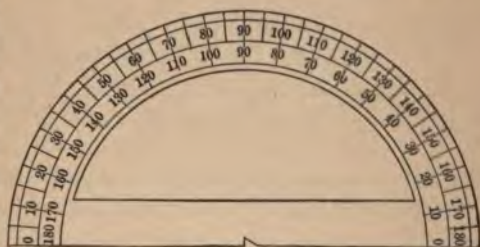


Fig. 315.

parts, that can be constructed by the compasses and ruler alone.

A convenient instrument for the rapid construction of the angles, 90° , 60° , and 30° , is a triangle made of wood, horn, hard rubber, or cardboard, whose angles are these respectively. Such a triangle may be easily made from a postal card as follows: Lay off on the short side of the card (Fig. 316) a distance a little less



Fig. 316.

than the width, as AB . Separate the points of the compasses a distance equal to twice this distance. Place one point of the compasses at B , and draw an arc cutting the adjacent side at C .

Cut the card into two parts along the straight line BC . The part ABC will be a right-angled triangle, having the longest side twice as long as the shortest side, with the larger acute

angle 60° and the smaller 30° . With this triangle and a straight edge the majority of the constructions required in elementary physics can be made.

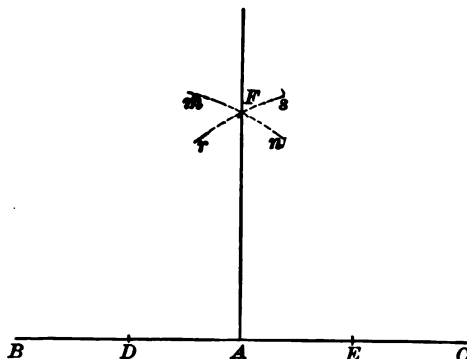


Fig. 317.

PROB. 1. — To construct an angle of 90° .

Let A be the vertex of the required angle (Fig. 317).

Through A draw the straight line BC . Measure off AD , a convenient distance; also make $AE = AD$. With a pair of compasses, using D as a centre, and a radius longer than AD draw the arc mn ; with E as a centre and the same radius, draw the arc rs , intersecting mn at F . Join A and F . The angles at A are right angles.

PROB. 2. — To construct an angle of 60° .

Let A be the vertex of the required angle (Fig. 318), and AB one of the sides. On AB take some convenient distance as AC . With a pair of compasses, using A as a centre and AC as a radius draw the arc CD . With C as a centre and the same radius, draw the arc mn , intersecting CD at E . Through A and E draw the straight line AE ; this line will make an angle of 60° with AB .

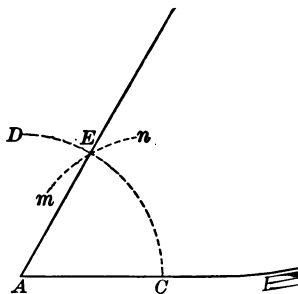


Fig. 318.

PROB. 3. — *To bisect an angle.*

Let BAC be an angle that it is required to bisect (Fig. 319). Measure off on the sides of the angle equal distances, AD and AE . With D and E as centres and with the same radius, draw the arcs mn and rs , intersecting at F . Draw AF . This line will bisect the angle BAC .

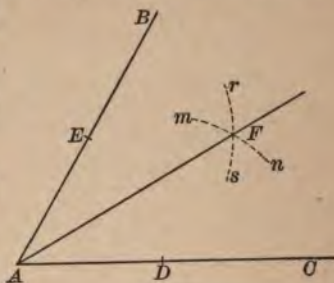


Fig. 319.

PROB. 4. — *To make an angle equal to a given angle.*

Let BAC be a given angle; it is required to make a second angle equal to it (Fig. 320).

Draw DE , one side of the required angle. With A as a centre and any convenient radius, draw the arc mn across the given angle. With D as a centre and the same radius, draw the arc rs . With s as a centre and a radius equal to the chord of

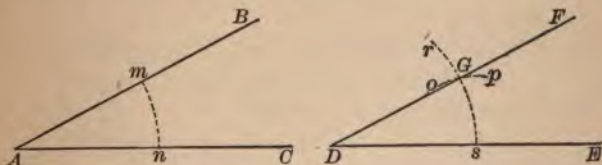


Fig. 320.

2, draw the arc op , cutting rs at G . Through D and G draw the line DF . This line will form with DE the required angle, as FDE .

PROB. 5. — *To draw a line through a point parallel to a given line.*

Let A be the point through which it is required to draw a line parallel to BC (Fig. 321). Through A draw ED ,

cutting BC at D . At A make the angle EAG equal to EDC . Then AG or FG is parallel to BC .

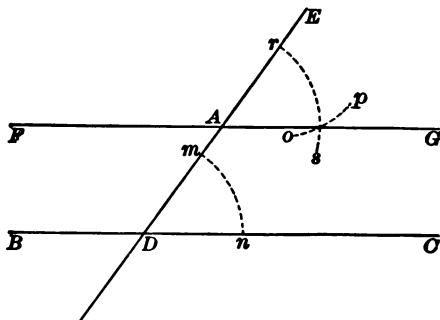


Fig. 321.

PROB. 6. — *Given two adjacent sides of a parallelogram to complete the figure.*

Let AB and AC be two adjacent sides of the parallelogram (Fig. 322). With C as a centre and a radius equal to AB ,

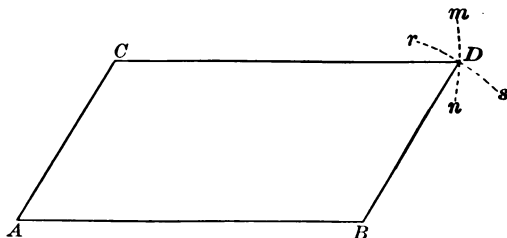
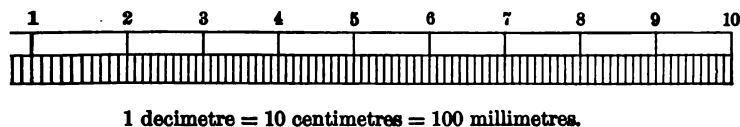
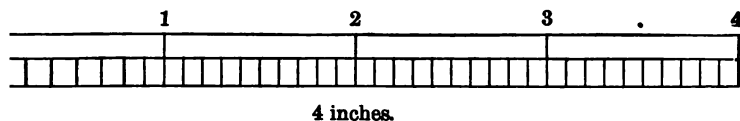


Fig. 322.

draw the arc mn . With B as a centre and a radius equal to AC , draw the arc rs , cutting mn at D . Draw CD and BD . Then $ABDC$ is the required parallelogram.



1 sq. decim. = 15.5 sq. in.
1 sq. in. = 6.45 sq. cm.

A cube of water at 4° C., one of whose faces is this square, has a mass of one kilogramme. The volume is a litre. A cube of water at 4° C., with each of its faces one sq. cm., has a mass of one gramme. A cubic inch of water at 4° C. has a mass of 0.03611 lb. or 0.58 oz.

1 sq.in.

1 sq.cm.

III. CONVERSION TABLES.

1. LENGTH.

To reduce	Multiply by	To reduce	Multiply by
Miles to km.	1.60935	Kilometres to mi. . . .	0.62137
Miles to m.	1609.347	Metres to mi.	0.0006214
Yards to m.	0.91440	Metres to yds.	1.09361
Feet to m.	0.30480	Metres to ft.	3.28083
Inches to cm.	2.54000	Centimetres to in. . . .	0.39370
Inches to mm.	25.40005	Millimetres to in. . . .	0.03937

2. SURFACE.

To reduce	Multiply by	To reduce	Multiply by
Sq. yards to m. ²	0.83613	Sq. metres to sq. yds. . .	1.19599
Sq. feet to m. ²	0.09290	Sq. metres to sq. ft. . . .	10.76387
Sq. inches to cm. ²	6.45163	Sq. centimetres to sq. in. .	0.15500
Sq. inches to mm. ²	645.163	Sq. millimetres to sq. in. .	0.00155

3. VOLUME.

To reduce	Multiply by	To reduce	Multiply by
Cu. yards to m. ³	0.76456	Cu. metres to cu. yds. . .	1.30802
Cu. feet to m. ³	0.02832	Cu. metres to cu. ft. . . .	35.31661
Cu. inches to cm. ³	16.38716	Cu. centimetres to cu. in. .	0.06102
Cu. feet to litres	28.31701	Litres to cu. ft.	0.03532
Cu. inches to litres	0.01639	Litres to cu. in.	61.02387
Gallons to litres	3.78543	Litres to gallons	0.26417
Pounds of water to litres .	0.45359	Litres of water to lbs. . . .	2.20462

4. WEIGHT.

To reduce	Multiply by	To reduce	Multiply by
Tons to kgm.	907.18486	Kilogrammes to tons . . .	0.001102
Pounds to kgm.	0.45359	Kilogrammes to lbs.	2.20462
Ounces to gm.	28.34953	Grammes to oz.	0.03527
Grains to gm.	0.064799	Grammes to grains	15.43236

5. FORCE, WORK, ACTIVITY, PRESSURE.

	Multiply by	To reduce	Multiply by
ht to dynes,	444520.58	Dynes to lbs.-weight,	22496×10^{-10}
kgm.-m. . . .	0.138255	Kgm.-m. to ft.-lbs. . . .	7.233
ergs	13549×10^8	Ergs to ft.-lbs. . . .	0.7381×10^{-7}
joules	1.3549	Joules to ft.-lbs. . . .	0.7381
r sec. to H.P.	18182×10^{-7}	H.P. to ft.-lbs. per sec. .	550
atts	745.196	Watts to H.P.	0.001342
sq. ft. to kgm.		Kgm. per m. ² to lbs. per	
sq. ft.	4.8824	sq. ft.	0.2048
sq. in. to gm.		Gm. per cm. ² to lbs. per	
sq. in.	70.3068	sq. in.	0.01422

calculated for $g = 980$ cm., or 32.15 ft. per sec. per sec.

6. MISCELLANEOUS.

	Multiply by	To reduce	Multiply by
ater to U.S. gal.	0.11983	U.S. gal. to lbs. of water,	8.345
U.S. gal. . . .	7.48052	U.S. gal. to cu. ft. . . .	0.13368
ater to cu. ft. at		Cu. ft. of water at 4° C.	
cu. ft.	0.01602	to lbs.	62.425
U.S. gal. . . .	0.004329	U.S. gal. to cu. in. . . .	231
res to lbs. per		Lbs. per sq. in. to atmos-	
res	14.69640	pheres	0.06737
res to gm. per		Gm. per cm. ² to atmos-	
res	1033.296	pheres	0.000968
es F. to calories,	252	Calories to lb.-degrees F.	0.003968
joules	4.18936	Joules to calories	0.2387
hour to ft. per		Ft. per sec. to miles per	
hour	1.46667	hour	0.68182
hour to cm. per		Cm. per sec. to miles per	
hour	44.704	hour	0.02237

IV. MENSURATION RULES.

Area of triangle	$= \frac{1}{2} (\text{base} \times \text{altitude}).$
Area of triangle	$= \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2} (a+b+c)$
Area of parallelogram	$= \text{base} \times \text{altitude}.$
Area of trapezoid	$= \text{altitude} \times \frac{1}{2} \text{sum of parallel sides}.$
Circumference of circle	$= \text{diameter} \times 3.1416.$
Diameter of circle	$= \begin{cases} \text{circumference} \div 3.1416. \\ \text{circumference} \times 0.3183. \end{cases}$
Area of circle	$= \begin{cases} \text{diameter squared} \times 0.7854. \\ \text{radius squared} \times 3.1416. \end{cases}$
Area of ellipse	$= \text{product of diameters} \times 0.7854.$
Area of regular polygon	$= \frac{1}{2} (\text{sum of sides} \times \text{apothem}).$
Lateral surface of cylinder	$= \text{circumference of base} \times \text{altitude}.$
Volume of cylinder	$= \text{area of base} \times \text{altitude}.$
Surface of sphere	$= \begin{cases} \text{diameter} \times \text{circumference}. \\ 4 \times 3.1416 \times \text{square of radius}. \end{cases}$
Volume of sphere	$= \begin{cases} \text{diameter cubed} \times 0.5236. \\ \frac{4}{3} \text{ of radius cubed} \times 3.1416. \end{cases}$
Surface of pyramid }	$= \frac{1}{2} (\text{circumference of base} \times \text{slant height}).$
Surface of cone }	
Volume of cone	$= \frac{1}{3} (\text{area of base} \times \text{altitude}).$

V. TABLE OF DENSITIES.

The following table gives the mass in grammes of 1 cm.³ of the substance:—

Water, at 0° C. and 76 cm. pressure	2.615	Human body	0.890
Alcohol, ethyl, 90%, 20° C.	0.818	Hydrogen, at 0° C. and 76 cm. pressure	0.0000896
Alcohol, methyl	0.814	Ice	0.917
Alumina, common	1.724	Iceland spar	2.723
Alumina, wrought	2.670	India-rubber	0.930
Antimony, cast	6.720	Iron, white cast	7.655
Asphaltum	0.964	Iron, wrought	7.698
Bitumen, cast	9.822	Ivory	1.820
Bitumen, cast	8.400	Lead, cast	11.360
Bitumen, hard drawn	8.700	Magnesium	1.750
Carbon, gas	1.89	Marble	2.720
Carbon disulphide	1.293	Mercury, at 0° C.	13.596
Coal	1.6	Mercury, at 20° C.	13.558
Anthracite	1.26 to 1.800	Milk	1.032
Bituminous	1.27 to 1.423	Nitrogen, at 0° C. and 76 cm. pressure	0.001255
Brass, cast	8.830	Oil, olive	0.915
Brass, sheet	8.878	Oxygen, at 0° C. and 76 cm. pressure	0.00143
Brass	0.14 to 0.24	Paraffin	0.824 to 0.940
Brass	3.530	Platinum	21.581
Brass	1.187	Potassium	0.865
Brass	3.900	Silver, wrought	10.56
Brass	0.736	Sodium	0.970
Brass	7.580	Steel	7.816
Brass-silver	8.432	Sulphuric Acid	1.84
Brass-crown	2.520	Sulphur	2.033
Brass-flint	3.0 to 3.600	Sugar, cane	1.593
Brass-plate	2.760	Tin, cast	7.290
Brass-rine	1.260	Water, at 0° C.	0.999
Brass	19.360	Water, at 20° C.	0.998
Brass	2.650	Water, sea	1.027
Brass	2.500	Zinc, cast	7.000
Brass, crys.	2.310		

VI. TABLE OF NATURAL SINES AND TANGENTS.

Angle.	Sine.	Tangent.	Angle.	Sine.	Tangent.	Angle.	Sine.	Tangent.
0	0.000	0.000	31	0.515	0.601	62	0.883	1.881
1	0.017	0.017	32	0.530	0.625	63	0.891	1.963
2	0.035	0.035	33	0.545	0.649	64	0.899	2.060
3	0.052	0.052	34	0.559	0.675	65	0.906	2.145
4	0.070	0.070	35	0.574	0.700	66	0.914	2.246
5	0.087	0.087	36	0.588	0.727	67	0.921	2.356
6	0.105	0.105	37	0.602	0.754	68	0.927	2.475
7	0.122	0.123	38	0.616	0.781	69	0.934	2.606
8	0.139	0.141	39	0.629	0.810	70	0.940	2.747
9	0.156	0.158	40	0.643	0.839	71	0.946	2.904
10	0.174	0.176	41	0.656	0.869	72	0.951	3.078
11	0.191	0.194	42	0.669	0.900	73	0.956	3.271
12	0.208	0.213	43	0.682	0.933	74	0.961	3.487
13	0.225	0.231	44	0.695	0.966	75	0.966	3.732
14	0.242	0.249	45	0.707	1.000	76	0.970	4.011
15	0.259	0.268	46	0.719	1.036	77	0.974	4.331
16	0.276	0.287	47	0.731	1.072	78	0.978	4.701
17	0.292	0.306	48	0.743	1.111	79	0.982	5.141
18	0.309	0.325	49	0.755	1.150	80	0.985	5.67
19	0.326	0.344	50	0.766	1.192	81	0.988	6.31
20	0.342	0.364	51	0.777	1.235	82	0.990	7.111
21	0.358	0.384	52	0.788	1.280	83	0.993	8.144
22	0.375	0.404	53	0.799	1.327	84	0.995	9.514
23	0.391	0.424	54	0.809	1.376	85	0.996	11.43
24	0.407	0.445	55	0.819	1.428	86	0.998	14.30
25	0.423	0.466	56	0.829	1.483	87	0.999	19.08
26	0.438	0.488	57	0.839	1.540	88	0.999	28.64
27	0.454	0.510	58	0.848	1.600	89	1.000	57.29
28	0.469	0.532	59	0.857	1.664	90	1.000	Infinity.
29	0.485	0.554	60	0.866	1.732			
30	0.500	0.577	61	0.875	1.804			

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